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Optimization methodology to target a vibration source natural frequency of energy harvesting cantilever

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Abstract— Targeting the frequency of a vibrating source efficiently is the main aim of this research. Also, using an intelligent optimization methodology to improve the power output per unit volume of the harvester is considered. Two case studies of engine vibration and car suspension energy harvesting were investigated. The two optimization methodologies were the genetic algorithm (GA) and COMSOL optimization module (BOBYQA solver). Then, a multi-physics COMSOL was utilized to simulate the results of the COMSOL optimization module and MATLAB was applied to simulate the results of the GA optimization methodology. The analytical simulation of GA and literature results were used to verify the COMSOL results. The power per unit volume of the first case study using COMSOL (FEM) was 67.25×10^{-3} mW/mm^3 versus 60.5×10^{-3} mW/mm^3 for analytical (GA). While, the power per volume of the second case study, using COMSOL (FEM) was 93.8×10^{-3} mW/mm^3, versus 81.5 mW/mm^3 for analytical (GA). From results we can prove that the COMSOL optimization module introduced about 12% improvement in the output power. Also, the studies of the stresses due to dynamic load were performed to avoid the failure.

Keywords—Optimization, piezoelectric, cantilever, energy harvesting .

I. Introduction

Due to the importance of energy harvesting, a lot of researches were published in this field. The harvested energy from piezoelectric harvesters can be utilized to power the small wireless sensors, actuators and small batteries. A lot of efforts, time and money due to the batteries replacing can be saved using the energy harvesting devices. Targeting the natural frequency of a vibrating source and maximizing the output power of the energy harvesting device are the goals of this research. During this work, genetic algorithm (GA) and COMSOL optimization module were utilized to maximize the output power. The COMSOL Multi-physics and MATLAB were used to simulate the optimized results. The COMSOL is a commercial software widely used in simulation with preset material properties that can be linked with other simulation tools like MATLAB.

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In 1995 Williams and Yates [1] introduced the idea of harvesting the energy using piezoelectric material. The importance of adding a resistance to the harvesting circuit to evaluate the efficiency of the harvester in both analytical and experimental studies was suggested [2-4]. The output voltage and power as functions of the base force was studied by [2 and 3]. Also, the importance of optimizing the harvester parameters was revealed [5, 6]. Hong et al.[7] showed the effect of the neutral axis position on the stresses and power generation of piezoelectric devices. Song et al. [8] optimized the harvester parameters to control the stresses and to maximize the output power.

The mutual weakness in previous studies is the concentrating on only one parameter; either mechanical or electrical. In this research we considered the electrical and mechanical parameters and made a stresses analysis during the optimization methodology targeting the vibrating source natural frequency.

II. The optimized Car engine compartment model

The main challenge in the design of the piezoelectric cantilever is to maximize the power output. Design and manufacturing the harvester require several prototypes and extensive experimentation (design process). This design approach can be expensive. So, the application of more non-traditional optimization techniques could be very valuable. In this paper two optimization methodologies were utilized and compared. The first, is the genetic algorithm (GA) and the second is the COMSOL built in optimization module. In this way, a collection of variables is defined as optimized parameters. By using the defined objective function, constraints, and one set of suitable parameter values, we can effectively execute the optimization process [9]. As the voltage and power output significantly depend on the dimensions of the harvester, there is a need to define the optimum dimensions of the piezoelectric energy harvester.

In this section, the GA effect on geometry optimization of the proposed energy harvester cantilever will be illustrated. Using the GA to maximize the power and to target the frequency of a car engine compartment is the aim of this section. The frequency of the car engine compartment is 200 Hz as revealed in table (I). From Table I we can realize that most vibration sources show a natural frequency lower than 200 Hz, and acceleration value of a dozen m/s^2. In the first step, the natural frequency of the proposed piezoelectric harvester was constrained to be 200 Hz. That natural frequency was calculated using MATLAB from (1) [17]. The output power was utilized as an objective function. We optimized the geometric variable of the cantilever (L-length; w-width; t_p-piezoelectric thickness of one layer; t_s-base plate thickness) in order to increase the output power. The power output can be calculated from (2) [10] . The piezoelectric material was PZT-5H with modulus of elasticity E_p=71.4 GPa and density=7650 kg/m^3 and the base material was structural steel with modulus of elasticity E_s=200 GPa and density=7800 kg/m^3. The damping ratio was selected to be 0.08.

TABLE I. NATURAL FREQUENCY AND ACCELERATION OF VARIOUS DIFFERENT SOURCES

Vibration source	***Frequency (Hz***	***Acceleration (m/s2)***
Car engine	200	12
Vehicles	5-2000	0.5~110

The natural frequency:

$$f_n = \frac{H\lambda^2}{4\pi L^2}\sqrt{\frac{E_p\,[1+3(1+2\alpha)^2+4\alpha^2]}{3\rho_p[4(1+\alpha)^2(\gamma+1)]}} \quad (1)$$

Where:
f_n: The natural frequency of the piezoelectric cantilever beam in Hz,
λ: The frequency coefficient,
H: the total thickness (of the two piezoelectric layers and steel layer) in mm,
L: The beam length in mm,
E_p: The elastic modulus of base material (steel) in MPa,
α: The thickness ratio of a single piezoelectric plate and the total thickness,
β: Ratio of the piezoelectric plate modulus of elasticity and that of the base board,
ρ_p: The density of the base material in kg/m^3, and
γ: The ratio of the piezoelectric plate density and the base layer density.

The objective function:

$$P = \frac{1}{2w_n^2}\frac{Rc_b^2\left(\frac{2E_p d_{31} t_p}{k_2 a\varepsilon}\right)A_{in}^2}{(4\zeta^2+k_{31})(Rc_b w_n^2)+4\zeta k_{31}(Rc_b w_n)+4\zeta^2} \quad (2)$$

Where:
A_{in} : The amplitude of the input acceleration (12m/s^2);
C_b: the Capacitance of the piezoelectric;
d_{31}: the Piezoelectric strain coefficient (-265*10^{-12} m/V)
K_2: tip deflection of the cantilever,
K_{31}: Coupling coefficient (0.36);
R: Resistive load is assigned as R_{opt} optimal resistance value);
ζ: damping coefficient 0.001,a: constant(=1series,=2 parallel) and ε dielectric Constant (3.36*10^{-8});

$$c_b = \frac{2\varepsilon w_s L}{t_p};\ R_{opt} = \frac{1}{c_b w_n}\frac{2\zeta}{\sqrt{4\zeta^2+k_{31}^4}};\ k_2 = \frac{4L^2}{3(t_p+t_s)} \quad (3)$$

The boundary condition is the natural frequency equal to 200 Hz.

The optimization study mentioned above, with GA, was programed using MATLAB. Table II shows the lower and upper bounds, and the results of the optimized harvester parameters. The total volume of the optimized harvester, using GA, was 332 mm^3.

The COMSOL optimization module was utilized as the second optimization methodology. The objective function, the geometric parameters, the constraints were the same in both optimization methodologies. In COMSOL optimization module, the utilized solver was the BOBYQA. The BOBYQA is an iterative algorithm for minimizing a function, in which each iteration employs a quadratic approximation. Powell [11] proved that BOBYQA had a lot of advantages to be used as an optimization algorithm. The lower and upper bounds, and the results of the COMSOL optimization module are demonstrated in table II. The total volume of the optimized harvester using COMSOL was 362.9 mm^3.

TABLE II. THE LOWER BOUND, UPPER BOUND AND THE RESULTS OF BOTH COMSOL AND GA AT 200 HZ

Harvester parameters	Lower Bound (mm)	Upper Bound (mm)	Results COMSOL (mm)	Results Analytical (mm)
Length(L)	30	90	54.373	54.4
width(w)	5	20	5.4789	5
Piezo thickness (t_p)	0.15	0.6	0.3936	0.39
Steel thickness (t_s)	0.2	0.8	0.431	0.43

A. *Optimized model simulation*

The simulation results of the optimized harvester using the COMSOL will be presented in this section. The design was built with acceleration of 12 m/s^2 as body force. Before the simulation using the COMSOL, some steps were performed to establish the model. The body load (base excitation) and fixed constraints boundary were chosen in solid mechanic. Also, in electro-statistics field, ground layers were selected to top and bottom part of piezoelectric bimorph. The terminals were applied to top and bottom layers of the base material. Fig. 1 reveals the meshing of the optimized model using physics-controlled mesh. As first step, to determine the resonance frequencies of the cantilever, Eigen frequency study was investigated. Fig. 2 shows that the resonance frequency of the first mode was 200.02 Hz and the tip deflection of the cantilever was 0.35mm. Fig.3 demonstrates the simulation of the optimized harvester where, the frequency response, resistance dependence and the acceleration dependence of the output power and the volt are shown. The frequency response was used to determine the maximum power and volt. The frequency response is the relation between the vibration source frequency and the output power. Fig. 3(a) shows the frequency response of the power and the volt. The results shown in Fig. 3(a), reveal that the maximum power was 24.5 mW at frequency 200Hz (resonance). While, the volt was 3.5 V at the same the frequency. The resistance dependence was utilized to determine the optimal resistance that maximizes the output power. Fig. 3(b) shows that the optimal resistance was 251.2 Ω. To indicate the effect of the acceleration on the power and volt, the acceleration dependence was investigated. Fig. 3(c) shows the acceleration dependence of the power and the volt, where the acceleration simulation proves that the increase the acceleration increases the excitation force, the stress and the power output. The resistance dependence and acceleration dependence were simulated at the resonance frequency (200 Hz). Fig. 4 shows the electric potential distribution at the resonance. Fig.5 shows the stress distribution of the harvester using Von Mises theory. The stress level was maximum at the fixed end. From the simulation results, we can deduce that the power output is proportional to the stress and strain. Fig. 5 demonstrates that a maximum stress, around 9.5 MPa, resulted from both stresses of base excitation and inertia force. The maximum stress was less than the strength of both steel and piezoelectric, where the strength of piezoelectric is 20 MPa and steel is 355 MPa.

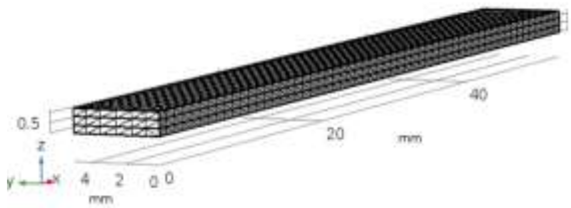


Figure1. Meshed design of the optimized harvester.

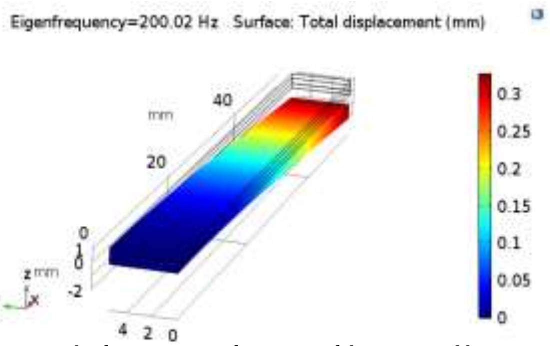
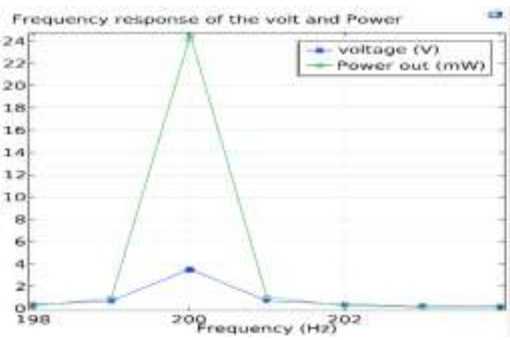
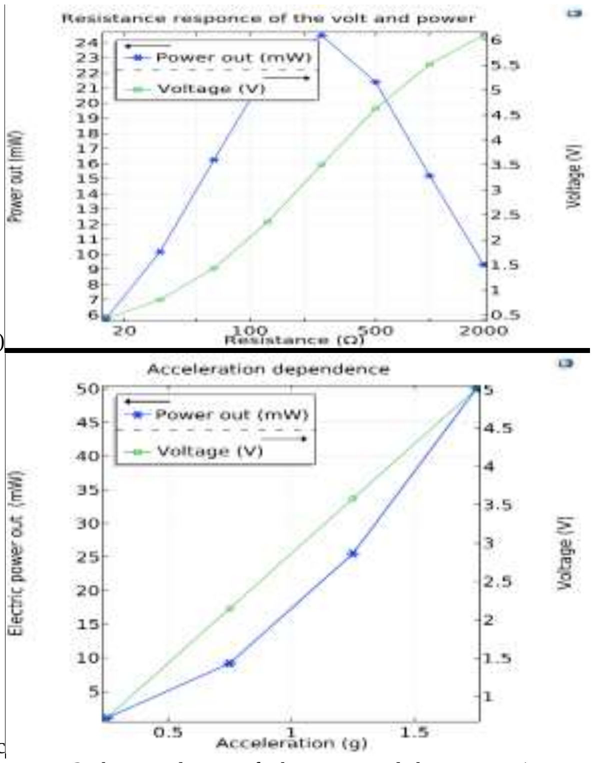


Figure2. The first resonance frequency of the optimized harvester.



a)

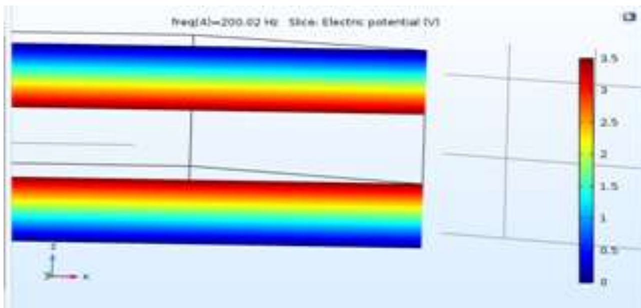
b)



c

Figure 3.The simulation of the optimized harvester. a) Frequency response of the power and volt b) the resistance dependence c) the acceleration dependence.

a)



b)

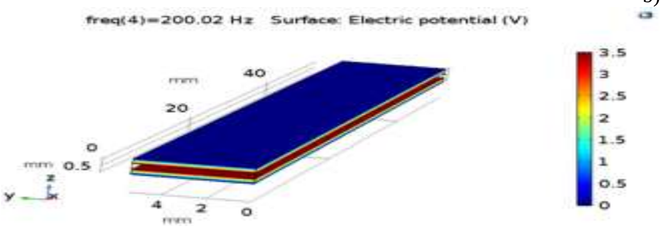


Fig. 4 The electric Potential distribution at resonance frequency a) potential slice b) surface potential.

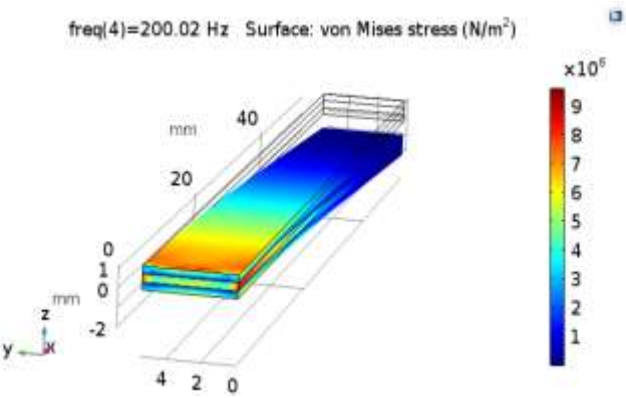


Figure 5. The stress distribution of the cantilever using Von Mises theory.

TABLE III. THE ENERGY HARVESTERS OUTPUT POWER OF LITERATURE COMPARED WITH THIS WORK

Author	year	Piezoelectric material	Frequency (HZ)	Base Excitation (g) or force	Mmax. Power(mw)	Volume (mm^3)	Power per volume(mW/mm^3)
Aktakka [13]	2011	PZT	154	1.5	205×10^{-3}	26.95	7.6×10^{-3}
Jauphuang [14]	2014	PZT	100	1	82.4×10^{-3}	47.82	1.723×10^{-3}
Liu [15]	2014	PVDF	20	0.2	8.1×10^{-3}	10	0.81×10^{-3}
Kim [12]	2005	PZT	100	70 N	52	1500	34×10^{-3}
Roundy [16]	2003	PZT	100	0.23	60×10^{-3}	1000	0.06×10^{-3}
Zhao [17]	2018	PZT	8.83	0.34 N	1.97	6000	0.328×10^{-3}
COMSOL	recent	PZT	200.02	1.2	24.5	362.9	67.5×10^{-3}
Analytical	recent	PZT	200	1.2	20.1	332.3	60.5×10^{-3}

B. *Verification of the results using the analytical simulation*

The analytical results were utilized to verify the COMSOL results. Also, the COMSOL results were compared with the results collected from literature that are listed in table III. The COMSOL was utilized to simulate the results of the optimization COMSOL module, while the MATLAB was utilized to simulate the GA results. Fig. 6 demonstrates the agreement between the COMSOL results and the analytical results. The power output of the COMSOL was 24.5 mW versus 20.1 mW for analytical solution. Due to the difference between of the two volumes resulted from the two optimization methodologies, the comparison should be based on the power per unit volume. The power per volume using COMSOL (FEM) was 67.25×10^{-3} mW/mm^3 versus 60.5×10^{-3} mW/mm^3 for analytical (GA). From the results we can prove that the COMSOL optimization module introduced about 12% improvement in the output power.

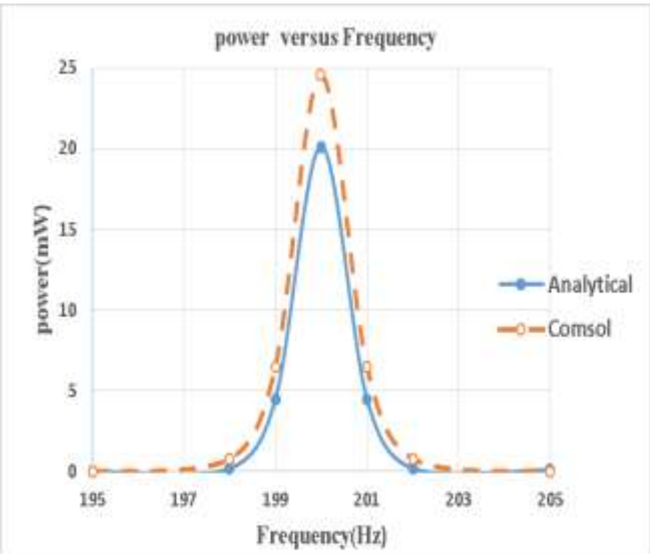


Figure 6. Power output of COMSOL simulation versus the analytical results of the optimized harvester.

III. Piezoelectric cantilever for a vehicle suspension vibration

The aim of this section is to design and optimize an energy harvester to target the natural frequency of a vehicle suspension. The harvested power can be utilized in powering the wireless sensors and engine monitoring devices. One of the advantages of the energy harvesting, using piezoelectric material, is saving time, effort and money of replacing the batteries. The energy harvesting in the car could cover a wide range of natural frequencies (In this section, the vibration covered range from 0 to 150 Hz has a major interest as shown in Fig. 7)[18]. The great interest in this section is to harvest the vibration energy of a vehicle suspension at frequency 150 Hz.

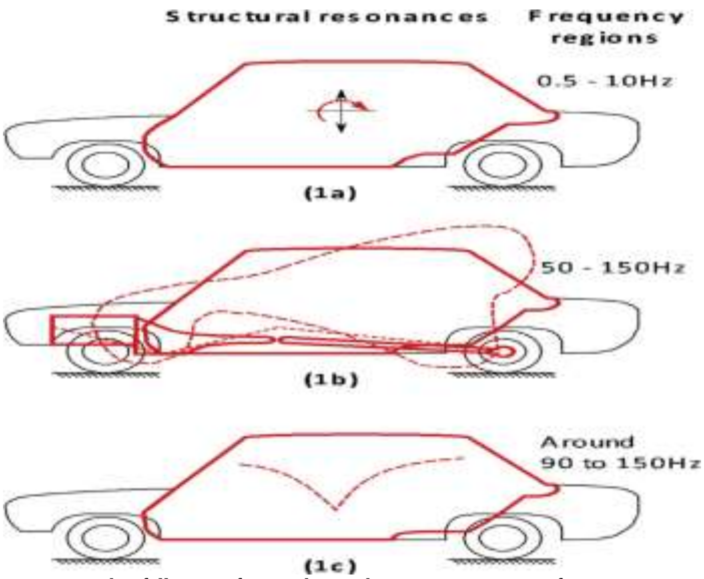


Figure 7. The following figure shows the main resonances frequencies available on a car during the drive. a) The vibration of suspension systems and wheels of a car, b) ring mode of the passenger compartment vibration. and bending vibration of driveline, and c) acoustic resonance of the cavity in the passenger compartment[18].

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A. *Optimization and simulation of the harvester*

The GA and the COMSOL optimization module were utilized to optimize the proposed energy harvester cantilever. The optimization methodologies were used to optimize the variables which improve the output power and grasp the suspension vibration frequency around (150Hz). The materials used for the base material and piezoelectric material are the same as the previous case study. The damping ratio was selected to be .08. The objective function, the optimization variables and the constrains were as before. The frequency was constrained to be 150 Hz. Table V shows the lower and upper bounds, and the results of both COMSOL and GA at 150 Hz.

The total volume of the optimized harvester using GA was 340.7 mm^3 while, the volume of optimized harvester using the COMSOL was 319.8mm^3. The simulated results of optimized model using the COMSOL are shown in Figures 8 to 13. Fig.8 shows that the maximum voltage was about 350 V. while Fig.11 shows that the maximum power output was about 30 mW. Figures 9 and 12 show the resistance dependence of the volt and power in order to determine the optimal resistance. Fig. 12 shows that the optimum resistance was about 2×10^6 Ω. The stress based on Von Mises theory was simulated at frequency 150 Hz to avoid the failure. Fig.10 demonstrates that maximum stress was around 6 MPa. Fig.13 shows the acceleration dependence of the power and the volt.

TABLE V. The lower bound, upper bound and the results of both COMSOL and GA at 150 Hz

variables	Lower Bound (mm)	Upper Bound (mm)	Results (mm) FEM	Results (mm) Analytical
Length(L)	30	90	59.9	59.9
width(w)	5	20	5	5
Piezo thickness (t_p)	0.15	0.6	0.35	0.337
Steel thickness (t_s)	0.2	0.8	0.3679	0.446

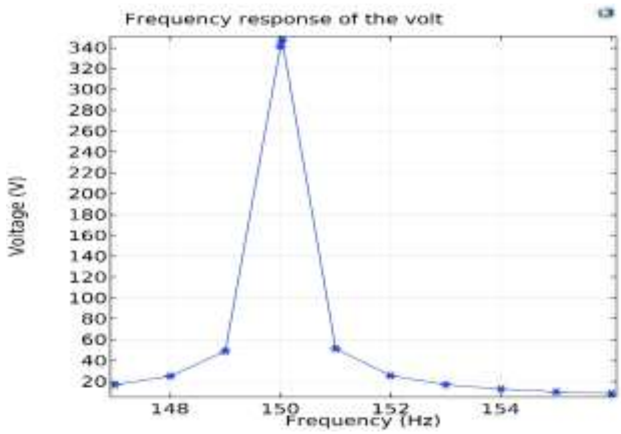

Figure 8. The frequency response of the volt for a vehicle suspension vibration model.

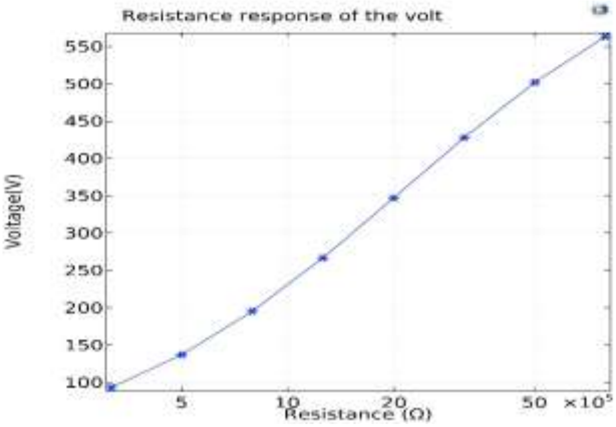

Figure 9. The resistance dependence of the voltage

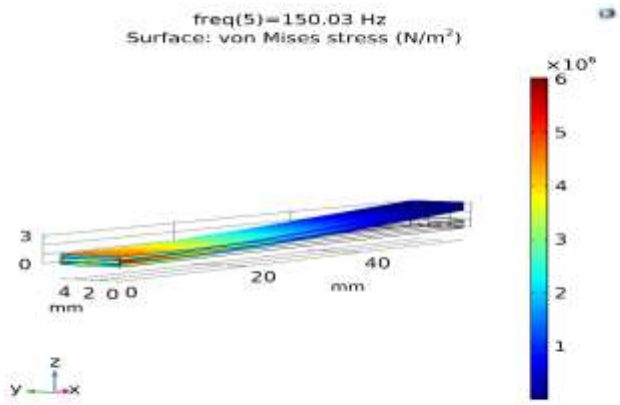

Figure 10. The von Mises stresses distribution along the harvester.

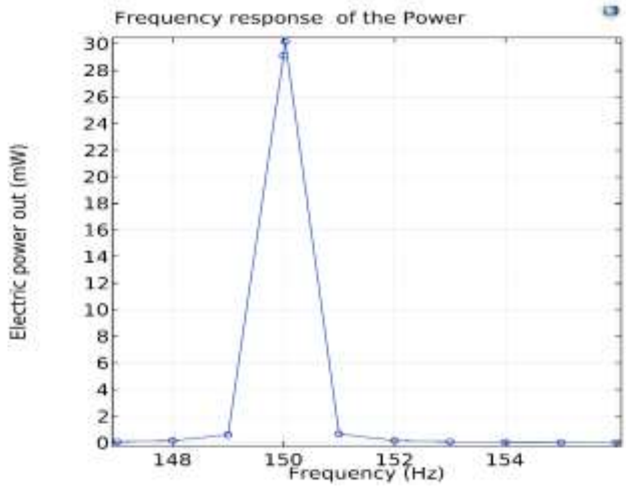

Figure11. The frequency response of the power for a vehicle suspension vibration model.

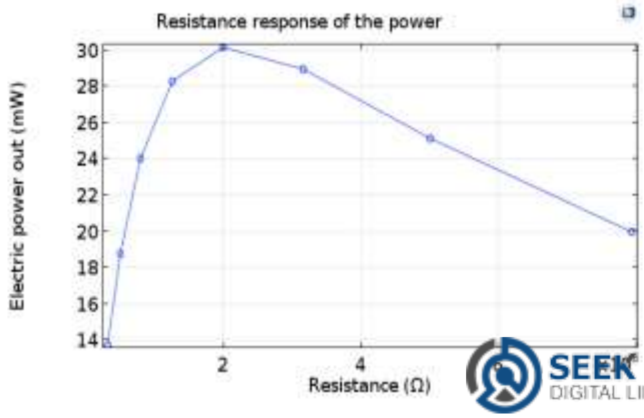

Figure 12. Resistance dependence of the power.

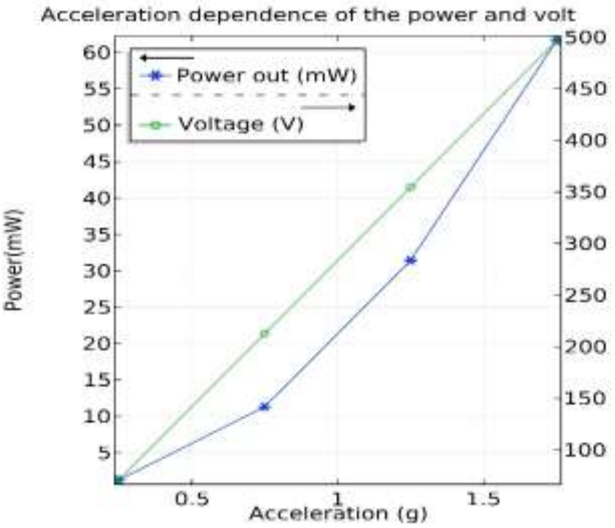

Figure 13. The acceleration dependence of the power and the volt.

B. *Verification of the COMSOL results*

The analytical results were utilized to validate the COMSOL results. Fig. 14 shows the good agreement between the COMSOL results and the analytical results. The power output of the COMSOL was 30 mW versus 27.7 mW for analytical solution. Due to the difference between of the two volumes resulted from the two optimization methodologies, the comparison should be based on the power per unit volume. The power per volume using COMSOL (FEM) was 93.8×10^{-3} mW/mm^3 versus 81.5×10^{-3} mW/mm^3 for analytical (GA). From the results we can see that the COMSOL optimization module introduced about 13.1 % improvement in the output power.

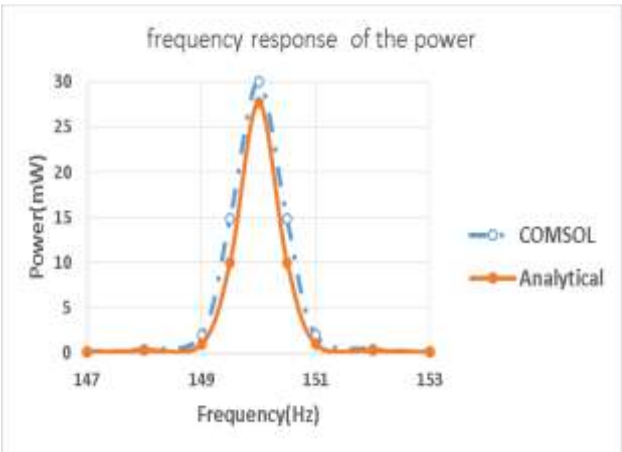

Figure 14. The frequency response of the power using the COMSOL and analytical.

IV. Conclusion

The genetic (GA) and COMSOL optimization module (BOBYQA) methodologies have been applied to optimize the vibration energy harvesting and to target the natural frequency of car suspension and engine vibration. The COMSOL has been utilized to simulate the results of the BOBYQA optimization algorithm while MATLAB has been utilized to simulate the results of the GA optimization methodology. The comparison between the results of the two optimization methodologies were performed. The results of the two optimization methodologies demonstrate that the BOBYQA optimization methodology was better than the GA optimization methodology for the same volume. Also, the stresses due to dynamic load were analyzed.

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