International Journal of Business and Management Study– IJBMS
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Volume 6 : Issue 2 - [ISSN : 2372-3955] - Publication Date: 27 Dec, 2019

An Iterative Mecahnism for Capacity Transfer among M/M/1 Queueing Systems with Private Information

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Abstract — **We study mechanism design problem for capacity transfer among M/M/1 queueing systems with private information. We propose an iterative mechanism, which is shown to simultaneously satisfy strategy-proofness, budget balance, individual rationality, and convergence. Finally, we conduct numerical studies to evaluate the performance of the proposed mechanism.**

***Keywords*—mechanism design, iterative mechanism, M/M/1 queueing systems, private information**

I. Introduction

Cooperation among different queueing systems through capacity transfer has served as an effective way for queueing systems to increase their capacity utilization. Most literature on cooperating queueing systems assume that information involved is public, for example, see Anily and Haviv (2010), Zeng et al. (2018). In some cases, however, the queueing systems involved may have private information that cannot be verified by other parties. For instance, in the example wherein different clinics share their staff members, each clinic may have different unit congestion cost and this information may be private. In these situations, appropriate mechanisms are needed to incentivize these systems to share information and exert cooperative efforts.

Some desirable properties of a mechanism include efficiency, strategy-proofness (or dominant strategy incentive compatibility), budget balance, and individual rationality. An efficient mechanism maximizes the total value over all agents. A strategy-proof mechanism can neutralize the complications that strategic misrepresentation creates in the situations with private information, since in a strategy proof mechanism, truth telling is every agent's dominant strategy. Budget balance of a mechanism ensures that no net payment made both from agents to the mechanism and from the mechanism to the agents. Finally, individual rationality makes sure that an agent achieves as much utility from in the mechanism as outside the mechanism.

In this paper, we focus on designing mechanisms satisfying these desirable properties. More specifically, we propose an iterative mechanism for capacity transfer among M/M/1 queueing systems with private information. This mechanism is shown to satisfy strategy-proofness, budget balance, individual rationality, and convergence, simultaneously. Finally, we conduct numerical studies to evaluate performance of the proposed mechanism.

II. Literature Review

Our study in this paper is related to the stream of study on the iterative mechanisms. Mechanism design literature is mainly focused on direct-revelation mechanisms like VCG mechanisms. Despite some attractive features of the VCG mechanisms, it has some drawbacks, such as requiring the agents to report their complete information and failing to simultaneously satisfy efficiency, strategy proofness, budget balance and individual rationality; it has been argued that an iterative version of the VCG mechanism would be preferable to its direct-revelation counterpart (Vohra, 2011).

One iterative mechanism is proposed by Dreze and de la Vallee Poussin (1971), in which each agent announces "gradient" information about his preferences for different outcomes. If the agents report truthful information, this iterative procure is Pareto optimal. Parkes (1999) proposes an efficient ascending-price (iterative) auction called iBundle for the combinatorial allocation problem, with myopic best-response agent strategies. The auction is weak budget-balanced, and individual-rational.

In the literature, it is also very common to design price-based iterative mechanisms to converge to competitive equilibrium (CE) prices. For example, Demange, Gale and Sotomayor (1986) show that for the multi-item auction, the minimum equilibrium price allocation can be achieved (approximately) by "progressive" auctions. They define a minimal update by gradually increasing the prices on a minimal over-demanded set of items, and these updates drive prices towards minimal CE prices. Later Gul and Stacchetti (2000) generalize the auction studied in Demange, Gale and Sotomayor (1986) for economies with substitutes. de Vries, Schummer and Vohra (2007) further generalize this to define minimal updates in terms of minimally under-supplied bidders, by gradually increasing the prices on the bundles in a minimally under-supplied set for which bidders submitted a bid.

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III. Problem Definition

We revisit the capacity transfer problem among M/M/1 systems in Zeng et al. (2018). In particular, in order to obtain non-trivial results, we focus on the special case wherein there is no capacity transfer cost. In Zeng et al. (2018), they study public information case, while in this paper we consider the case with private information. The specific problem is described as follows.

Let N = {1, … , n} be a set of n service providers, each having its own service rate and customer arrival rate. The incoming stream of customers to service provider $i \in N$ is assumed to be a Poisson process with rate λ_i. Service time for service provider $i \in N$ is exponentially distributed with mean $1/\mu_i$, and the service rate is μ_i accordingly, where it is assumed that $\mu_i > \lambda_i$ for stability. The unit congestion cost for service provider i is h_i. If all information involved is public, the central planner solves the following capacity transfer problem for the grand coalition N:

$$c^0(N) = \min_{m_i, i \in N} \frac{h_i \lambda_i}{m_i - \lambda_i}$$
$$\text{s.t. } \sum_{i \in N} m_i = \sum_{i \in N} \mu_i,$$
$$m_i > \lambda_i, i \in N,$$

where m_i is the service capacity of queue i after capacity transfer.

The solution to this problem is calculated as

$$m_{i,N} = \lambda_i + \frac{\sqrt{h_i \lambda_i} \sum_{k \in N} (\mu_k - \lambda_k)}{\sum_{k \in N} \sqrt{h_k \lambda_k}}, \text{i} \in \text{N}. \tag{1}$$

Recall that in Zeng et al. (2018), they propose a core cost allocation rule for capacity transfer among M/M/1 queueing systems. This cost allocation rule is defined as

$$x_i = \frac{\lambda_i}{m_{i,N} - \lambda_i} + \alpha_N \left(m_{i,N} - \mu_i \right), \text{i} \in \text{N}, \tag{2}$$

where α_N is the Lagrange multiplier w.r.t the first constraint,

$$\alpha_N = \frac{\left(\sum_{k \in N} \sqrt{h_k \lambda_k} \right)^2}{\left(\sum_{k \in N} (\mu_k - \lambda_k) \right)^2}. \tag{3}$$

This cost allocation rule in fact is a competitive allocation mechanism which calculates the market-clearing (competitive equilibrium) price to select a feasible, Pareto optimal allocation that varies with service providers' information. The problem with this allocation mechanism is that each agent can misreport his information to the mechanism if the information is private to him and not verifiable. Therefore, he will tend to misreport whenever he realizes that doing so will result in a more preferable allocation than a truthful report. Hence, truth-telling mechanisms are desirable in this case. In this paper, we seek to design strategy-proof mechanisms for queueing systems with private information.

More specifically, in this paper, we consider the case wherein service provider i's unit congestion cost h_i is his private information. For instance, in the example of transferring capacity among different clinics, each clinic's congestion cost h_i may not be verifiable, and thus can be treated as private information. We assume that the service and arrival rate of each service provider is still public information. This assumption is realistic as service providers often advertise their service capacity publicly while arrival rates can usually be verified by other servers, and thus falsifying these information is difficult. If some information is private to each service provider, it is conceivable that a self-interested service provider may act strategically and misreport their private information if doing so is beneficial. Our goal is to design mechanisms such that service providers cooperate in a way to maximize the social welfare (minimizing the total cost). In the following, we design an iterative mechanism.

IV. An Iterative Mechanism

In this section, we seek to implement indirect mechanisms in which budget balance and individual rationality are satisfied, and the strategy-proofness and efficiency are maintained as well. Towards this end, we focus on price-based approaches, in which the central planner sets price iteratively to coordinate the process. The price we are interested in here is the competitive equilibrium (CE) price.

A. *Competitive Equilibrium (CE) Price*

The CE prices for a market with transferable payoff are equilibrium prices given rise to by freely interactions of agents in this market. Formally, a market with transferable payoff consists of a finite set N of agents, a positive integer l denoting the number of input goods, an endowment vector $\omega_i \in R_+^l$ for each agent i, and a production function $f_i: R_+^l \to R_+$ for each agent i. An profile $(z_i)_{i \in N}$ of input vectors for which $\sum_{i \in N} z_i = \sum_{i \in N} \omega_i$ is an allocation.

A CE of a market with transferable payoff is defined as a pair $(p^*, (z_i^*)_{i \in N})$ consisting of a price vector p^* for the resources and an allocation $(z_i^*)_{i \in N}$ such that for each agent i, z_i^* solves the problem

$$max_{z_i \in R_+} (f_i(z_i) - p^*(z_i - \omega_i)).$$

The price vector p^* is then called the competitive prices. The idea of CE prices is that the agents in the market can trade resources at fixed prices, and these prices generates a CE if, when each agent chooses his trades to maximize his payoff, the resulting profile $(z_i^*)_{i \in N}$ is feasible in the sense that it is an allocation. That is, this price p^* clears the market, wherein the quantity supplied is equal to the quantity demanded. In another sense, given the price vector p^*, the decentralized decisions of the agents coincide with the centralized decision.

In summary, a CE should satisfy the following requirements:

- Market Clearance: the demand equals the supply:

$$\sum_{i \in N} z_i = \sum_{i \in N} \omega_i$$

- Individual Rationality: all agents are better-off after the trade than before the trade:

$$f_i(z_i^*) - p^*(z_i^* - \omega_i) \geq f_i(\omega_i), \forall i \in N.$$

- Budget Balance: the total transfer payment is zero.

$$\sum_{i \in N} p^*(z_i^* - \omega_i) = 0.$$

Because of these desirable properties of CE, we are interested in developing an price-based iterative mechanism to converge to CE prices.

B. *An CE Price-based Iterative Mechanism*

The scenario of capacity transfer in the M/M/1 setting can be modeled as a market with transferable payoff, which consists of: (1) a finite set of agents N ; (2) a positive integer l = 1 (the number of input goods); (3) for each agent $i \in N$ an endowment of resource μ_i ; (4) for each agent $i \in N$ a continuous, non-increasing, and convex function $H_i: R_+ \rightarrow R_+$ (the congestion cost function of agent i). A profile $(m_i)_{i \in N}$ for the resource is called an allocation if $\sum_{i \in N} m_i = \sum_{i \in N} \mu_i$.

The CE of the M/M/1 capacity transfer market is a pair $(p^*, (m_i^*)_{i \in N})$ consisting of a price p^* for the resource and an allocation $(m_i^*)_{i \in N}$) which satisfies $\sum_{i \in N} m_i^* = \sum_{i \in N} \mu_i$ such that for each agent $i \in N$, m_i^* solves the problem

$$\min_{m_i > \lambda_i} \left(\frac{h_i \lambda_i}{m_i - \lambda_i} + p^*(m_i - \mu_i) \right)$$

It can be easily calculated that the CE price of the capacity transfer problem in the M/M/1 setting for coalition N is exactly its Lagrangean multiplier defined by (3).

Now we design a price-based iterative mechanism which converges to this CE price. In particular, the goal is to develop an iterative mechanism in which incremental revelation of truthful information is a dominant strategy in each step for every service provider.

The CE price-based Iterative Mechanism:

- 1. Set an initial price $p^1 > 0$, independent from all service providers' private information. Set $k = 1$.

- 2. In each round k, given the price p^k, the service providers submit the amount of resources that they would like to sell/buy to the central planner.

- 3. If the total supplied amount > the total demanded amount, lower the price by ε_k, i.e., set $p^{k+1} = p^k - \varepsilon_k$; otherwise, increase the price by ε_k, i.e., set $p^{k+1} = p^k + \varepsilon_k$. Set $k = k + 1$.

- 4. Repeat Steps 2 and 3 until the total supply is equal to the total demand. Then, match the supply and the demand, and stop.

This iterative mechanism is convergent when the price incremental ε_k is small enough. In particular, when the price incremental is set to be diminishing, e.g., $\frac{c}{\sqrt{k}}$ or $\frac{c}{k}$ where c is a constant, then the iterative mechanism is guaranteed to be convergent.

Theorem 1 With diminishing price incremental, e.g., $\varepsilon_k = \frac{c}{\sqrt{k}}$ or $\frac{c}{k}$, where c is a constant, the iterative mechanism is guaranteed to be convergent. In particular, the price p^k converges to the CE price α_N.

The proofs of this paper are omitted here and can be provided upon request. Besides convergence, this iterative mechanism has several other desirable properties as shown by the following theorem.

Theorem 2 The proposed iterative mechanism satisfies efficiency, strategy-proofness, budget balance, individual rationality, and convergence.

Hence, this iterative mechanism simultaneously satisfies the desirable properties such as efficiency, strategy-proofness, budget balance, and individual rationality. Moreover, it does not require the service providers provide their direct information (e.g., the unit congestion cost), only indirect information (e.g., the amount of resources to buy/sell is required).

V. Numerical Studies

In this section, we conduct numerical studies to evaluate the performance of the iterative mechanism, mainly looking at the speed of convergence.

The speed of convergence for the iterative mechanism depends on two parameters: the initial price p^1 and the price incremental ε_k. The selection of a good initial price will depend on a good estimation of the unit congestion cost. For the price incremental, there is a trade-off between the calculating time of the mechanism and the convergence. A large price incremental may speed up the convergence of the iterative mechanism but it also exposes the mechanism to the risk of not being convergent. In this numerical study, we focus on this trade-off and gain some insights on how to choose the price incremental.

Suppose there is a set of M/M/1 systems N = {1, 2, 3}. The unit congestion cost vector is h = (100, 200, 300), the arrival rate vector is λ = (1, 2, 3), and the service rate vector is μ = (1.25, 2.2222, 4.2857). If all information is public, the competitive equilibrium price of the capacity transfer market for these M/M/1 systems is

$$\alpha_N = \frac{\left(\sum_{k \in N} \sqrt{h_k \lambda_k}\right)^2}{\left(\sum_{k \in N} (\mu_k - \lambda_k)\right)^2} = 1164.9.$$

International Journal of Business and Management Study– IJBMS
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Volume 6 : Issue 2 - [ISSN : 2372-3955] - Publication Date: 27 Dec, 2019

Now we implement the iterative mechanism to obtain this competitive price. We set the initial price p^1 = 1000 and set the precision to be five decimals in the code. We test two types of price incremental: constant price incremental and diminishing price incremental.

A. *Constant Price Incremental*

Constant price incremental means that the price incrementals are the same in each iteration. For example, if the price incremental is set to be 0.1 at the very beginning, then it will remain unchanged throughout the mechanism.

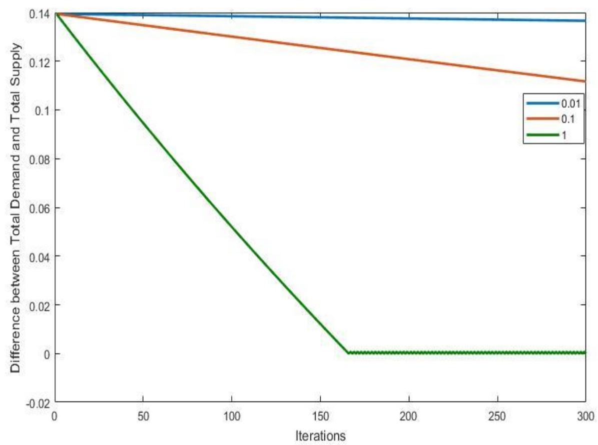

Figure 1. Descent Rate of the Mechanism under Different Price Incrementals

Fig. 1 shows the descent rate of the iterative mechanism when the price incremental is set to be 0.01, 0.1 and 1, respectively. This figure describes the differences between the total supply and the total demand in the first 300 iterations, in which the slope of the curve can be viewed as the descent rate. It can be easily seen from this figure that roughly in the first 160 iterations the descent rate is the largest when the price incremental is set to be 1. However, as the mechanism proceeds, the value of the difference between total demand and total supply fluctuates around zero, and thus the mechanism fails to converge in this case. Hence, it is not appropriate to set price incremental to 1.

B. *Diminishing Price Incremental*

As shown above, with constant price incremental, the iterative mechanism may not converge if the constant price incremental is not set appropriately. However, with diminishing price incremental, the iterative mechanism is guaranteed to converge. The question of interest now is how different schemes of diminishing price incremental affect the speed of convergence. We test two schemes of diminishing price incremental: $\varepsilon_k = \frac{c}{\sqrt{k}}$ and $\varepsilon_k = \frac{c}{k}$, where c is a constant, and we refer to these two schemes as linear diminishing scheme and square root diminishing scheme, respectively. We also test how different choices of constant c will affect the speed of convergence. The following three figures show the speed of convergence of the iterative mechanism under the two schemes of diminishing price incremental when the constant c is equal to 10, 100, and 1000, respectively.

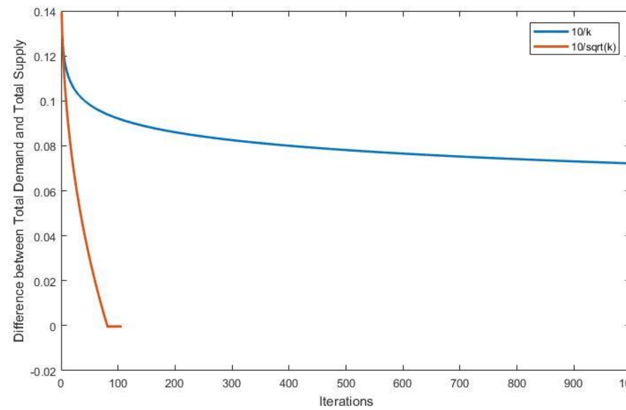

Figure 2 Speed of Convergence when c=10

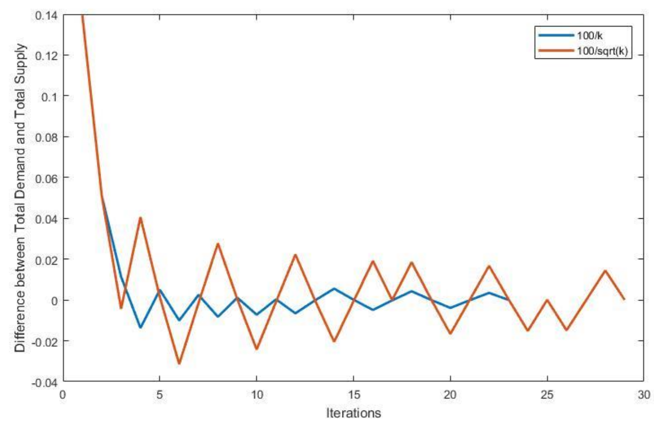

Figure 3 Speed of Convergence when c=100

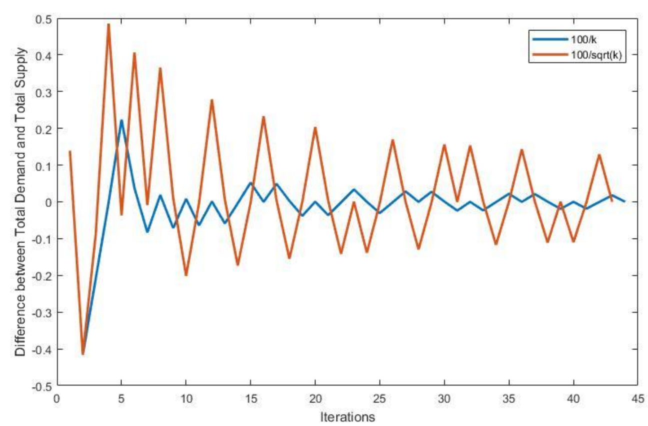

Figure 4 Speed of Convergence when c=1000

Note that in the legends of these three figures, *sqrt(k)* denotes $\sqrt{k}$. From these three figures we obtain the following insights.

(1) For the two schemes $\varepsilon_k = \frac{c}{\sqrt{k}}$ and $\varepsilon_k = \frac{c}{k}$, the square root one leads to significantly faster convergence when the iteration number required for the linear scheme is large. For

example, in Fig. 2, with the same constant c = 10, while the number of iterations for the liner scheme is over 1000 (in fact, it is over 300,000 iterations as we tested), the number of iterations for the square root scheme is only a little more than 100, which means the square root scheme significantly speed up the convergence. In contrast, when the number of iterations for the linear scheme is less than 100, then the square root scheme does not speed up the convergence that much (as illustrated by Fig. 4) or may slower the speed of convergence (as illustrated by Fig. 3). Hence, if the expected number of iterations is large, it is preferable to choose the square root scheme; otherwise, the choice of the two schemes will not make much difference.

(2) For the choice of the constant c, in both schemes, it is not the case that the larger c is, the better. When the number of iterations required is large, increasing c will speed up the convergence significantly. For example, for the linear scheme, changing c from 10 to 100 reduces the number of iterations significantly. However, when the number of iteration required is small, increasing c may not be beneficial. For example, for the square root scheme, increasing c from 100 to 1000 slows the convergence. Hence, when we find that convergence is very slow, we can try increasing c to speed up the convergence.

(3) Compared with the iterative mechanism with constant price incremental, the one with diminishing price incremental does not only guarantee convergence, but also generally has a faster speed of convergence.

VI. Conclusions

In this paper we consider the scenario where agents have their unit congestion costs as private information. Truth-telling mechanisms are important to prevent the cooperation from being jeopardized by self-interested players. For this scenario, we propose an iterative mechanism, which is shown to simultaneously satisfy strategy-proofness, budget balance, individual rationality, and convergence. Moreover, this mechanism does not require the agents to report their direct private information, e.g, the unit congestion cost; only the indirect information, e.g., demand/supply, is needed to report. Finally, we conduct numerical studies to evaluate the performance of the iterative mechanism. We test two types of price incrementals: constant price incremental and diminishing price incremental. Compared with the constant price incremental, the diminishing price incremental does not only guarantee conver- gence, but also generally has a faster speed of convergence.

Acknowledgment

This paper is adapted from one chapter of the first author's PhD thesis. The authors acknowledge the supports of: (i) National Natural Science Foundation of China (No. 71671146) to Yinlian Zeng, and (ii) National Natural Science Foundation of China (Nos. 71531003, 71432004), Research Grants Council of Hong Kong (No. T32-102/14), and the Leading Talent Program of Guangdong Province (No. 2016LJ06D703), to Xiaoqiang Cai.

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