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Non-linear control for a group of omni-wheel robots

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***Abstract*— The paper presents a solution of the problem of non-linear control law design for a group of mobile robots. The solution consists of the new approach to control of a group of mobile robot based on principals and methods of synergetic control theory. The design of robots control is considered with the analysis of mathematic model and with specified aim. In the synergetic control theory the set of criteria for the control system is usually expressed in the form of an appropriate system of invariants. Invariants play the role of control objectives, which are enforced a given technological aims. The synergetic synthesis procedure reduces to process of finding the non-linear control law on which these given invariants are satisfied. The synthesized control law provides an asymptotically stable movement of a group of omni-wheel robots with specified type of formation.**

***Keywords*— group control; omni-wheel robot; non-linear mathematical model; synergetic control theory**

I. Introduction

Nowadays the power of single mobile robot is not enough to solve the technological problems such as sensing and exploring of large areas, the assembly of non-trivial structures in the extreme conditions, performing fire-fighting or other large-scale action. [1] It is obvious that for such tasks we need to involve a large number of homogeneous [1, 2] or heterogeneous [3, 4] (depending on the task) of autonomous mobile robots. In this case, the task of control a group of such robots while achieving the global goal is called as group control.

There are many benefits of this approach to solving large-scale problems. These include coverage of large areas [1] in less time as compared to single autonomous robots. This advantage is achieved by assigning workspaces to each robot in the group. Also one of the advantages of the group is to expand the robots functionality due to the seting of individual hardware for each group robot. It is obvious that a single autonomous robot could not hold the set of technical tools that can be used for group use of robots. However, the most valuable feature of robots working in a group, is the system overall reliability and high probability of achieving the goals by the group robotic system (GRS). This property is achieved by redistribution of roles between the robots in the event of failure of one or several group units [5-7].

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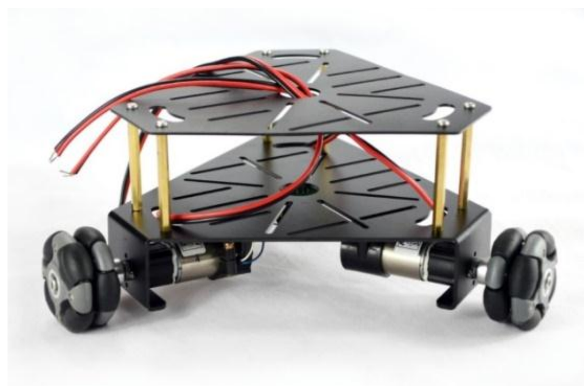
With a variety of advantages of the use of the GRS, there are a number of problems arising from the need to organize a "swarm" behavior of robots, solving the problem of a unified process. One of these problems is to create algorithms for effective interaction between robots at all levels of the hierarchy, which achieves the ultimate goal (or target situation) in minimum time and at minimum cost.

II. The Mathematical Model of the Robot Group

To determine the mathematical model of the robot group it is necessary to determine the dynamics of behavior of one robot. Let consider that the robot (Fig. 1) is a type of omnidirectional transport vehicle [8].

Figure 1. Omni-wheel mobile robot



To design of high level control strategies, we need to build robot mathematical model as a rigid body. Robot motion may be explored as a sum of linear motion of center of masses and spherical motion of body against center of masses [8]. Assume that the robots of the group have the same design and mass-inertial characteristics, with the center of mass of each robot is located at the center of the platform. Driving mode of robot group is described by the following system of differential equations:

$$
\begin{aligned}
&\dot{x}^k = V_x^k; \quad \dot{y}^k = V_y^k; \quad \dot{\psi}^k = \omega_\psi^k; \\
&\dot{V}_x^k = \frac{\sin\psi^k u_1^k + \sin(\psi^k + k_1)u_2^k + \sin(\psi^k + k_2)u_3^k - hV_x^k - m_d\omega_\psi^k V_y^k}{m}; \\
&\dot{V}_y^k = \frac{-\cos\psi^k u_1^k - \cos(\psi^k + k_1)u_2^k - \cos(\psi^k + k_2)u_3^k - hV_y^k + m_d\omega_\psi^k V_x^k}{m}; \\
&\dot{\omega}_\psi^k = \frac{-a(u_1^k + u_2^k + u_3^k) - 2a^2 h\omega_\psi^k}{I},
\end{aligned}
\quad (1)
$$

where x^k, y^k are coordinates of center of masses of k-th robot of the group; V_x^k, V_y^k, are projections of linear velocity vector

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of k-th robot; ψ^k is a yaw angle of the k-th robot, ω_ψ^k is a yaw angular velocity of k-th robot; u_1^k, u_2^k, u_3^k are the control channels of k-th GRS; *m* is a mass of robot; *I* is a moment of inertia around axis *Z*.

As the main task for control system was selected an asymptotically stable movement of robot group to a given position while holding the specified formation.

Designing a control system for a group of mobile robots is not a trivial task. This task is required using classical control theory methods, which are based on linearization of nonlinear components, which inevitably leads to problems of stability of the entire system. Therefore, for the synthesis of the laws of group control in this paper we propose to use the methods and approaches of synergetic control theory (SCT) [9-14].

III. Design Procedure

In the SCT the set of criteria for the system control is usually expressed in the form of an appropriate system of invariants, which act as control objectives. They ensured the implementation of a given technological problem, and the procedure for the synergetic synthesis comes to the search of control laws under which these stated invariants are implemented. Following by the centralized approach of group control [6], as the first invariants of the system it is necessary to select the movement of the center of mass of the first (k = 1) or master of GRS to the point with given coordinates x^*, y^* and to set retention of orientation angle ψ^1 in a given direction ψ^*. Thus a subset of the objectives for the master of GRS has the form:

$$\Sigma_1^1 = \left\{x^1 = x^*, \quad y^1 = y^*, \quad \psi^1 = \psi^*\right\}. \qquad (2)$$

In turn, the slave of GRS (k = 2, 3, ..., n, where n is number of GRS) should perform a task tracking system, so a subset of the control goals of the slave of GRS will have the following form:

$$\Sigma_1^k = \left\{x^k = x^1 + \Delta_x^k, \quad y^k = y^1 + \Delta_y^k, \quad \psi^k = \psi^1 + \Delta_\psi^k\right\} \qquad (3)$$

where $\Delta_x^k, \Delta_y^k, \Delta_\psi^k$ are factors of the slave of GRS offset along master of GRS, k is number of slave GRS.

To implement the synthesis by ADAR method [9] we express specific subsets of control objectives of master of GRS (2) by setting the following macro variables:

$$\begin{aligned} \Psi_1^1 &= V_x^1 - V_{x_{\max}}^1 \tanh(x^* - x^1); \\ \Psi_2^1 &= V_y^1 - V_{y_{\max}}^1 \tanh(y^* - y^1); \\ \Psi_3^1 &= \omega_\psi^1 - k_1^1(\psi^* - \psi^1), \end{aligned} \qquad (4)$$

where $V_{\max}^1 = \left[V_{x_{\max}}^1, V_{y_{\max}}^1\right]$ is vector speed limit leading GRS, k_1^1 – a positive constant.

System of macro variables (4) in this case must satisfy the solution Ψ_i^1=0 (i = 1, 2, 3) system of functional equations:

$$\dot{\Psi}_i^1 + \lambda_i^1 \Psi_i^1 = 0, \quad i = \overline{1,3}. \qquad (5)$$

where $\lambda_i^1, i = \overline{1,3}$ are positive constants.

From system (5) we determine the "external" control laws of master of GRS u_1^1 u_2^1, u_3^1 which are responsible for master of GRS moving to a given point $[x^*, y^*]$.

Furthermore, for the implementation the tracking of master GRS we express a subset of control objectives for slave GRS (3) through the following set of macro variables

$$\begin{aligned} \Psi_1^k &= V_x^k - V_x^1 - V_{x_{\max}}^k \tanh(x^1 + \Delta_x^k - x^k); \\ \Psi_2^k &= V_y^k - V_y^1 - V_{y_{\max}}^k \tanh(y^1 + \Delta_y^k - y^k); \\ \Psi_3^k &= \omega_\psi^k - \omega_\psi^1 - k_1^k(\psi^1 + \Delta_\psi^k - \psi^k), \end{aligned} \qquad (6)$$

where $V_{\max}^k = \left[V_{x_{\max}}^k, V_{y_{\max}}^k\right]$ is master of GRS max allowable motion speed vector, k_1^k is a positive constant. According to synergetic control theory [15-18] the system of macro variables (6) must satisfy the solution $\Psi_i^k = 0$ (i = 1, 2, 3) of system of functional equations:

$$\dot{\Psi}_i^k + \lambda_i^k \Psi_i^k = 0, \quad i = \overline{1,3}. \qquad (7)$$

Solution of the system of functional equations (7) are "external" control laws of slave of GRS u_1^k, u_2^k, u_3^k, which provide a solution of specified manifolds system $\Psi_i^k = 0$ (i = 1, 2, 3), which results in a dynamic decomposition of the original system (1). As a result, the behavior of a group of mobile robots at the intersection of invariant manifolds $\Psi_i^1 = 0$ and $\Psi_i^k = 0$ (i = 1, 2, 3) is described by the following system:

$$\begin{aligned} &\dot{x}^1 = V_{x_{\max}}^1 \tanh(x^* - x^1); &&\dot{x}^k = V_x^1 + V_{x_{\max}}^k \tanh(x^1 + \Delta_x^k - x^k); \\ &\dot{y}^1 = V_{y_{\max}}^1 \tanh(y^* - y^1); &&\dot{y}^k = V_y^1 + V_{y_{\max}}^k \tanh(y^1 + \Delta_y^k - y^k); \\ &\dot{\psi}^1 = k_1^1(\psi^* - \psi^1); &&\dot{\psi}^k = \omega_\psi^1 + k_1^k(\psi^1 + \Delta_\psi^k - \psi^k). \end{aligned} \qquad (8)$$

In order to closed-loop system (8) be asymptotically stable it is necessary that $k_1^1 > 0$, $k_1^k > 0$, and the projection of the maximum allowable speed for the master and slave of GRS are calculated as follows:

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$$X_1 = x^* - x^1; \qquad X_k = x^1 + \Delta_x^k - x^k;$$
$$Y_1 = y^* - y^1; \qquad Y_k = y^1 + \Delta_y^k - y^k;$$
$$n_1 = \sqrt{X_1^2 + Y_1^2}; \qquad n_k = \sqrt{X_k^2 + Y_k^2};$$
$$V_{x_{\max}}^1 = \frac{\operatorname{sign}(X_1) X_1 V_0^1}{n_1}; \quad V_{x_{\max}}^k = \frac{\operatorname{sign}(X_k) X_k V_0^k}{n_k};$$
$$V_{y_{\max}}^1 = \frac{\operatorname{sign}(Y_1) Y_1 V_0^1}{n_1}; \quad V_{y_{\max}}^k = \frac{\operatorname{sign}(Y_k) Y_k V_0^k}{n_k},$$

where V_0^1 and V_0^k are desired speeds of master and slave GRS motion.

Thus taking into account the mathematical model of mobile robot (1) and "external" controls of master of GRS u_1^1, u_2^1, $u_3^1$and slave of GRS u_1^k, u_2^k, u_3^k, u_4^k we can get a group control strategy implementing a centralized behavior, coherent motion of the slave of GRS with respect to master of GRS moving to any point in the workspace.

IV. Computer Simulation

Let simulate the behavior of synthesized close loop system of robot group. Assumed that one master and two slaves (k = 3) robot are used to line up the group in the space.

Let set a limit of speed of the master of GRS movement with threshold of V_0^1 = 0.3 m/s. Let define the target coordinates of master of GRS as follows: x^* = 15 m, y^* = 0 m, and yaw angle ψ^* = 0 rad. Let limit the speed of the slave of GRS by threshold of V_0^k = 0.8 m/s.

Let take as initial conditions for master and slave of GRS zero values of the projections of the vector of the linear ($V_x^k(0) = V_y^k(0) = 0$) and angular velocities ($\omega_\psi^k(0) = 0$) of robot. As the initial location of the robot in the workspace let take the following values:

$$x^1(0) = 0, \quad y^1(0) = 0, \quad \psi^1(0) = 0;$$
$$x^2(0) = -10, \quad y^2(0) = 0, \quad \psi^2(0) = 0;$$
$$x^3(0) = -5, \quad y^3(0) = 0, \quad \psi^3(0) = 0.$$

At Fig. 2 – 5 presented the process of coordinates changing for obtained closed-loop system. Figure legend: 1 is master of GRS; 2, 3 are slaves of GRS.

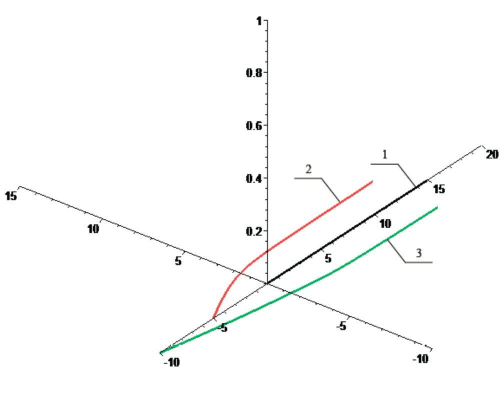

Figure 2. Trajectory of the robots group movement

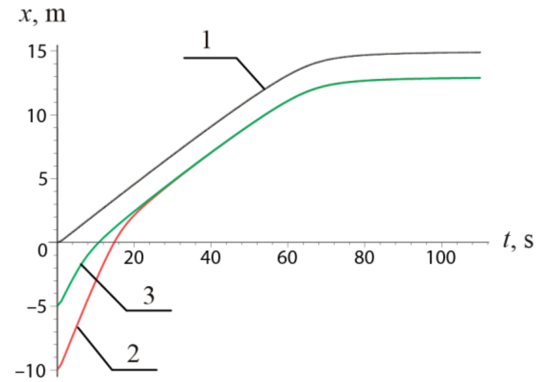

Figure 3. Changing the projection of the group robots movement on the X-axis

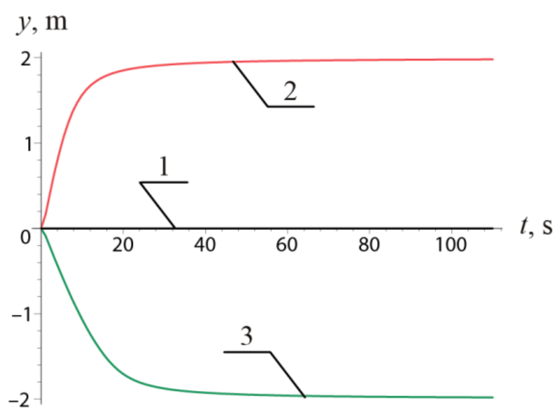

Figure 4. Changing the projection of the group robots movement on the Y-axis

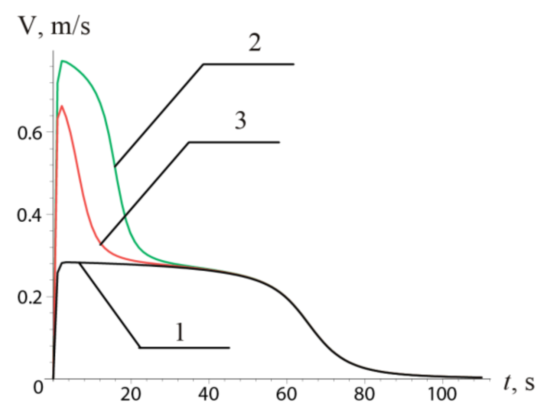

Figure 5. Change in the linear velocity vector of mobile robots group

Presented simulation results confirm that in a closed system, the synthesized control of robots group (4) - (10), ensure that the system introduced invariants: movement of master of GRS to a given point (see Fig. 3 and Fig. 4), holding slaves of GRS specified offset to the master of the GRS (Figure 4), as well as performing all the GRS limit for maximum speed of movement (see Fig. 5).

V. Conclusion

The article presents the procedure for analytical synthesis of strategy of mobile robots group control by using the full non-linear models of motion. This control strategy provides asymptotical stability of closed-loop systems and precise implementation of defined invariants. The use of non-linear components in the synthesis of the laws of group control allows accurately maintain the desired offset of slave robot respect of the master. Variation of displacement of slave of GRS allows to create different types of group of robots line up. In this paper we propose the basic control laws for mechanical subsystems of mobile robots group. To build a complete control system we further intend to include dynamics of actuators into the model of the system as well as the dynamics of relationships between robots to enhance their efficiency and overall system stability.

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