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Control of dynamic stability of stays using passive viscous dampers

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***Abstract*—Since cable vibrations cause cable-stayed bridge users discomfort and may also lead to bridge collapse, assessment and control of dynamic behavior of stays are key aspects in designing such a bridge type. Mounting viscous dampers close to deck anchorages is an efficient way to control all kind of cable vibrations. In the paper, relevant issues as parametric excitation, external excitation and cable-structure interaction are investigated in order to define the required damping ratio to control the dynamic stability of stays. The study ends with a damper design example, referring to a relevant case study.**

***Keywords*—cable-stayed bridge, cable vibrations, cable-structure interaction, viscous damper.**

I. Introduction

Assessment and control of dynamic behavior of stays are key aspects in designing cable stayed bridges, as demonstrated by several vibration episodes occurred in all over the world as those described in [1]. Moreover, since the stays are very flexible elements, with small damping coefficients ($\xi \leq 0.50\%$), they are very vulnerable to vibrations induced by dynamic actions.

Aim of this paper is to investigate the dynamic stability of stays under indirect excitation and to define a possible countermeasure for its improvement.

Considering that mounting viscous dampers close to deck anchorages is an efficient passive measure to control all kind of cable vibrations, this control technique is discussed and the design of a suitable viscous damper is illustrated referring to a relevant cases study.

II. Cable vibration phenomena

In cable stayed bridges, dynamic actions induced by wind, traffic and earthquakes produce also an *indirect excitation* of cables through the motion of their anchorages. Generally two kind of excitations are distinguished: *external* and *parametric*. The former corresponds to a motion of anchorages perpendicular to the cable chord, the latter corresponds to a motion of anchorages in the direction of cable chord. Designers also have to verify *cable-structure interaction*: if some cable frequencies are close to bridge global frequencies, an internal resonance could occur.

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III. The case study

The considered case study refers to a cable-stayed bridge designed to cross the river Arno in Figline Valdarno, a town near Florence [2]. This self-anchored cable-stayed bridge has a total length of 281 m with three spans respectively of 44 m, 37 m and 200 m (Fig. 1). The S355 steel box girder is trapezoidal, 2.00 m in depth and 14.96 m width (Fig. 2), and the 12 mm thick orthotropic steel deck plate is reinforced with trapezoidal stiffeners spaced 600 mm center to center.

The deck is suspended to a pylon via 17 pairs of locked-coil strands forming an asymmetric semi-fan; these cables are 15.0 m spaced on the girder and 3.75 m on the pylon. The concrete A-shaped pylon has a total height of 96.00 m. All substructures are founded on piles with a diameter of 1.60 m.

Structural analysis has been carried out with SAP2000®.

IV. Case analysis

A. *Natural frequencies of cables*

Natural frequencies of cables have been evaluated with Mehrabi and Tabatabai formula [3] as reported in a companion paper [4]. Table I summarizes these results, together with cable inclination (θ), cable chord length (L), cable sag at midpoint (f), cable diameter (D) and cable tension at the dead load configuration (T). These cables are sensibly stretched, so in the analysis an equivalent stiffness has been taken into account through the Irvine parameter [5], without resorting to more refined theoretical models [6].

TABLE I. NATURAL FREQUENCIES OF CABLES.

Stay	θ (°)	L (m)	f (m)	D (mm)	T (kN)	f_1 (Hz)
1	48	122	0.750	230	7335	0.717
2	51	106	0.247	55	942	1.129
3	54	88	0.198	55	798	1.264
4	58	69	0.172	230	10 318	1.459
5	66	52	0.069	60	964	2.181
6	65	51	0.078	65	965	2.059
7	54	62	0.156	65	707	1.451
8	46	74	0.193	65	821	1.291
9	40	88	0.264	70	971	1.106
10	37	102	0.325	70	1062	0.995
11	34	116	0.401	70	1123	0.895
12	31	131	0.493	75	1334	0.809
13	30	146	0.589	75	1383	0.740
14	28	161	0.694	75	1428	0.683
15	27	176	0.791	75	1500	0.640
16	26	191	0.935	75	1499	0.591
17	25	206	0.872	70	1628	0.605

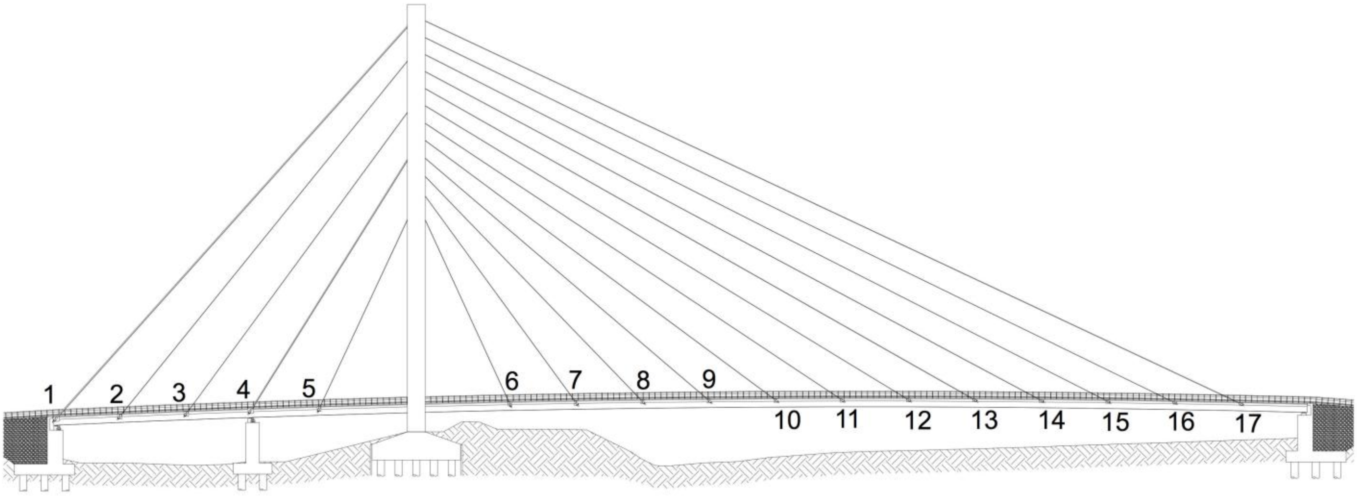

Figure 1. The cable-stayed bridge analyzed.

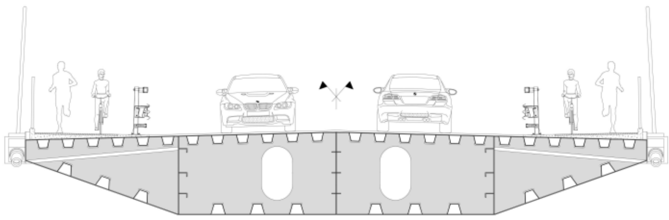

Figure 2. The bridge deck.

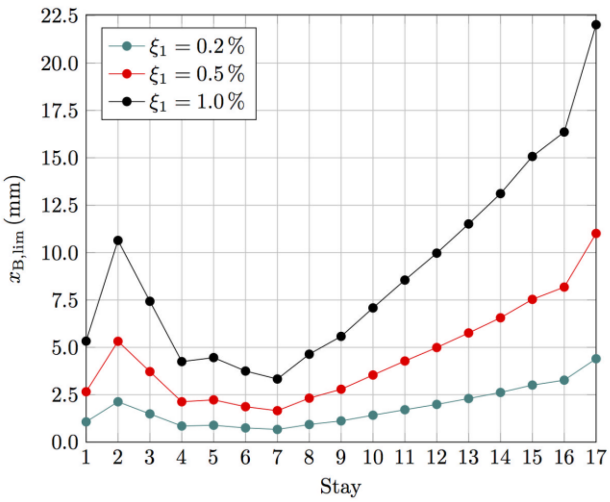

Figure 3. Threshold amplitude of parametric oscillation for δ=1.

B. *Parametric excitation*

As known, the usual approach to the study of parametric excitation consists of an evaluation of the cable resonance condition from dynamic equilibrium equations of a single cable under harmonic motion of its supports.

The response to a harmonic of frequency ω is not increased exclusively at resonance (when the fundamental frequency of cable $\omega_1=\omega$), but also at specific ratios $\beta=\omega/\omega_1=2/\delta=1/r=\{1/2;1/3;2;3\}$ [1].

Considering the *first parametric resonance* in correspondence of a subharmonic of order 1/2 (δ^2=1, r=1/2, β=2), the threshold amplitude for occurrence parametric excitation is given by

$$x_{\mathrm{B,lim}} = 4\xi_1 X_0 \qquad (1)$$

where $X_0 = TL/(EA_0)$ is the elastic elongation of the cable [1]. The threshold amplitude of oscillation has been calculated for bridge stays considering three possible damping coefficients: 0.2 %, 0.5 % and 1.0 % (Fig. 3). These values are lower for stiffer stays (#4 to #7).

Fig. 4 shows amplitudes of steady-state oscillation $a(x_B)$

$$a(x_B) = \frac{4}{\pi}\sqrt{\frac{X_0 L}{3}}\sqrt{1-\delta^2+\sqrt{\delta^4\left(\frac{x_B}{2X_0}\right)^2-4\delta^2\xi_1^2}} \qquad (2)$$

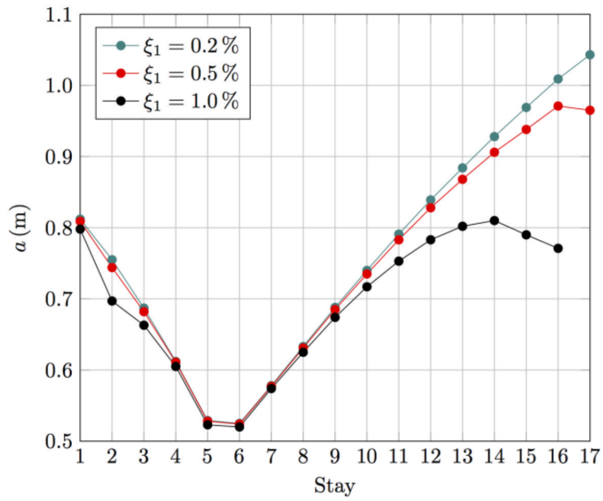

Figure 4. Amplitudes of steady-state parametric oscillation for x_B = 20 mm.

for a sinusoidal excitation of 20 mm amplitude, at twice the cable frequency (δ=1) [1].

A damping ratio of 1.0 % affects only the behavior of weaker stays. In general, damping is important only to prevent parametric excitation: once oscillation sets up the amplitude is almost damping-independent [1], as clearly shown in Fig. 5 for stay #7.

C. *External excitation*

External excitation induces amplitudes of vibration almost halving those induced by parametric excitation, which are therefore considered the most important [1]. This aspect appears clearly comparing diagram in Fig. 6, referring to amplitude of steady state external excitation of stay #7, with the previously discussed Fig. 5.

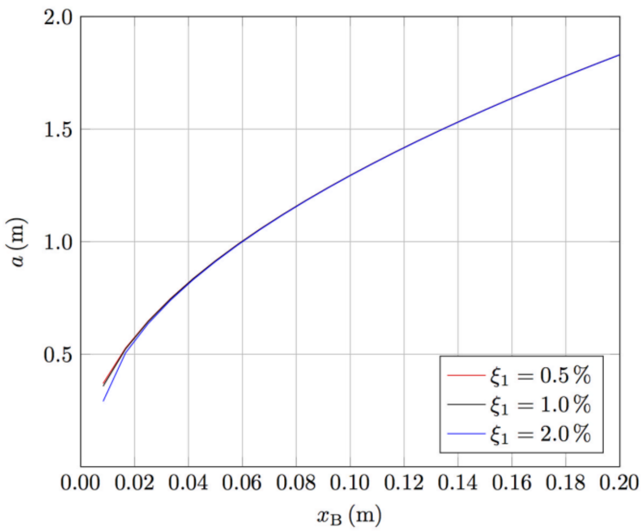

Figure 5. Amplitudes of steady-state parametric oscillation for stay #7.

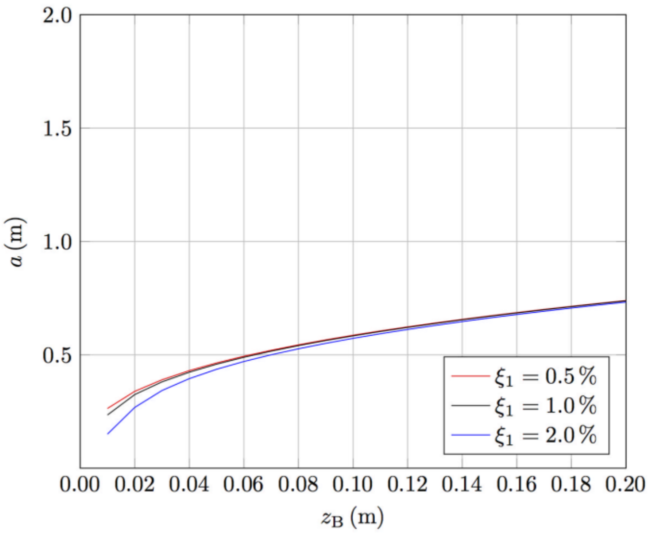

Figure 6. Amplitudes of steady-state external oscillation for stay #7.

Amplitudes of steady-state external oscillation for stay #7 have been calculated through the solution developed by Nayfeh and Mook in the vicinity of *primary resonance* ($r \approx 1$) [7]:

$$a(z_B) = \frac{z_B}{\pi \xi_1} C \sin\gamma \qquad (3)$$

$$C = \frac{1 - \dfrac{\lambda^2}{\pi^2}}{\sqrt{1 + \dfrac{1}{2}\left(\dfrac{2}{\pi}\right)^4 \lambda^2}}. \qquad (4)$$

The phase of response γ is obtained by numerical solution of

$$\sin^2\gamma \tan\gamma = \frac{32}{3} X_0 L \frac{\left[1 + \dfrac{1}{2}\left(\dfrac{2}{\pi}\right)^4 \lambda^2\right]^4}{\left[1 - \dfrac{\lambda^2}{\pi^2}\right]\left[1 - 32\dfrac{\lambda^2}{\pi^4}\right]} \frac{\xi_1^3}{z_B^2}. \qquad (5)$$

D. *Cable-structure interaction*

Cable-structure interaction can be studied through the ratio r of fundamental cable frequency f_{1c} to bridge global frequency f_b. The value $r \approx 1$ defines the region where global modes may provide external excitation. Subharmonic ($r \approx 0.5$) and superharmonic ($r \approx 2$) resonance conditions bound two nonlinear interaction regions, where global modes may provide parametric and angle variation excitation of local modes respectively [8,9]. The excitation from angle variation between cable tension and bridge girder is a phenomenon detailed by Gattulli and Lepidi and Gattulli et al. [9,10].

Fig. 7 and Fig. 8 show values of r considering each cable frequency (Table I), ten first vertical global modes and first torsional mode (Table II). It emerges that: stays from #3 to #11, #16 and #17 are vulnerable to *external excitation* of six first vertical global modes; all stays are vulnerable to *parametric excitation* of higher global modes; and stays #2 and #9 are vulnerable to angle variation excitation.

TABLE II. BRIDGE GLOBAL FREQUENCIES.

Mode	f_b (Hz)	Description
1	0.543	1° vert. (V1)
2	0.870	1° lat. (L1)
3	0.913	2° lat. (L2)
4	0.946	2° vert. (V2)
5	1.311	3° vert. (V3)
6	1.556	4° vert. (V4)
7	1.921	5° vert. (V5)
8	1.923	1° tors. (T1)
9	1.993	3° lat. (L3)
10	2.107	6° vert. (V6)
11	2.820	7° vert. (V7)
12	2.892	4° lat. (L4)
13	2.995	8° vert.(V8)
14	4.142	9° vert. (V9)
15	4.678	5° lat. (L5)
16	4.809	10° vert. (V10)
17	5.179	1° long. (Lo1)
18	5.284	2° tors. (T2)
19	5.474	11° vert. (V11)
20	5.786	6° lat. (L6)

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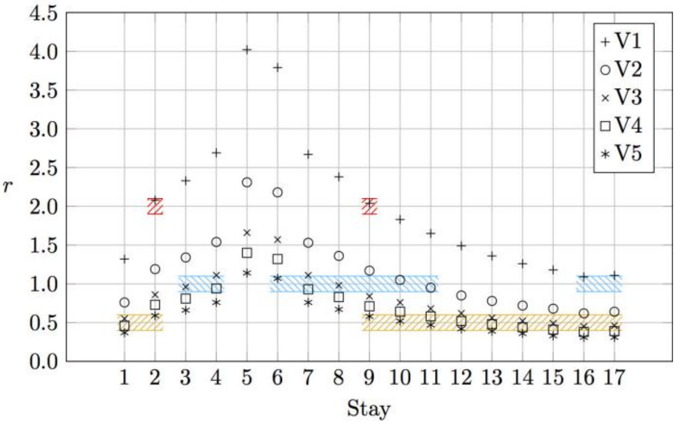

Figure 7. Ratio $r = f_{1c}/f_{\text{bridge}}$ for modes V1-V5.

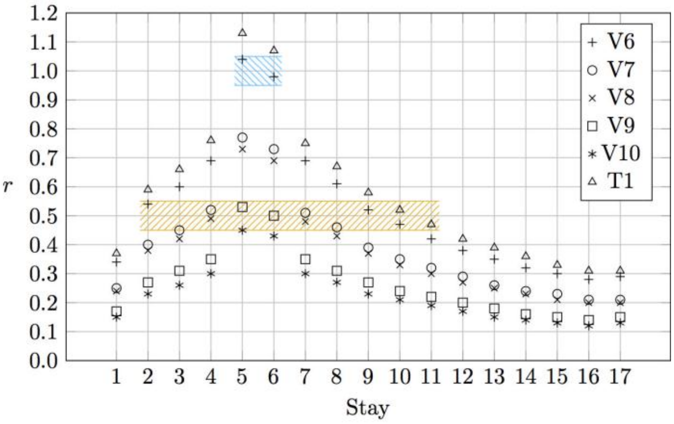

Figure 8. Ratio $r = f_{1c}/f_{\text{bridge}}$ for modes V6-V10 and T1.

V. Damper design

As already anticipated, mounting viscous dampers close to deck anchorages is an efficient way to control all kind of cable vibrations. Moreover this is the least expensive technique in retrofitting existing bridge suffering from wind/rain-wind induced vibrations. Therefore, the evaluation of damping needed is crucial in preventing large cable vibrations.

For all stays a damping ratio $\xi_1 > 0.3$ % is generally recommended as appropriate to control dry inclined galloping and rain wind induced vibrations [4].

However for cable stays longer than 80 m provisions should be made for the installation of dampers to ensure that critical damping ratios exceed 0.5 % in the first mode [11].

Focusing on the longest stay (#17, L=206 m), neglecting intrinsic structural damping and taking account of all vibration modes in the range (0.3÷3) Hz [1], a damping ratio $\xi > 0.3\%$ is requested for the first five modes:

$$\left(\mathbf{f} = [0.6\,\text{Hz} \quad 1.19\,\text{Hz} \quad 1.78\,\text{Hz} \quad 2.38\,\text{Hz} \quad 2.97\,\text{Hz}]^T\right).$$

The simplified formula derived by Fujino and Hoang [1,12] was used in the design of an optimal passive damper:

$$\frac{\xi_n}{x_c/L} = R_n\, R_{EJ}\, R_{k\text{EJ}} \frac{\eta_n \eta_{k\text{EJ}}}{1+(\eta_n \eta_{k\text{EJ}})^2} \tag{6}$$

$$R_n = \begin{cases} 1 & \text{if } n \text{ is even,} \\ \dfrac{\left[\tan\left(\dfrac{k_n\pi}{2}\right) - \left(\dfrac{k_n\pi}{2}\dfrac{x_c}{L}\right)\right]^2}{\tan^2\left(\dfrac{k_n\pi}{2}\right) + \dfrac{12}{\lambda^2}\left(\dfrac{k_n\pi}{2}\right)^2} & \text{if } n \text{ is odd.} \end{cases} \tag{7}$$

$$R_{EJ} \approx \frac{(1-q)^2}{1-q-0.5rq^2} \tag{8}$$

$$r = \zeta \frac{x_c}{L} \tag{9}$$

$$q = \frac{1-\exp(-r)}{r} \tag{10}$$

$$R_{k\text{EJ}} = \frac{\overline{\eta_k}\,\eta_{\text{EJ}}}{1+\overline{\eta_k}\,\eta_{\text{EJ}}} \tag{11}$$

where

ζ is a bending stiffness coefficient appearing in the Mehrabi and Tabatabai formula;

x_c is the distance of the damper from the cable anchorage;

$\eta_n = \eta k_n \pi x_c / L$ is a non-dimensional damping parameter;

$k_n = \omega_n/\omega_{01} = f_n/f_{01}$ is the ratio of n-th cable frequency to the fundamental frequency of corresponding taut string $\left(\omega_{01} = \dfrac{\pi}{L}\sqrt{\dfrac{T}{m}}\right)$;

R_n is a reduction factor due to sag effect;

R_{EJ} is a reduction factor due to bending effect;

$\eta_{k\text{EJ}} = \eta_{\text{EJ}} + 1/\overline{\eta_k}$ is a non-dimensional damping parameter related to the stiffness k of support and to the bending stiffness of damper;

$\eta_{\text{EJ}} = 1 - q - 0.5rq^2$ is a non-dimensional damping parameter related to the bending stiffness of the damper;

$\overline{\eta_k} = (k\,x_c)/T$ is a non-dimensional damping parameter related to the stiffness k of support;

$R_{k\text{EJ}}$ is a reduction factor associated with the support stiffness.

Assuming that viscous damper is linear, the maximum modal damping ratio is obtained by

$$\frac{\xi_{n,\max}}{x_c/L} = 0.5 R_n\, R_{EJ}\, R_{k\text{EJ}} \tag{12}$$

and occurs for

$$\eta_{n,\text{opt}} = \frac{1}{\eta_{k\text{EJ}}\, k_n}. \tag{13}$$

With damper inserted in a deviation guide of height h=2.5 m and an inclination of cable chord θ=25°, $x_c = h/\sin\theta = 6$ m and $x_c/L = 0.03$. For stay #17 ζ=609, so R_{EJ}=0.97 and

$$\mathbf{k} = [1.020 \quad 2.007 \quad 3.010 \quad 4.013 \quad 5.017]^T$$

$$\mathbf{R_n} = [0.94 \quad 1.00 \quad 0.87 \quad 1.00 \quad 0.45]^T.$$

Neglecting support stiffness, the achievable maximum modal damping ratio is

$$\boldsymbol{\xi}_{\mathbf{max}} = [1.313\% \quad 1.397\% \quad 1.215\% \quad 1.397\% \quad 0.629\%]^T$$

occurring for

$$\boldsymbol{\eta}_{\mathbf{opt}} = [1.07 \quad 0.54 \quad 0.36 \quad 0.27 \quad 0.22]^T.$$

Setting $\xi_5 = 0.3$ in (6) and solving for η, it results $\eta = 0.61$. Knowing that

$$\eta = \frac{\pi c}{m L \omega_{01}} \tag{14}$$

and that for stay #17 ω_{01}=3.54 rad/s, the requested damper size is

$$c = 59627 \text{ Ns/m} \approx 60 \text{ kNs/m}.$$

Finally, referring to the so-called *universal curve* reported in Fig. 9, the abscissas

$$x_n = \frac{\eta n \pi x_c/L}{\pi^2} \tag{15}$$

can be evaluated,

$$\mathbf{x} = [0.88 \quad 1.72 \quad 2.58 \quad 3.44 \quad 4.31]^T,$$

so obtaining the modal damping ratios

$$\boldsymbol{\xi} = [1.28\% \quad 1.26\% \quad 0.87\% \quad 0.81\% \quad 0.30\%]^T,$$

which demonstrate that a critical damping ratio ξ_1=1.28 % is sufficient to control all kind of cable vibrations of stay #17.

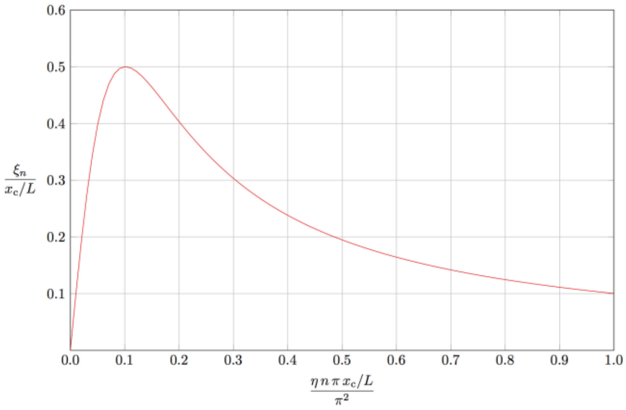

Figure 9. Universal curve relating modal damping ratio ξ_n with damper size c, location of damper x_c and cable parameters m, L and ω_{01} [1].

VI. Concluding remarks

The dynamic stability of stays under indirect excitation has been investigated for a relevant case study. In particular, parametric excitation, external excitation and cable-structure interaction have been considered.

The results confirm that damping is very significant to prevent parametric excitation, while external excitation is less important because it induces smaller amplitudes of vibration. Moreover, cable-structure interaction is a relevant matter for the considered bridge. At the end, a viscous damper has been designed as possible countermeasure for the improvement of dynamic stability of stays.

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