International Journal of Advancements in Mechanical and Aeronautical Engineering – IJAMAE 2018
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Volume 5 : Issue 1- [ISSN : 2372-4153] - Publication Date: 25 June, 2018

Advanced Design and Manufacture of Spiral Bevel, Hypoid, and Worm Gears

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***Abstract*—A method is presented for the optimization of design and manufacture of spiral bevel, hypoid, and worm gears. The aim of the optimization is to improve the operating characteristics of the gear pair, namely, to increase the service life of gears and to reduce the dynamic effects of gear mesh. Optimal modifications are introduced into the tooth surfaces to simultaneously reduce the maximum tooth contact pressure and transmission errors. These modifications are introduced by applying optimal machine tool settings and geometry of cutting tool in the manufacturing process. The method is demonstrated on spiral bevel, hypoid, and worm gears. The obtained results have shown that significant improvements in the operating characteristics of the gear pairs are achieved.**

***Keywords*—gears, design, manufacture, optimization**

I. Introduction

Gear drives are one of the most widely used modes in power transmission system with the advantages of large power, high efficiency, and long service life.

In the gear design process, the traditional method only gives basic parameters of gear structure. The modern power transmission industries need an optimum design of high quality gears as to have high load capacity, longer service life, low weight, and size. Any attempt to improve the functional attributes of a gear pair in terms of strength, quality, and noise requires an optimization of its design and manufacture. Nowadays, in order to improve the operating characteristics of gears and to reduce the influence of tooth errors and misalignments on tooth mesh, optimal tooth surface modifications are introduced into the teeth of one or both members of the gear pair. The aim of these tooth surface modifications is to simultaneously reduce the tooth contact pressure and the transmission errors. The tooth surface modifications are introduced by the optimization of the machine tool settings and the geometry of the cutting tool used in the manufacturing procedure. The new CNC machines have made it possible to perform varying correction motions during the manufacturing process.

Spiral bevel, hypoid, and worm gears can be considered as a more general case on gear geometry. That is the reason that these types of gears will be used to demonstrate the optimization procedure of the design and manufacture of gears.

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In the last decades many research works have been directed towards the improvements in gear design and manufacture. Because of the limited lebgth of this presentation, only small part of the corresponding publications can be referenced [1-16].

II. Optimal Machine Tool Settings for the Manufacture of Spiral Bevel and Hypoid Gears

In order to improve the operating characteristics of spiral bevel and hypoid gear pairs, modifications are introduced into the teeth surfaces of the pinion. The modifications are introduced by the variation of machine tool settings and the profile of the head-cutter's cutting edges. The machine tool settings used for pinion tooth finishing are specified in Fig. 1: sliding base setting (c), basic radial (e), basic offset (g), tilt angle (β), and swivel angle (δ). The other manufacture parameters are the velocity ratio in the kinematic scheme of the machine tool for the generation of the pinion tooth surface (i_{g1}) and the radii of the head-cutter profile ($r_{prof\,1}, r_{prof\,2}$). The tooth surface of the pinion is defined by the following system of equations:

$$\vec{r}_1^{(1)} = \boldsymbol{M}_4(i_{g1}) \cdot \boldsymbol{M}_3(c,g) \cdot \boldsymbol{M}_2(e) \cdot \boldsymbol{M}_1(\beta,\delta) \cdot \vec{r}_{T1}^{(T1)}\left(r_{prof\,1}, r_{prof\,2}\right) \quad \text{(1a)}$$

$$\vec{v}_0^{(T1,1)} \cdot \vec{e}_0^{(T1)} = 0 \quad \text{(1b)}$$

where $\vec{r}_{T_1}^{(T_1)}$ is the radius vector of head-cutter surface points, matrices $\boldsymbol{M}_1$, $\boldsymbol{M}_2$, $\boldsymbol{M}_3$, and $\boldsymbol{M}_4$ provide the coordinate transformations from system K_{T1} (rigidly connected to the cradle and head-cutter T_1) to system K_1 (rigidly connected to the being generated pinion). The second equation describes mathematically the generation of pinion tooth surface by the head-cutter ; $\vec{v}_0^{(T1,1)}$ is the relative velocity vector of the head-cutter to the pinion and $\vec{e}_0^{(T1)}$ is the unit normal vector of the generator surface of the head-cutter.

The new CNC hypoid generators have made it possible to perform the variation of sliding base setting, basic radial, basic offset, tilt and swivel angles, and the ratio of roll conducted by polynomial functions of fifth-order:

$$\begin{aligned} \Delta c &= c_{10} + c_{11} \cdot (\psi - \psi_0) + c_{12} \cdot (\psi - \psi_0)^2 \ldots + c_{15} \cdot (\psi - \psi_0)^5 \\ \Delta e &= c_{20} + c_{21} \cdot (\psi - \psi_0) + c_{22} \cdot (\psi - \psi_0)^2 \ldots + c_{25} \cdot (\psi - \psi_0)^5 \\ \Delta g &= c_{30} + c_{31} \cdot (\psi - \psi_0) + c_{32} \cdot (\psi - \psi_0)^2 \ldots + c_{35} \cdot (\psi - \psi_0)^5 \\ \Delta \beta &= c_{40} + c_{41} \cdot (\psi - \psi_0) + c_{42} \cdot (\psi - \psi_0)^2 \ldots + c_{45} \cdot (\psi - \psi_0)^5 \quad \text{(2)} \end{aligned}$$

$$\Delta\delta = c_{50} + c_{51}\cdot(\psi-\psi_0) + c_{52}\cdot(\psi-\psi_0)^2 \ldots + c_{55}\cdot(\psi-\psi_0)^5$$
$$\Delta i_{g1} = c_{60} + c_{61}\cdot(\psi-\psi_0) + c_{62}\cdot(\psi-\psi_0)^2 \ldots + c_{65}\cdot(\psi-\psi_0)^5$$

where ψ is the angle of rotation of the cradle in pinion tooth surface generation.

Therefore, the maximum tooth contact pressure and maximum transmission error depend on 32 manufacture parameters:

$$p_{max}(\boldsymbol{mp}) = p_{max}\left(r_{prof\,1}, r_{prof\,2}, \sum_{i=1}^{i=6}\sum_{j=0}^{j=5} c_{ij}\right)$$
$$\Delta\phi_{2max}(\boldsymbol{mp}) = \Delta\phi_{2max}\left(r_{prof\,1}, r_{prof\,2}, \sum_{i=1}^{i=6}\sum_{j=0}^{j=5} c_{ij}\right) \qquad (3)$$

An optimization method is applied to systematically define optimal head-cutter geometry and machine tool settings to simultaneously minimize maximum tooth contact pressure and angular displacement error of the driven gear. The goal of the optimization is to minimize tooth contact pressure and transmission errors while keeping the loaded contact pattern inside the physical tooth boundaries of the pinion and the gear. The optimization problem to be solved is as follows:

$$\min_{mp} f(\boldsymbol{mp}) = \min_{mp}\left[c_p \cdot \frac{p_{max}(\boldsymbol{mp})}{p_{max0}} + c_\phi \cdot \frac{\Delta\phi_{2max}(\boldsymbol{mp})}{\Delta\phi_{2max0}}\right] \qquad (4)$$

$$\text{subject to } C(\boldsymbol{mp}) = 0$$

where $C(\boldsymbol{mp})$ represents the constraints, p_{max0} and $\Delta\phi_{2max0}$ are the maximum tooth contact pressure and transmission error obtained for the initial values of manufacture parameters; c_p and c_ϕ are non-negative weight coefficients, expressing the relative importance of p_{max} and $\Delta\phi_{2max}$, respectively.

Functions $f(\boldsymbol{mp})$ and $C(\boldsymbol{mp})$ are not available analytically, they exist numerically through the load distribution calculation. Therefore, the computer simulation of load distribution must be run, repeatedly, in order to compute the various quantities needed by the optimization algorithm. The load distribution calculation is based on a highly nonlinear system of equations [17]. An iterative technique was used to solve this system of equations. This causes that the calculation of partial derivatives for gradient-based optimization algorithms to be quite impractical. For this reason, a nonderivative method is selected to solve this particular optimization problem. The pattern search method is used.

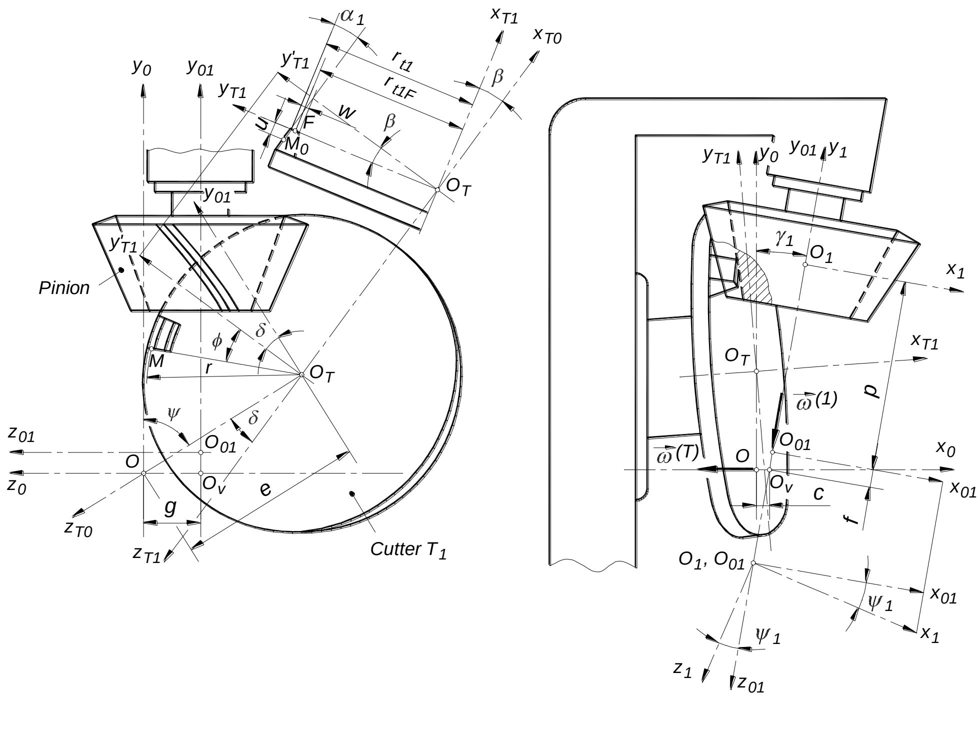

Figure 1. Machine tool setting for pinion teeth finishing

The CNC machine for generation of spiral bevel and hypoid gears is provided with six degrees-of-freedom for three rotational motions (θ, ζ, η), and three translational motions (*X, Y, Z,* Fig. 2). The six axes of CNC generator are directly driven by the servo motors and able to implement prescribed functions of motions. The face-hobbing method requires simultaneous six-axis control, the face-milling method requires only five-axis control. The following coordinate systems are applied to describe the relations and motions in the CNC generator (Fig. 2): Coordinate systems $K_t(x_t, y_t, z_t)$ and $K_i(x_i, y_i, z_i)$ are rigidly connected to the head cutter and the pinion/gear, respectively. The coordinate transformation from system K_t to system K_i performs the following equation:

$$\vec{r}_i = M_i(\eta)\cdot M_{ti0}(\zeta, X, Y, Z)\cdot M_t(\theta)\cdot \vec{r}_t = M_{ti}^{CNC}\cdot \vec{r}_t \quad (5)$$

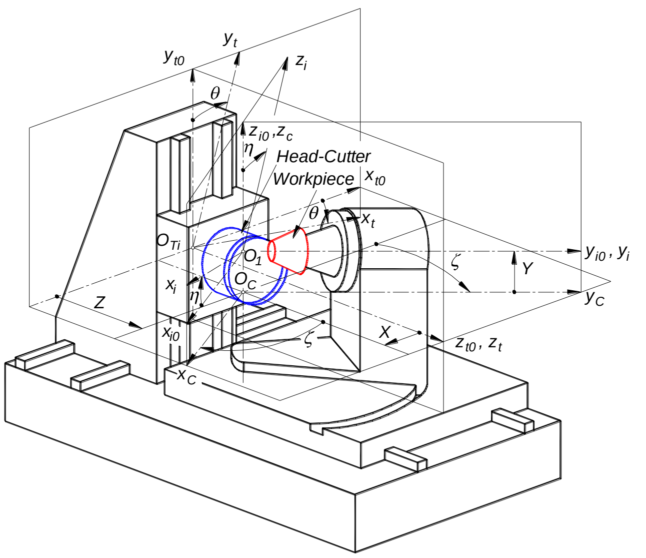

Figure 2. Machine-tool setting for pinion tooth-surface finishing on CNC generator

The location and the orientation of the tool with respect to the pinion/gear are given in coordinate systems that are represented for a conventional, cradle-type generator (Fig. 1). An algorithm is developed for the execution of motions on the CNC generator using the relations valid for the cradle-type machine. This algorithm is based on the conditions that the relative position of the axes of the head-cutter and the pinion rotations, z_{t0} and y_{i0} , and the axial relative position of the head cutter and the pinion/gear should be the same whether the pinion/gear is cut on a cradle-type or a CNC hypoid generator.

To ensure the same relative position of the two axes, z_{t0} and y_{i0} , on both the cradle-type and CNC hypoid generating machines, the elements of the coordinate transformation matrices should be equal. On the basis of Eqs. (1) and (5) the following condition should be satisfied:

$$\vec{e}_{i0}^{(z_{t0})} = M_{i2}\cdot M_{i1}\cdot M_{c4}\cdot M_{c3}\cdot M_{c2}\cdot \vec{e}_{t0}^{(z_{t0})} = M_{ti0}(\zeta, X, Y, Z)\cdot \vec{e}_{t0}^{(z_{t0})} \quad (6)$$

The same relative position of the head cutter and the pinion along their axes in the case of both machines, is satisfied by applying the following condition

$$\vec{r}_{i0}^{(O_t)} = M_{i2}\cdot M_{i1}\cdot M_{c4}\cdot M_{c3}\cdot M_{c2}\cdot \vec{r}_{t0}^{(O_t)} = M_{ti0}\cdot \vec{r}_{t0}^{(O_t)} \quad (7)$$

A computer program was developed to implement the formulation provided above. By applying this program the optimal profile of the head-cutter and the optimal values of machine tool settings were calculated. The main design data of the example hypoid gear pair used in this study are given in Table 1. The transmitted torque was *20 Nm*.

TABLE 1. Pinion and gear design data

Design parameters	Pinion	Gear
Number of teeth	10	41
Module, mm	3.4	
Pinion offset, mm	35	
Pressure angle, deg	20	
Mean spiral angle, deg	52	27.3
Face width, mm	37.8	31

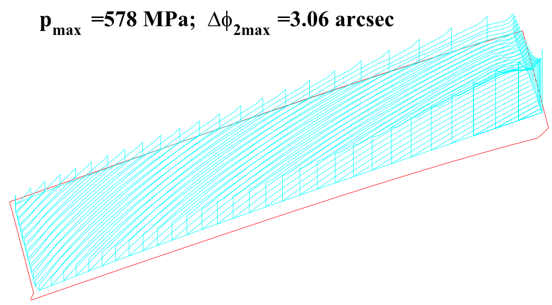

Figure 3. Tooth contact pressure distributions when the pinion and gear tooth surfaces are fully conjugate

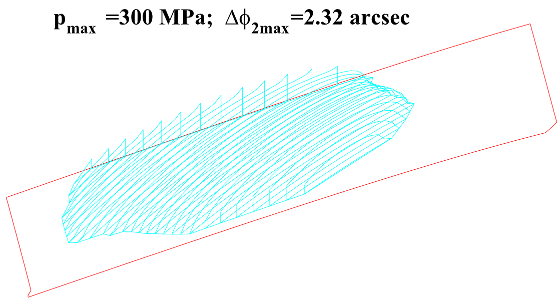

Figure 4. Tooth contact pressure distributions when the optimized head-cutter profile and machine tool settings are applied

The load distribution calculation was performed for 21 instantaneous positions of the pinion and the gear rolling through a mesh cycle. The tooth contact pressure distributions along the potential contact lines for 21 instantaneous positions and for all the adjacent tooth pairs engaged for a particular position of the mating members, for the case when no modifications are introduced into the pinion teeth, namely straight-lined head-cutter profile and the basic values of machine tool settings are applied, are shown in Fig. 3. In this case the pinion and gear tooth surfaces are fully conjugate. The obtained maximum tooth contact pressure is *578 MPa* and the maximum angular displacement error of the driven gear is *3.06 arcsec*. The tooth contact pressure distribution for the case when the pinion teeth are manufactured by the head-cutter of optimized profile and by optimal values of machine tool settings is shown in Fig. 4. It can be observed that the maximum tooth contact pressure is reduced to $p_{max} = 300\,MPa$ and the maximum transmission error to $\Delta\phi_{2max} = 2.32\ arc\sec$.

III. Improvements in the Precision of Worm Gear Manufacture

The worm gears can be manufactured by a hob or by a flying tool. Because of the finite number of cutting edge of the hob and in the case of the use of a flying tool, geometrical errors will occur on the processed gear tooth surface. The possibilities to reduce these tooth surface errors will be investigated in this section.

The kinematics of worm gear teeth processing by a hob is shown in Fig. 5. The coordinate systems in the case of flying tool are the same as presented in Fig. 5.
The gear tooth surface, shaped by the cutting edge of the hob or by the flying tool is defined by the following equation

$$\vec{\boldsymbol{r}}_g^{(c.e.)} = \boldsymbol{M}_{hg} \cdot \vec{\boldsymbol{r}}_h^{(c.e.)} \tag{8}$$

where matrix $\boldsymbol{M}_{hg}$ transforms the coordinates from system K_h (attached to the hob) to system K_g (attached to the gear) and $\vec{\boldsymbol{r}}_h^{(c.e.)}$ is the position vector of the points of the cutting edge of the hob or of the flying tool.

The gear tooth surface, which will be fully conjugated with the worm surface, is defined by the system of equations

$$\begin{aligned} &\vec{\boldsymbol{r}}_g^{(g)} = \boldsymbol{M}_{wg} \cdot \vec{\boldsymbol{r}}_w^{(w)} \\ &\vec{\boldsymbol{n}}_w^{(w)} \cdot \vec{\boldsymbol{v}}_w^{(w,g)} = 0 \end{aligned} \tag{9}$$

where matrix $\boldsymbol{M}_{wg}$ transforms the coordinates from system K_w (attached to the worm) into the system K_g (attached to the gear), $\vec{\boldsymbol{r}}_w^{(w)}$ is the position vector of worm surface points, $\vec{\boldsymbol{n}}_w^{(w)}$ is the normal vector of the worm surface, and $\vec{\boldsymbol{v}}_w^{(w,g)}$ is the relative velocity vector of the worm to the gear.

The tooth surface errors of a worm gear processed by a hob of finite number of cutting edges or by a flying tool can be calculated by the equation

$$f_t = \sqrt{\left(x_g^{(g)} - x_g^{(c.e.)}\right)^2 + \left(y_g^{(g)} - y_g^{(c.e.)}\right)^2 + \left(z_g^{(g)} - z_g^{(c.e.)}\right)^2} \tag{10}$$

The calculations were made for the commonly used thread-ground worm gear drive. The investigations have shown that a considerably gear tooth surface error reduction can be achieved by the increase of the number of hob teeth (Fig. 6). But, to ensure the strength of hob teeth and the tool life, the number of hob teeth is usually limited to 6, eventually for larger hob diameters it can be 8, 10, or more. In the case of worm gear teeth processing by a flying tool, the role of hob tooth number is replaced by the axial shift of the flying tool. It is desirable a relatively small amount of the axial shift of the flying tool (Fig. 7), but by the reduction of this shift the gear tooth processing time and the cost of gear manufacture are increased. A compromise should be found.

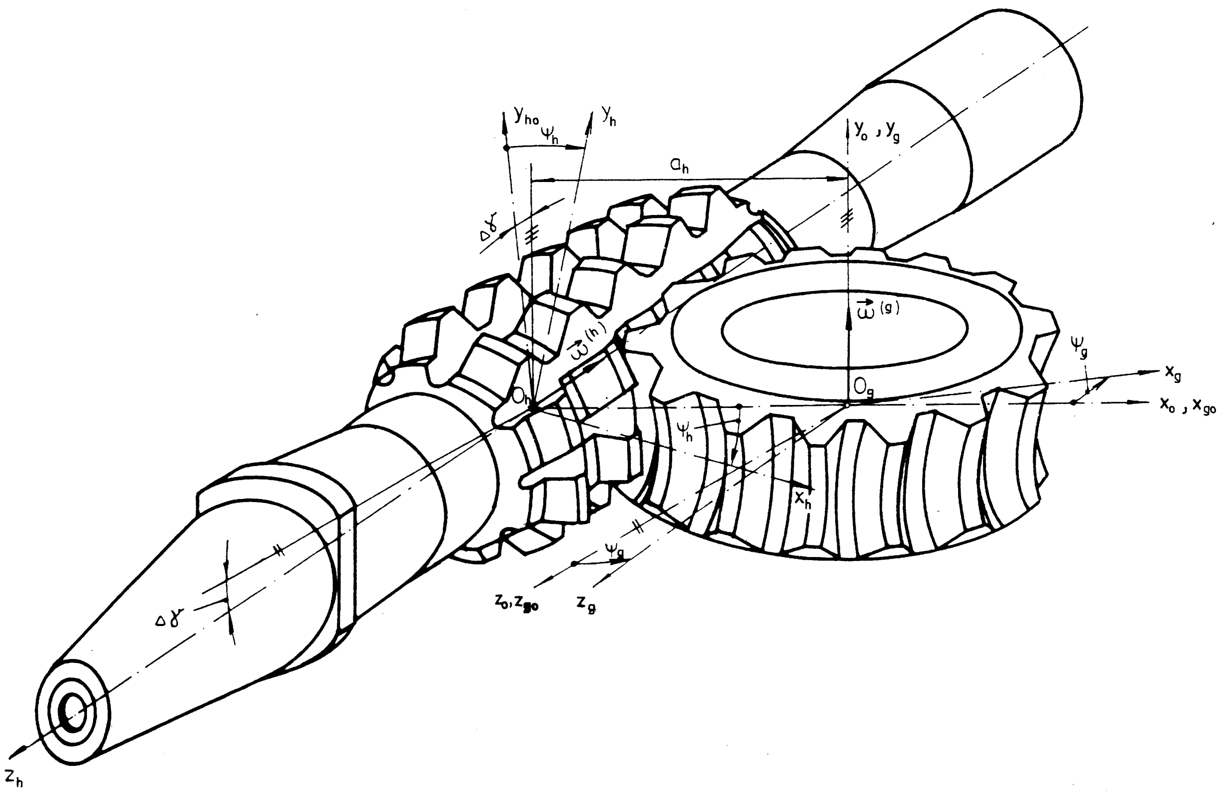

Figure 5. Worm gear teeth processing by hob

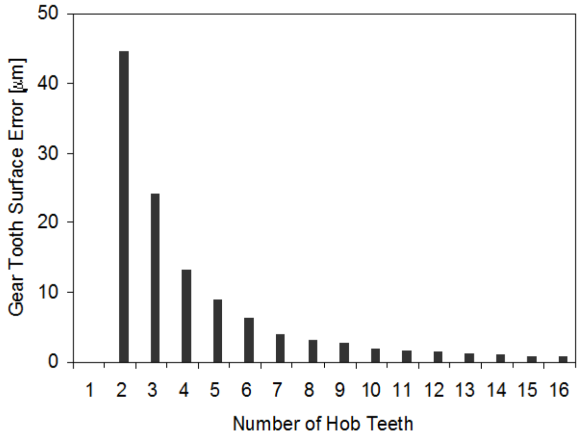

Figure 6. Influence of hob tooth number on maximal gear tooth surface errors

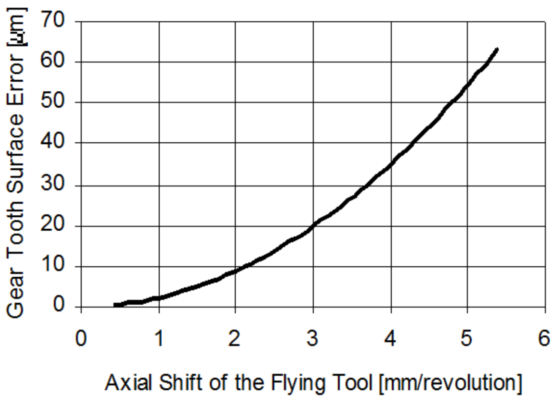

Figure 7. Influence of the axial shift of flying tool on maximal gear tooth surface errors

IV. Conclusions

Based on the obtained results the following conclusions can be made:

1. In the case of the hypoid gear pair, manufactured by the head-cutter with optimal profile and optimal machine tool settings, considerable reduction in the maximum tooth contact pressure of 48% and in the transmission error of 24% were obtained.

2. In the case of worm gears, the errors of processed gear tooth surface can be considerably reduced by the increase of the number of hob teeth (N_h) and by the reduction of the axial shift of the flying tool.

References

[1] V. Simon, "Optimal Tooth Modifications for Spur and Helical Gears", ASME J. Mech. Trans. Automat. Design, vol., 111, pp. 1989, 611-615.

[2] J. Bruyère, P. Velex, "A Simplified Multi-Objective Analysis of Optimum Profile Modifications in Spur and Helical Gears", Mech. Mach. Theory, vol. 80, 2014, pp. 70-83

[3] C. Lohmann, M. Walkowiak, P.J. Tenberge, Gear Techn., "Optimal Modifications on Helical Gears for Good Load Distribution and Minimal Wear", Gear Technology, 2015(6), pp. 54-59.

[4] Y. P. Shih, Z. H. Fong, "Flank Correction for Spiral Bevel and Hypoid Gears on a Six-Axis CNC Hypoid Generator", ASME J. Mech. Des., vol. 130, 2008, Art. No. 062604, pp. 1-8.

[5] A. Artoni, M. Kolivand, A. Kahraman, "An Ease-off Based Optimization of the Loaded Transmission Error of Hypoid Gears", ASME J. Mech. Des., vol. 132, 2010, Art. No.011010, pp. 1-9.

[6] S. Mo, Y. Zhang, "Spiral bevel gear true tooth surface precise modeling and experiments studies based on machining adjustment parameters", Proc. Inst. Mech. Eng. Part C – J. Mech. Eng. Sc., vol. 229(14), 2015, pp. 2524-2533.

[7] Z. C. Chen, M. Wasif, "A generic and theoretical approach to programming and post-processing for hypoid gear machining on multi-axis CNC face-milling machines", Int. J. Adv. Manu. vol. 81, 2015, pp. 135-148.

[8] K. Kawasaki, T. Isamu, H. Gunbara, H. Houjoh, "Method for Remanufacturing Large-Sized Skew Bevel Gears Using CNC Machining Center", Mech. Mach. Theory, vol. 92, 2015, pp. 213-229.

[9] R. Tan, B. Chen, C. Peng, "General Mathematical Model of Spiral Bevel Gears of Pure-Rolling Contact", Proc. Inst. Mech. Eng. Part C – J. Mech. Eng. Sc., vol. 229(15), 2015, pp. 2810-2826.

[10] G.F. Bär, "On Optimizing the Basic Geometry of Hypoid Gears", Mech. Mach. Theory, vol. 104, 2016, pp. 274-286.

[11] W. Guo, S. Mao, Y. Yang, Y. Kuang, "Optimization of Cutter Blade Profile for Face-Hobbed Spiral Bevel Gears", Int. J. Adv. Manu. Tech., vol. 85, 2016, pp. 209-216.

[12] H. Ding, J.Y. Tang, Z.Y. Zhou, W. Cui, "Tooth Flank Reconstruction and Optimization after Simulation Process Modeling for the Spiral Bevel Gear", Proc. Inst. Mech. Eng. Part C – J. Mech. Eng. Sc., vol. 230(13), 2016, pp. 2260-2272.

[13] P. Wang, Y. Zhang, M. Wan, "Global Synthesis for Face Milled Spiral Bevel Gears with Zero Transmission Errors", ASME J. Mech. Des., vol. 138, 2016, Art. No. 033302, pp. 1-9.

[14] Y. Hiltcher, M. Guingand, J.P. de Vaujany, "Numerical Simulation and Optimisation of Worm Gear Cutting", Mech. Mach. Theory, vol. 41, 2006, pp. 1090-1110.

[15] J. Sohn, N. Park, "Geometric Interference in Cylindrical Worm Gear Drives Using Oversized Hob to Cut Worm Gears", Mech. Mach. Theory, vol. 100, 2016, pp. 83-103.

[16] L. Skoczylas, P. Pawlus, "Geometry and Machining of Concave Profiles of the ZK-Type Worm Thread", Mech. Mach. Theory, vol. 95, 2016, pp. 35-41.

[17] V. Simon, "Load Distribution in Hypoid Gears", ASME J. Mech. Des., vol. 122, 2000, pp. 529-535.

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