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Study of Eutrophication of a greek lake via fuzzy linear regression

Eutrophication and fuzzy linear regression

[G. Ellina]

***Abstract*—The phenomenon of eutrophication of water bodies and specially the lakes, demand the proper investigation of the physical, chemical and biological factors that influence these ecosystems. The aim of this study is to construct a fuzzy logic system with the most suitable fuzzy decision rules in order to predict the ecological state of our study case.**

***Keywords*—fuzzy linear regression, Orestiada lake, eutrophication prediction, linguistic variables**

I. Introduction

One of the most important objects of Water Resources Management is lake ecosystems that are environmental goods of particular importance. The lakes are associated with aquatic ecosystems with multiple significance and value, both for man and for the natural environment. Their water is used in a variety of ways (water, irrigation, industry), but very often they become the final recipients of urban, industrial and agricultural waste. This results in pollution of their waters and disturbance of the ecological balance of the lake ecosystem. Many lakes, both in Greece and the rest of the world, face significant water quality problems, which are the subject of intense study by scientists. In this paper we are going to investigate some of the most influential factors for the phenomenon of eutrophication.

II. Methodology

A. *Case of Study*

Lake Orestiada, which encircles the city of Kastoria, giving its wonderful natural landscapes, will be the subject of this work. This lake is a very important natural ecosystem with diverse and rare individual habitats that support a large biodiversity including many rare and endangered species. The lake is also vital for the city of Kastoria and the surrounding municipalities, since it determines their physiognomy and, on the other hand, is an important natural resource. The supply of water for irrigation from the lake is important, as there are several agricultural crops in the prefecture of Kastoria, many of which are located around the lake. As natural wealth, plays a very important role in recreation. The lake is at an altitude of 630 meters, with a rectangular shape, due to the penetration of a hilly peninsula, in the isthmus of which is built the city of Kastoria, and is a remnant of an old extended lake. Today, its area is about 28 km^2 and the maximum depth of about 9 meters. Its water volume is approximately 100,000,000 cubic meters and its coastline is 30,8 km. The lake is fed by many lakeside sources and by the rainwater that either falls directly to its surface or ends up with it with the surface runoff through the torrents located mainly in its northern and eastern parts. In the southern part, a canal (Guilli stream) connects the lake with the Aliakmon River.

The sets of data coming from the Department of Environment and Water Policies, Region of Western Macedonia and concern the following factors in a monthly base: water temperature, dissolved oxygen, pH and chlorophyll-a from the year of 2015 to 2017.

B. *Fuzzy linear regression*

The general form of the model is as follows [1],[5] and [6]:

$$Y = A_0 + A_1 x_1 + \cdots A_n x_n$$

where $A_i,\ i = 1,2,...,n$ are symmetric triangular fuzzy numbers. The function involving fuzzy numbers A_i can be regarded as a probability distribution function. In this respect the fuzzy linear regression becomes a linear model in which differences between the actual values and the estimated values can be derived from uncertainty of the system. In fuzzy linear regression the fuzzy numbers are considered:

$$\begin{aligned} Y_1 &= A_0 + A_1 x_{11} + A_2 x_{21} + ... + A_n x_{n1} \\ Y_2 &= A_0 + A_1 x_{12} + A_2 x_{22} + ... + A_n x_{n2} \\ &\ldots\ldots\ldots\ldots\ldots\ldots \\ Y_j &= A_0 + A_1 x_{1n} + A_2 x_{2,\mathrm{n}} + ... + A_n x_{nj} \end{aligned}$$

where $A_0,\ A_1, \ldots,\ A_n,\ i = 1,2,...,n$ are symmetrical triangular fuzzy numbers. In this application $n = 3$ and $j = 24$.

We determined the degree h to which we expect the data $((x_{1j}, x_{2j}, x_{3j}), y_j)$ to be included in the referred number Y_i that is:

$$\mu_{Y_i}(y_i) \geq h,\ i = 1,2,...,24$$

We also want the spread of chlorophyll-a's fuzzy numbers $Y_i,\ i = 1,2,...,24$ be as low as possible. Since the fuzzy numbers, $A_i,\ i = 0,\ 1,\ 2, 3$ are symmetric, are in the form:

$$\mu_{A_i}(x) = L\left(\frac{x - r_i}{c_i}\right),\ c_i > 0$$

where r_i is the center and c_i the spread, $i = 0,1,2,\ldots,n$ and the function L(x) is defined:

$$L : R \to R,\ L(x) = \max\{0,\ 1 - |x|\}$$

The calculation of numbers A_i was performed by computing the numbers r_i and c_i , where $i = 0,1,2,\ldots,n$. This gives the following linear regression:

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$$\min\left\{24c_0+\sum_{i=1}^{24}\sum_{j=1}^{3}c_j\left|x_{ji}\right|\right\}$$

$$y_i \geq -(1-h)\left(c_0+\sum_{j=1}^{3}c_j\left|x_{ji}\right|\right)+r_0+\sum_{j=1}^{3}r_jx_{ji},\ i=1,2,\cdots,24$$

$$y_i \leq (1-h)\left(c_0+\sum_{j=1}^{3}c_j\left|x_{ji}\right|\right)+r_0+\sum_{j=1}^{3}r_jx_{ji}, i=1,2,\cdots,24$$

Finally in this paper we are going to construct with three linguistic variables (low, medium and high) via fuzzy inference system an adaptive. This classification of the independent variables will be done with the most representing classes, due to the amount of statistical data. In this way, they will approach the problem concerning the chl-a prediction by setting as inputs the values of the independent parameters.

III. Results

In Table 1 it can be easily observed the fuzzy numbers of each examined couple of independent and dependent variable. These fuzzy numbers are presenting all the possible values that can be represented by our parameters in this ecosystem.

At this point we can say that the three selected independents variables can predict with safety the parameter of chlorophyll-a. In fig.1, fig.2 and fig.2 it is pictured the fuzzy numbers of every value of chl-a in relation with the independent parameters of water temperature, DO_2 and pH.

TABLE I. FUZZY NUMBERS OF CHLOROPHYLL-A

Independent parameters	**Fuzzy numbers**
Water Temperature	$y_{temp}=(18.42,\ 13.23)+(-0.19,\ 0)x$
DO_2	$y_{DO_2}=(1.99,\ 0)+(1.37,\ 1.18)x$
pH	$y_{pH}=(-24.7,\ 13.23)+(5.05,\ 0)x$

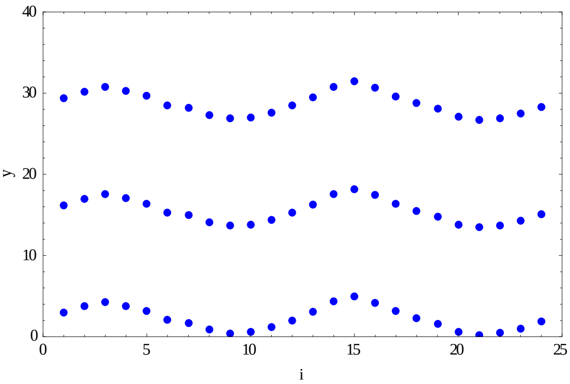

Figure 1. Fuzzy numbers of chl-a computed by water temperature

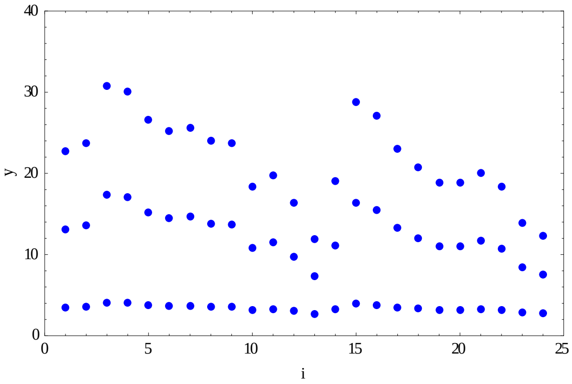

Figure 2. Fuzzy numbers of chl-a computed by DO_2

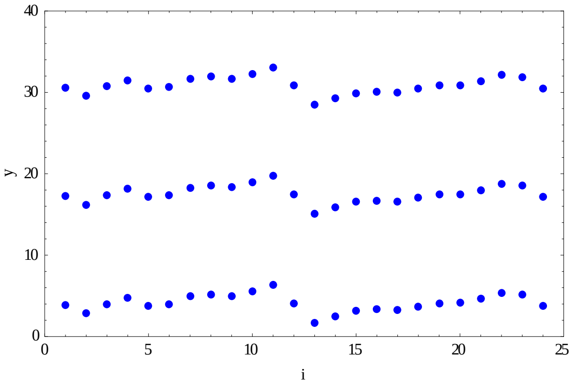

Figure 3. Fuzzy numbers of chl-a computed by pH

Since we noticed that the fuzzy numbers above have a very big range, our next step is to classify all of the independent parameters (temperature, DO_2 and pH) to three linguistic classes, low, medium and high. At this method we will still use triangular fuzzy numbers. So, in figures 4, 5 and 6 these fuzzy numbers can be pictured.

It should be mentioned that, this method is a useful tool for the prediction of the levels of our dependent parameter chl-a. The error derived from this is 0.00189. This leads us to a hopeful result that, our prediction will be almost a hundred percent accurate.

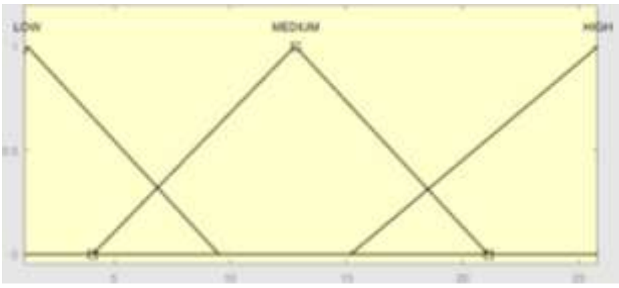

Figure 4. Classification of temperature as triangular fuzzy numbers

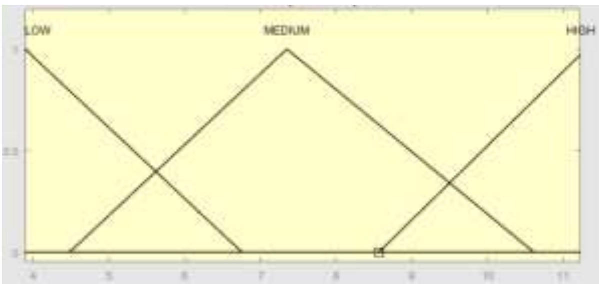

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Figure 5. Classification of DO_2 as triangular fuzzy numbers

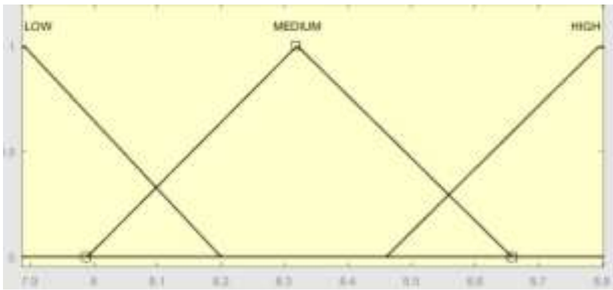

Figure 6. Classification of pH as triangular fuzzy numbers

IV. Discussion and Conclusion

Primary production models in water bodies involve the quantification and estimation of chlorophyll-a concentrations, based on empirical relationships, dynamic models and the fuzzy logic approach [2]. In this study we applied the fuzzy linear regression using data that have a strong relation with the phenomenon of the eutrophication. The selected water body in this study is the Orestiada lake and it is characterized as hypertrophic lake Using the linguistic variables we succeeded to minimize the error of prediction. Since the "ageing" of a reservoir is a complicated process lasting several years [3] the estimation of parameters which express this process such as the chlorophyll a as a proxy of primary production is considered of high importance [4]. The results of this study can help us understand better the mechanisms of this ecosystem. In a previous paper [1], studying another Mediterranean lake and using less data sets, we found a stronger relation between the parameter of water temperature and chl-a instead of the others.

Orestiada lake is a site of Community interest of the Pan-European Ecological Network Natura 2000 (GR1320001-BioGreece'99). According to Directive 92/43 / EEC, a special management plan should be established for this area. This will identify specific actions and management measures that will aim and should ensure the good ecological status of conservation of the types of vegetation and flora species and fauna that occur in the region and have been the main reasons for accession of the area on the Network.

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G. Ellina
Democritus University of Thrace
Greece

G. Papaschinopoulos
Democritus University of Thrace
Greece

B.K. Papadopoulos
Democritus University of Thrace
Greece