

Taxation, Corruption and Punishment: A Game Equilibrium Model

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Abstract— This work examines the issue of tax evasion through underreporting activity. We assume that the decisions are described by an evolutionary dynamic process. The game considers that both citizens and officers can either ask for a bribe or be asked for it. The structure of the evasion-corruption game explicitly distinguishes between the probability to detect and punish a dishonest citizen (q) and a dishonest officer (p). These two parameters strongly characterize the game and can be considered as proxy of the efficacy of the institutional system and supply an interesting indication about the corruption and evasion behaviour in a certain country. We analyse the relationship between tax evasion and efficacy of the institutional country through the basins of attraction of the game strategies.

Keywords—evolutionary game, replicator dynamic, corruption model, bribery.

I. Introduction

Recently, the economics of corruption has attracted considerable attention among academics and policy makers. Over the last decades a vast body of theoretical and empirical literature has investigated the causes of corruption, with the aim of identifying policy measures that might be successful in its reduction. A widespread consensus has emerged on the detrimental effects that corruption has on the economic system, and more attention is being given to the social costs of corruption. The incidence and the degree of corruption differs substantially across countries and societies, depending on institutions and morality. According to [1], [2] for instance, countries with higher tax revenues achieved by lower tax rates, fewer laws and regulations and less bribery facing enterprises have smaller shadow economies. Similarly, countries with a better rule of law, which is financed by tax revenues, also have smaller shadow economies. Their overall conclusion is that “wealthier countries” accomplish a “good equilibrium” of relatively low tax and regulatory burden, sizeable revenue mobilization, good rule of law and corruption control. By contrast, there are a number of countries (particularly in Latin American and the former Soviet Union) which accomplish a “bad equilibrium” with high levels of tax and regulatory discretion and burden on the firms. These countries have weak rule of law and a high incidence of bribery and shadow economy.

Moreover, the quality and efficiency of the public apparatus also affect the size of the illegal economy because it impacts tax morale. Tax morale commonly refers to the fraction of tax compliance which cannot be explained by standard choice motivations and deterrence policies.

According to [3] tax compliance is the result of a complex interaction between tax morale and deterrence measures. These latter ought to be transparent and fair to taxpayers ([4], [5]). Poorly weak or excessively complex tax systems in a country could lead to market failures, resulting from lack of contract enforcement, expensive negotiation and compliance costs, and inefficient search and information asymmetries ([6]). This could easily incentivise the agents to resort to lawless ways to comply ([7], [8], [3]). Tax morale improves if citizens feel the public services received in exchange for their tax payments are worth it and if political decisions and the rule of law is perceived to be equitable ([9]). Hence tax morale is an intrinsic part of the system, as it is influenced by deterrence, the quality of public institutions and the legal system. This indeed makes the differences among countries.

For instance, according to the Transparency International Corruption Perception Index (CPI), Finland is among the most virtuous country in the world in terms of public sector integrity, and minute corruption is considered practically non-existent (Group of European States against corruption). By contrast, Italy is world 69th in the world, and corruption is deeply rooted in civil society, public administration and private sector. One can expect that between these two countries there is a substantial difference in terms of criminal prosecution. However, they appear very similar in terms of the density of judicial corruption investigation (0.4 per 100,000 inhabitants). They also appear very similar in terms of frequency of convictions per inhabitants. This picture represents a robust indication of the inefficiency of the judicial apparatus to make effective the Sanctions/Fines in Italy, where a citizen has roughly the same probability of a Finnish of being investigated and condemned for corruption but where illegal activities are more diffused.

This work examines the issue of tax evasion through underreporting activity. We develop a view of this phenomenon as an equilibrium game between citizens and officeholders, in which the formers can hide part of his profit and offer bribes to public officials who can be either, corrupt or honest.

The model is based on evolutionary game theory, which assumes that individuals adjust dynamically their behaviour in response to the competing strategies in the population. The time evolution of the game is described by the standard replicator dynamics (see [10]), a learning-by-imitation model of evolution widely used in economics. The replicator dynamics postulate that players are boundedly rational and update their choices by adopting the relatively more rewarding behaviour that emerges from available observations of others' behaviours.

Table 1: Payoff matrix.

C	O	D	H
	E	$(1 - \tau)\pi_D + \pi_E - b_e;$ $w + b_e - i - pS_d$	$(1 - \tau)\pi_D + \pi_H - qS_e;$ $w + i$
	NE	$(1 - \tau)(\pi_D + \pi_H) - b_d;$ $w + b_d - pS_d$	$(1 - \tau)(\pi_D + \pi_H); w$

The structure of the evasion-corruption game explicitly distinguishes between the probability to detect and punish a dishonest citizen (q) and a dishonest officer (p). Therefore, these two parameters strongly characterize the game and can be considered as proxy of the efficacy or the commitment of the institutional system in fighting illegality, and supply an interesting indication about the corruption and evasion behaviour in a certain country. From this respect the model helps the comprehension of the different corruption and evasion behaviour observable in the real world, where countries with similar level of taxation may have different levels corruption. We analyse the basins of attraction of the game strategies when varying the probability of the effectiveness of the penalty. We also analyse the stability properties of the stationary states. Finally, we analyse the optimal path to reduce the number of dishonest citizens and dishonest officers through the use of q and p as control variables. This may represent a useful policy indication for the social planner.

II. Analysis of the model

We propose an evolutionary game between a population of physicians and a population of patients. In each instant of time $t \in [0, +\infty)$ there is a large number of random pairwise encounters between citizens and officials (see Tab. 1).

- There are strategic complementaries, i.e a tax evading e -citizen prefers matching a dishonest d -official and a non-tax evading ne -citizen prefers matching an honest h -official.
- The citizen receives profit $\pi > 0$ and faces flat tax rate $0 < \tau < 1$. However, the citizen can choose to either hide part of his profits (strategy E) or disclose the profit truthfully (strategy NE). That is, the profit is decomposed into the “disclosed” and ‘hidden’ parts, so that $\pi = \pi_D + \pi_H$. With some probability the d - official is caught and fined by court. If the official is detect by the court, he is charged a fine $S_d > 0$. The probability to detected a dishonest officer is denoted by $p \in [0,1]$.
- For each citizen inspected, the officer receives a monetary reward $w > 0$, which does not change according to the level of tax evading or whether the citizen is corrupt or not.
- A tax-evading citizen found guilty in court is charged a fine denoted by $S_e > 0$. The probability to detect a dishonest officer is denoted by $q \in [0,1]$.
- A non tax-evading citizen inspected by an honest official does not pay the fine.
- A tax-evading citizen inspected by a dishonest official pays a bribe b_e and avoids the fine.
- A non-tax evading citizen inspected by dishonest official must pay a bribe b_d , in order to have a fair report. That is non-tax evading firm must pay a bribe when inspection is done by a dishonest official.

The 2×2 game between the inspection officer O and a firm C is introduced in the following payoff matrix.

The natural choice of the parameter is $\tau\pi_H > b_e$.

Since we can add a constant to every column of payoff matrices in Tab.1 (see [10]), we can rewrite, without loss of generality, the payoff matrix in the following form:

Table 2: Payoff matrix of Citizens and Officials.

	D	H
E	$\alpha; \delta;$	$0; 0$
NE	$0; 0$	$\beta; \gamma$

If agents have no dominant strategies that is, if $\alpha\beta > 0$ and $\delta\gamma > 0$ hold, then there exists a mixed-strategy Nash equilibrium:

$$(e^*, d^*) = \left(\frac{\beta}{(\alpha+\beta)}, \frac{\gamma}{(\delta+\gamma)} \right) = \left(\frac{qS_e - \tau\pi_H}{qS_e - b_e + b_d}, \frac{pS_d - b_d}{b_e - 2i - b_d} \right)$$

in which the citizen plays strategy E with probability e^* (and strategy NE with probability $1 - e^*$), while the official plays strategy D with probability d^* (and H with probability $1 - d^*$). This equilibrium exists if one the following conditions hold:

$$q > q_1 := \frac{\tau\pi_H}{S_e} \quad \text{and} \quad p_1 := \frac{b_d}{S_d} < p < p_2 := \frac{b_e - 2i}{S_d}$$

$$q > q_1 := \frac{\tau\pi_H}{S_e} \quad \text{and} \quad p_2 := \frac{b_e - 2i}{S_d} < p < p_1 := \frac{b_d}{S_d}$$

A strict Nash equilibrium (E, D) and (NE, H) also exit, in addition to the mixed-strategy Nash equilibrium (e^*, d^*) .

Now, we assume that the time evolution of e and d is described by the standard replicator dynamics [16], a learning-by-imitation model of evolution widely used in economics. The replicator dynamics postulate that players are boundedly rational and update their choices by adopting the relatively more rewarding behavior that emerges from available observations of others’ behaviors. The growth or decline in the adoption rate of a strategy will be proportional to the difference between its payoff and the population average payoff. Accordingly, we obtain the following dynamic system:

$$\dot{e} = e(1 - e)(-\beta + d(\alpha + \beta))$$

$$= e(1 - e)(-(qS_e - \tau\pi_H) + d(qS_e - b_e + b_d))$$

$$\dot{d} = d(1 - d)(-\gamma + e(\delta + \gamma))$$

$$= d(1 - d)(-pS_d - b_d) + e(b_e - 2i - b_d)$$

where \dot{e} and \dot{d} represent the time derivatives of the shares e and d , respectively.

To analyse the relationship between tax τ and the parameters q and p , we need to introduce the definitions of risk dominance and centroid dominance.

A. Risk dominance

As introduced by [11] and [12], the risk dominant (RD) equilibrium is the Nash equilibrium with the largest Nash product, where the term Nash product refers to the product of the deviation losses of both players at a particular equilibrium. This implies that players are more strongly attracted by the risk dominant equilibrium when they are uncertain about the actions of other players. In our coordination game ($\alpha, \beta, \delta, \gamma$, strictly positive) pure strategy pairs (E, D) are said to risk dominate pure strategy pairs (NE, H) if the Nash products satisfy $\alpha\delta > \beta\gamma$, or equivalently $e^* + d^* < 1$.

Now, we can state the following proposition:

Proposition 1. Assume $\alpha, \beta, \delta, \gamma$ strictly positive (coordination game), so

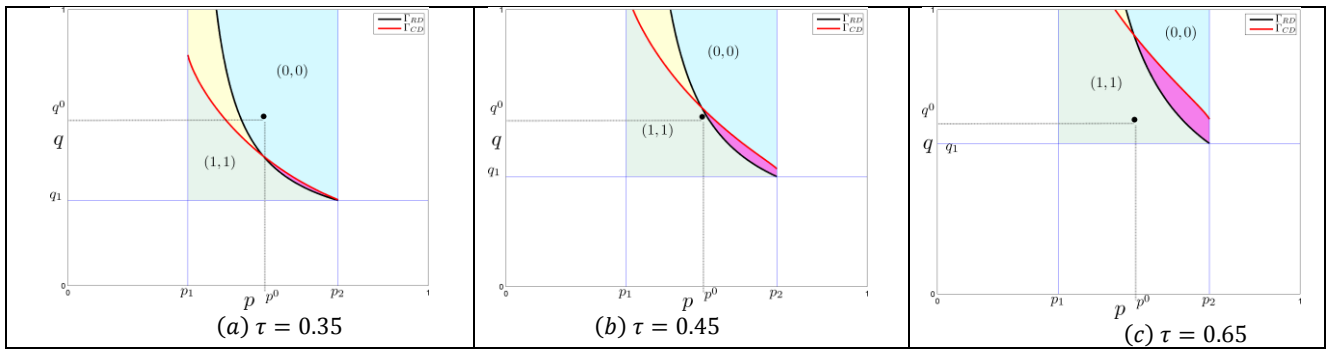


Figure 1. Relationship between τ and the basin of attraction of pure strategies (0,0) and (1,1).

iii. Conclusion

- If $q < \phi(p) = \frac{b_e - b_d}{S_e} + \frac{\alpha(\delta + \gamma)}{S_e S_d (p - p_1)}$ then the Nash equilibrium (1,1) is risk dominant.
 - Viceversa, if $q > \phi(p)$ then the Nash equilibrium (0,0) is risk dominant.

Proof. We define the curve of equation:

$$\Gamma_{RD}(p, q) = e^* + d^* - 1 = 0$$

By simple calculation, we obtain

$$q = \phi(p) = \frac{\alpha(\delta + \gamma)}{S_e S_d (p - p_1)} + \frac{b_e - b_d}{S_e}$$

It is evident that it has got an asymptote in the line $p = p_1$, moreover it is easy verified that $q(p_2) = \frac{\tau \pi_H}{S_e} = q_1$ (see Fig. 3). Hence (1,1) is risk dominant if and only if q is below the curve $\phi(P)$, otherwise (0,0) is risk dominant one.

B. Centroid dominance

For a 2x2 bimatrix coordination game, solutions of (12)-(13) includes a unique path starting for $t = 0$ at the centroid (1/2,1/2) and converging to a Nash equilibrium as $t \rightarrow \infty$ ([10]). This implies that we can define a unique selection from the set of Nash equilibria by tracing the trajectory of the replicator dynamics. An equilibrium is called centroid dominant (CD) if the solution of (12)-(13) with initial value at the centroid converges to it. The equilibrium (0,0) is CD if and only if $(1/2,1/2) \in R_0$ and (1,1) is CD if and only if $(1/2,1/2) \in R_1$. In other words, which equilibrium is selected is decided by the position of the separatrix. If it is above the centroid, (0,0) is CD, and if it is below the centroid, (1,1) is CD.

Zhang and Hofbauer (2015) prove that a equilibrium that is both RD and CD must be basin dominance (BD) and that the BD equilibrium must be either RD or CD or both.

Fig. 1 depicts the relationship between τ and the basin of attraction of pure strategies (0,0) and (1,1). It emerges that for increases in the tax rate, the basin of choices of the parameters p, q , which makes (1,1) BD, gets larger with respect to the region where (0,0) is BD. This has a strong implication since, increasing taxes incentivizes evasion and corruption progressively, driving the system towards the “all bad” equilibrium. This also means that, for given initial conditions, to ensure the dynamic towards the “all good” equilibrium, sufficiently high levels of the parameters p, q , are required. As it is depicted in Fig. 3(a) the combination (p^0, q^0) lies in the region (0,0) which is BD, CD and RD. Progressive increases in τ drive the combination (p^0, q^0) in the region (1,1).

This work investigates the issue of tax evasion through underreporting activity. We develop a view of this phenomenon as an evolutionary game between citizens and officeholders, in which the former can hide part of his profit and offer bribes to public officials who can be either, corrupt or honest. The game explicitly distinguishes between the probability (1) detect and punish a dishonest citizen (p) and a dishonest officer (q). These two parameters strongly characterize the game, then and supply an interesting indication about the corruption and evasion behaviour in a certain country. Thus in this work, they can be considered as proxy of the efficacy or the commitment of the state system in fighting illegality. In this respect the model helps the comprehension of the different corruption and evasion behaviour observable in the real world, where countries with similar levels of taxation exhibit different levels of corruption.

The analysis shows that for increases in the tax rate, the basin of choices of the parameters p and q , which makes (1,1) basin dominance, gets larger with respect to the region where (0,0) is basin dominance. This has a strong implication since, increasing taxes incentivize evasion and corruption progressively, driving the system towards the “all bad” equilibrium. This also means that, for given initial conditions, to ensure that the trajectories converge to “all good” equilibrium, sufficiently high levels of the parameters p, q , are required. All this may represent a useful policy indication for the social planner.

This may presumably explain why some countries manage to innovate and grow at sustained rates with taxes that are high and highly progressive, while other countries do not. Moreover, illegality will most easily spread throughout the society when the share of the potentially corruptible officials reaches a certain level, beyond which the proportion of agents hiding profit will start increasing. It is commonly recognized that, in a clean society the profitability of being corrupt is below that of being honest and that overall corruption makes individual corruption profitable (e.g., there is no loss of individual reputation). All this indeed makes the differences among countries and they are crucial arguments that governments ought to deal with.

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