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# Inspections and Assembiels:

## When two negative effects turn positive

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***Abstract*— No process is perfect and defective items are produced in any process, but in commercial operations there are sales targets and/or orders to deliver. Therefore, a very basic rule is the compensation principle: a compensating unit must be produced for any defective unit. A common method for coping with defective items is to perform cleaning – inspect the items and remove defective units. Inspections however are error prone and the destructive effect of false rejections in serial processes is presented. In assemblies, there are mutual relationships among the components and the yield – the ratio of the number of conforming units to the total number of units diminishes very rapidly in the presence of defective components. Yet, it turns out that cleaning of defective assembly components prior to the assembly, improves quality and reduces waste simultaneously.**

***Keywords*—Assembly, defect rate, inspection errors, yield.**

## I. Introduction

No process is perfect and defective items are produced in any process but in commercial operations there are sales targets and/or orders to deliver. "One of the customer's highest priorities is timely delivery of usable material."[7] Therefore, a very basic rule is the compensation principle: whenever a unit is defective in a sense that it cannot be used as intended, another unit must be produced to replace it (e.g., [1]). To make these additional units requires to enlarge the production facility and infrastructure to make extra production capacity. These extensions were termed the *hidden plant* [3].

A common method for coping with defective items is to perform cleaning – inspect the items and remove defective units. "The potentially dire consequences of not detecting an electrical fault early during the process have motivated technology managers in the semi-conductor industry to introduce an inspection step after about every ten process steps."[9] When inspections are introduced, inspection errors appear – missing defective unit and false rejection of conforming units; e.g., [5]. Each falsely rejected unit cannot be used as intended and hence requires compensation just as a defective unit. This addition to the process's input adds defective units, too. The attempt to reduce waste results in more waste, the costs associated with all these additional units can easily outdo the saving.

In an assembly, two or more entities – components are joined to form a new entity – a (sub)assembly. Consequently, an assembly is deemed conforming only when all its component are conforming. What is more, an assembly will be disqualified even when all but one of its components are conforming – one flat tire grounds a car, with the yield – the ratio of the number of conforming units to the total number of units – diminishing very rapidly.

## II. Assumptions and Basic Relations

The four assumptions of Eben-Chaime [2] are assumed here as well, plus one additional:

1. The work station for each activity/operation have already been selected;
2. Activities/operations are independent;
3. The processing of items in a station are independent;
4. Inspections are independent of each other and of anything else;
5. Long term averages are proper performance measures to use.

Assumption 1 is needed because the defect rate, the ratio of the number of defective units to the total number of processed units, is determined by both the activity and the work station where this activity is performed. Assumption 2 is rather reasonable because successive operations are performed in different work stations. Assumption 3 is justified by the prior (implicit) assumption that the process is in control, namely, defects are due to random causes, only. Assumption 4 is the additional assumption and is common in studies of inspections; e.g., [4], [6].

Under assumption 3, the mean number of conforming units produced by an operation with defect rate $d$, is the number of units processed less the defective units: $Q^{out} = Q^{in} \cdot (1-d)$. Consequently and following the compensation principle, in order to produce $Q^{out}$ conforming units, the minimal number of units that should be processed by each of the operations in a serial process of $n$ operations is:

$$Q_j^{in} = Q^{out} \Big/ \prod_{i=j}^{n} \left(1-d_i\right), \qquad j = 1, 2, ..., n. \tag{1}$$

These are minimal numbers as (1) is built on the assumption that each defective unit is detected and removed as soon as it becomes defective. This, of course, is an unrealistic assumption and, as pointed out by Eben-Chaime [2] and as implied by Rodriguez-Verjan et al. [8], many defective units

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continue through the system before they are detected and the quantities are inflated accordingly. Numerical examples can be easily constructed, which demonstrate the colossal effects of defective items - items are processed, knowing in advance that many, and even most of them will turn defective. Even when each defect rate is small, they accumulate vary rapidly.

## III. Inspections in Serial Processes

Paradoxically, attempts to fix the situation may worsen it! A defective item can be detected either in a coincidental manner or by inspection. Inspections, however, are, too, imperfect and involve errors. Type II errors miss nonconforming items and let them slip through to proceeding operations. These items were referred to in noting that the numbers of (1) are minimal. Type I error - disqualifying conforming units, is of interest here. As noted, both defective and falsely rejected units cannot be used as intended and more units must be produced to replace them. Suppose the type I error probability is $\alpha$, then $\alpha \cdot (1- d)$ of them are falsely disqualified. To illustrate, let $d$ = 1% and $\alpha$ = 5%. Then, of 1,000 units, 10 are defective and another 49.5 units, about 5 times more, are falsely rejected, on average! No wonder, thus, that Type I errors are termed the producer's risk (e.g., [6]). Further, there is another similarity between false rejections and defective items - both accumulate along the production processes. Namely, if $k_j$ inspections are introduced between operations $j$ and $n$ with type I error rate $\alpha_l$, $l$=1, ..., $k$, then (1) should be modified as follows:

$$Q_j^{in} = Q^{out} \Big/ \prod_{i=j}^{n}\left(1-d_i\right)\cdot\prod_{l=1}^{k_j}\left(1-\alpha_l\right), \quad j = 1, 2, ..., n. \qquad (2)$$

Let the yield of a process be the ratio of conforming to the input units, then the yield, $y$ of a serial process of $n$ operations with no inspections is obtained by dividing both sides of (1) by $Q^{out}$ and equals $\left[\prod_{i=1}^{n}\left(1-d_i\right)\right]^{-1}$, while with $K$ inspections both sides of Eq. (2) are divided and by $Q^{out}$ and $y = \left[\prod_{i=1}^{n}\left(1-d_i\right)\cdot\prod_{l=1}^{K}\left(1-\alpha_l\right)\right]^{-1}$.

Assuming that at least a final inspection is performed, $K \geq 1$, the effects of inspections can be examined by comparing other $K$ values with this base line. Consider for instance, a process of 70 operations, with a defect rate of 1%, each. The yield of this process is about 50%. This implies that if 1,000 units are required, 2,021 units should enter the process. Note by passing the magnitude of nonconformance. Further notice the contribution to costs of this part of the *hidden plant* [3]. With a single, final inspection with $\alpha$ = 5%, the yield decreases to 47% and the input grows to 2,127.22 units. Suppose six inspections are added, say after each ten operations, as in; e.g., [8], [9], with $\alpha$ = 5%, for each inspection. Then, the good intention results in a yield decrease to 34.55%. Consequently, the number of units that should enter the process, to yield 1,000 conforming units, grows from 2,127 to 2,894, knowing in advance that 1,894 units will not be used after the first pass through the process. Of the 2,894 units that enter the process, $[1-(1-0.01)^{10}]\cdot 2894 \sim 276.72$, on average, are defective after 10 operations - upon arrival to the first inspection. If the probability of type II error is $\beta$ = 5%, too, then about 13.84 of these defective units slip through the screening, while other $0.05\cdot(2894-276.72) \sim 130.86$ units are falsely rejected by this inspection. Thus, on average, only 2,500 units, including the few defective units, continue. Of the 2,486 conforming units, some 2,249 are still conforming upon arrival to the next inspection. This pattern continues, as shown in Table 1, until the last inspection which falsely rejects 52.63 units before passing the required 1,000 units, plus 6 defective units that managed to slip through all seven inspections.

With a single final inspection 56.36 defective units slip through. Namely, the additional inspections reduced this number by almost 10 times – an order of magnitude. However, this came at the expense of additional 767 units (2,894- 2,127), 36% that are wasted and 6 more inspections, which might be extremely expensive. Another dimension is the operations costs. 2,127 units flow through the process with a single final inspection. With more inspections, the number varies – it is first larger but decreases until it becomes smaller, as listed in the right column of Table 1. An accurate and complete comparison should account for the materials costs (wasted units), the costs associated with the defective units that slip through, inspection costs and the processing cost. To get an impression on the latter, consider the average number of units that flow through the process (the average of the right column of Table 1, as if the processing cost are equal in all stations). With seven inspections it is about 1,930 units, compared to the 2,127 units of the single final inspection.

Next, consider a longer process, with 100 operations. With a single final inspection, the yield is 34.77% – 2,876 units should enter the process to output 1,000 conforming units and about 94 defective units will slip through. The pattern of the flow through the process when 9 inspections are added, one after each ten operations as before, is portrayed in Table 2 and is very similar to that of Table 1. The number of defective units that slip through at the end is 6, as in Table 1. The number of units that should enter the process is 4,563 and 2,542 units flow through the process, on average.

In sum, outgoing quality increases with the additional inspections but at the expense of more waste – material loss, and more inspections. That is, increased costs.

## IV. Mutual Relationships among Assembly components

As noted in the introduction, an assembly is deemed conforming only when all its component are conforming. Consider, for example an item which is assembled from components of three types: three units of one type, two units of another type. Suppose the defect rate of all three component types is 1% while the defect rate of the assembly operation is 0.26%. Then, following [2], the average yield of the assembly is $0.99^{(3+2+1)}\cdot 0.9974 \sim 0.939$. Consequently, some 1,000/0.939 ~ 1,065 units should by assembled for 1,000 to be conforming,

TABLE I. A SERIAL PROCESS WITH 70 OPERATIONS AND 7 INSPECTIONS

| **Inspection #** | **# Conforming** | **# Defective** | **#False rejections.** | **# Slip through** | **# Conforming remained** | **Total # moved on** |
|---|---|---|---|---|---|---|
| 1 | 2617.12 | 276.70 | 130.86 | 13.84 | 2486.26 | 2500.10 |
| 2 | 2248.53 | 251.57 | 112.43 | 12.58 | 2136.10 | 2148.68 |
| 3 | 1931.85 | 216.83 | 96.59 | 10.84 | 1835.26 | 1846.10 |
| 4 | 1659.78 | 186.33 | 82.99 | 9.32 | 1576.79 | 1586.10 |
| 5 | 1426.02 | 160.09 | 71.30 | 8.00 | 1354.72 | 1362.72 |
| 6 | 1225.18 | 137.54 | 61.26 | 6.88 | 1163.92 | 1170.80 |
| 7 | 1052.63 | 118.17 | 52.63 | 5.91 | 1000.00 | 1005.91 |

TABLE II. A SERIAL PROCESS WITH 100 OPERATIONS AND 10 INSPECTIONS

| **Inspection #** | **# Conforming** | **# Defective** | **# False rejections.** | **# Slip through** | **# Conforming remained** | **Total # moved on** |
|---|---|---|---|---|---|---|
| 1 | 4126.64 | 436.30 | 206.33 | 21.81 | 3920.31 | 3942.12 |
| 2 | 3545.46 | 396.67 | 177.27 | 19.83 | 3368.18 | 3388.02 |
| 3 | 3046.12 | 341.89 | 152.31 | 17.09 | 2893.82 | 2910.91 |
| 4 | 2617.12 | 293.80 | 130.86 | 14.69 | 2486.26 | 2500.95 |
| 5 | 2248.53 | 252.42 | 112.43 | 12.62 | 2136.10 | 2148.72 |
| 6 | 1931.85 | 216.87 | 96.59 | 10.84 | 1835.26 | 1846.10 |
| 7 | 1659.78 | 186.33 | 82.99 | 9.32 | 1576.79 | 1586.10 |
| 8 | 1426.02 | 160.09 | 71.30 | 8.00 | 1354.72 | 1362.72 |
| 9 | 1225.18 | 137.54 | 61.26 | 6.88 | 1163.92 | 1170.80 |
| 10 | 1052.63 | 118.17 | 52.63 | 5.91 | 1000.00 | 1005.91 |

on average. That is, 3,195 units of the first component type, 2,130 of the second, and 1,065 of the third. However, only 1% ~ 32 units of the first component type, only 21.3 units of the second component type and only 10.65 units of the third type are defective, on average. The additional units are required because as also noted an assembly is disqualified even when all but one of its components are conforming. The additional units: 163 of the first type, 108.7 of the second type and 54.35 units of the third component type are conforming units which are assembled with defective units of component of the same or other types, or damaged during the assembly.

To further demonstrate the mutual relationships among assembly components, suppose the second component type is a subassembly which is assembled from three units of component type 2.1with a unit of component 2.2 and the defect rates of these components are 0.5% and 2.5%, respectively. Then, the conforming rate of the sub-assembly – the second component type is about 96% and, with no intervention to improve the situation, the yield of the final assembly drops to 88.4%. Consequently, the required input is for additional 66 units – 1131final assemblies: 3,393 units of the first component type, 2,262 sub-assemblies – the second component type, and 1,131 units of the third component type. Accordingly, 6,786 unit of component 2.1 and 2,262 units of component 2.2 are required as well. One can easily imagine how these patterns continue as complexity grows in terms of both the number of components and the number of steps in the manufacturing process.

## V. Inspections and Assemblies

Inspections, as demonstrated earlier, can improve quality. The simple way is to add a single, final inspection. With $\alpha = \beta = 5\%$, the outgoing quality of a final inspection in the first example above increases from 0.939 to $0.95 \cdot 0.939/[0.95 \cdot 0.939 + 0.05 \cdot (1-0.939)] \sim 0.9966$ – the defect rate went from 6.1% down to 0.34%. On the other hand, the required input went up to 1,121– 56 additional units, on average. In the second example, the yield increases to $0.95 \cdot 0.884/[0.95 \cdot 0.884 + 0.05 \cdot (1-0.884)] \sim 0.993$ (from 0.884) at the expense of an increase of the required input from 1,131 to 1,191assemblies: 3,573 units of the first component type, 2,382 units of the second, and 1,191 units of the third component type.

Considering the mutual relationships among assembly components, it seems to be of advantage to inspect the components prior to the assembly. To allow comparisons, the same data will be used. In the first example, the defect rate of each component type decreases from 1% to $0.05 \cdot 0.01/[0.95 \cdot 0.99 + 0.05 \cdot 0.01] = 0.000531$. The corresponding conforming rate – the yield, is about 0.9995. With this number, the yield of the assembly operation is $0.9995^6 \cdot 0.9974 \sim 0.994$, compared to 0.9966 above. However, only 1,006 units should be assembled to yield 1,000 conforming assemblies on average, compared to 1,121! $3 \cdot 1{,}006/0.95 \cdot 0.99 \sim 3{,}208$ units of the first component type, 2,139 units of the second component type and 1,070 units of the third component type. These numbers account for both the defective units among the components and the units that will be falsely rejected. Similarly, in the second example, the conforming rate of the first and third component type remains 0.9995, while that of the second component type is $0.95{\cdot}0.96/(0.95{\cdot}0.96 + 0.05{\cdot}0.04) \sim 0.998$. Consequently, the yield of the final assembly is $0.9995^{(3+1)}{\cdot}0.998^{2}{\cdot}0.9974 \sim 0.991$, compared to 0.993 with a final inspection. The required input, however, is only for 1,009 final assemblies, rather than 1,191! 3,029 units are required of the first component type, 2,212 unit of the second component type and only 1,010 units of the third component type, compared to 3,573, 2,382 and 1,191, with only final inspection. In both examples, the yield – the conforming rate is only slightly lower while the required input is much lower with inspections prior to as opposed to post assembly.

Moreover, in the second example, Inspections can be added also prior to the sub-assembly of the second component type, in which case the conforming rate of component 2.1 is $0.95{\cdot}0.995/[0.95{\cdot}0.995 + 0.05{\cdot}(1\text{-}0.995)] \sim 0.9997$ and the conforming rate of component 2.2 is $0.95{\cdot}0.975/[0.95{\cdot}0.975 + 0.05{\cdot}(1\text{-}0.975)] \sim 0.99865$, upon arrival to the first assembly and the yield of the sub-assembly is $0.9997^{3}{\cdot}0.99865$, even closer to 0.998. Accordingly, the yield of the final assembly with prior inspections of the other component types is about the same, too: 0.991. The required input is the same for the final assembly as well as for the first and the third component types. However, the addition of an inspection and their earliness, result in a decrease of the required input of the second component type to only 2,022 units, 6,419 (rather than 6,636) units of component 2.1 and 2,184 (rather than 2,212) units of component 2.2!

## VI. Summary and Conclusions

The discussion and the example above clearly demonstrate the advantage of cleaning – inspection and removal of defective component units prior to assemblies. Adding inspections in serial processes improve the yield but have a destructive effect in terms of waste. Similar effect in terms of waste are due to the mutual relationships among the components of assemblies. Yet, cleaning the defective components prior to assembly weakens the mutual relationships among them. This results in both: improved quality and reduced waste.

Before closing, a negative effect of early cleaning should be noted. With no or only a final inspection, the relative increase in the required input of all part types – elementary components and (sub) assemblies, are the same. This keeps the quantitative relationships in accordance with the assembly ratios. When components are cleaned prior to assembly and the yield/defect rates differ, mismatch is created by type II errors – the defective units that sleeps through the inspections. To illustrate, consider the second example when all three component types are cleaned prior to the final assembly. The required inputs are 3,029, 2,212, and 1,010 units of each part type (see above), which do not match the 3:2:1 assembly ratios. Prior cleaning of components 2.1 and 2.2 of the sub-assembly – the second type component, improves this matter – only 2,022 units of this component type are required because the yield of this component type is closer to those of the other component types. However, the required quantities of components 2.1 – 6,419 units, and component 2.2 – 2,184 units, do not match. This issue is left for future examination.

## References

[1] J.M. Apple, *Plant layout and material handling* (2nd Edition), The Ronald Press Company, New York, USA, 1963.

[2] M. Eben-Chaime, Mutual Effects of Defective Components in Assemblies, *J. of Manufacturing System,* 36, 2015, pp. 1-6, DOI: 10.1016/j.jmsy.2015.02.008.

[3] A.V. Feigenbaum, *Total quality control,* 3rd ed., revised. McGraw-Hill, New York, USA, 1991.

[4] F., M. Franceschini, Galetto, G. Genta and D. Maisano, Evaluating quality-inspection effectiveness and affordability in short run productions. In: proceedings of the 2nd International Conference on Quality Engineering and Management, 2016: A better world with Quality, Guimarães (Portugal),, July 13-15, 2016, pp. 420-432.

[5] S.S. Mandroli, A.K. Shrivastava and Y. Ding, A survey of inspection strategy and sensor distribution studies in discrete-part manufacturing processes, *IIE Transactions*, 38(4), 2006, pp. 309–328.

[6] D.C. Montgomery, *Introduction to statistical quality control*, 6th ed. Wiley, New York, USA, 2008.

[7] C.A. Ptak, *ERP: tools, techniques and applications for integrating the supply chain*, St. Lucie Press, Boca Raton, USA, 2003.

[8] G.L. Rodriguez-Verjan, J. Pinaton and S. Dauzere-Peres, Impact of Control Plan Design on Tool Risk Management: A Simulation Study in Semiconductor Manufacturing". *Proceedings of the 2011 Winter Simulation Conference*, 2011, pp. 1913-1920.

[9] C. Weber, V. Sankaran, K. Tobin and G. Scher, Quantifying the value of ownership of yield analysis technologies, *IEEE Trans. Semicond. Manuf.*, 15 (4), pp. 411-419, 2002.

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