International Journal of Fluid Mechanics & Machinery– IJFMM
Volume 1 : Issue 2 [ISSN 2374-1481]
Publication Date : 25 June 2014

Application of Some Chaotic Signal Processing Tools for the Diagnosis of Rotor-to-Stator Rub in Rotating Machinery

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***Abstract*—This paper reports on the application of correlation dimension D_2, largest Lyapunov exponent λ_1 and maximum approximate entropy $ApEn_{max}$ to diagnose the severity of rotor-to-stator rub in rotating machinery. The method of time delay was employed to reconstruct the vibration signal obtained from numerical simulation of a Jeffcott rotor subjected to rotor-to-stator rub. Numerical results showed clear correlation between the increase of rub severity with the invariant quantities, D_2, λ_1 and $ApEn_{max}$, investigated in this work.**

Keywords*— Correlation dimension, largest Lyapunov exponent, maximum approximate entropy, rotor-to-stator rub, time delay, attractor reconstruction, Jeffcott rotor.

I. Introduction

It is a well-known fact that simple deterministic systems with only a few degrees of freedom could display complex chaotic behavior. Such complex chaotic behavior has been observed in various fields including, among others, medicine, biology, psychology, chemistry, robotics, physics, economics and engineering. In the specific field of engineering, evidence of chaotic vibrations has been observed in rotating machinery subjected to certain faults such as loose pedestal, cracked rotors, and rubbing between rotor and stator. Advancement in the development of nonlinear dynamic theory in recent years has brought about new and effective methods for the analysis of complex signals, which appear to be stochastic, but is actually inherently deterministic.

Rotor-to-stator rub has been known to cause serious damage to rotating machinery. Rub is usually considered as a secondary fault, which is often the result of some other primary faults. Muszynska ref.[1] has covered comprehensively on rub related vibration phenomena. Lin et al. ref.[2] treated the nonlinear behavior of rub related vibrations in rotating machines by considering its friction and damping coefficients, angular speed and clearance between rotor-to-stator. Sun et al. ref.[3] discussed on diverse attractor configurations when investigating different regions of chaos with the assistance of the rotor trajectory, bifurcation diagrams and Poincaré section. Hu and Wen ref.[4] reconstructed the attractor in pseudo space based on delay coordinates and use short-term predictability to identify its chaotic properties with the help of the correlation dimension and the largest Lyapunov exponent. Base on the numerical analysis done by Chu and Zhang, ref.[5,6] periodic, quasi-periodic and chaotic vibrational motion was observed when the rub-impact model being investigated was supported by oil-film bearings. The vibration that was due to the variation in the eccentricity parameter showed the importance of effective diagnosis of rotor-rub fault. Li et al. ref.[7] considered the nonlinear dynamics of a rub-impact rotor system that was supported by oil film bearings and elastic stator. Choi ref.[8] demonstrated different orbital configurations as the severity of rub increases. He also detected an impact every two revolutions, which indicated the presence of subharmonic response in the rotor system.

Many of the nonlinear methods used to diagnose rotor-to-stator rub in rotating machinery, as presented in the preceding brief review, are qualitative and therefore are not effective for fault diagnosis purposes. In the present paper, quantitative invariant measures based on the theory of chaos are exploited for the assessment of rotor-to-stator rub severity in rotating machinery. The specific invariants utilized in this work are correlation dimension D_2, largest Lyapunov exponent λ_1 and maximum approximate entropy $ApEn_{max}$.

II. Theoretical Treatment

The Jeffcott rotor, which is a simplified representation of rotating machinery, is the most widely utilized model for the study and understanding of rotor dynamics phenomena. The Jeffcott rotor is characterized by a single rigid disk that is mounted at the mid-span of a massless and flexible shaft, which in turn is supported by two identical infinitely stiff bearings. The motion of the bearing system can be described by the displacements *x* and *y* of the geometric center of disk. Considering the external forces acting on the rotor that are due to rotor-to-stator contact, rotor imbalance and gravity, the equations of motion of the rotor-bearing system can be expressed as

$$\begin{aligned} m\ddot{x} + c\dot{x} + kx &= F_x + mu\omega^2\cos\omega t \\ m\ddot{y} + c\dot{y} + ky &= F_y + mu\omega^2\sin\omega t - mg \end{aligned} \qquad (1)$$

m is the mass of the disk, *c* is the damping coefficient at the disk, *k* is the stiffness of the shaft, *u* is the eccentricity of the rotor's center of mass, ω is the angular speed of the rotor, *g* is the gravity constant, *t* is the time, F_x and F_y are the rotor-to-stator contact forces in the *X* and *Y*- directions respectively. The contact forces between the rotor and the stator is modeled based on the Hertzian theory ref.[2,9] as shown in Fig. 1 below,

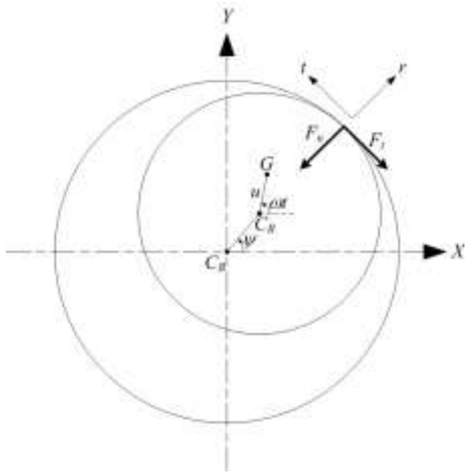


Fig. 1. Rotor-to-stator contact model

where the normal contact force F_n is a function of the contact stiffness k_c and rotor penetration depth, *e*. The friction force F_t, which acts perpendicular to the normal force, is a function of the normal force F_n and the Coulomb sliding friction coefficient between the rotor and stator, μ. The rotor-to-stator contact forces in the *X* and *Y*- directions are given in Equation (2), where the radial clearance between the rotor and stator is denoted by c_r.

$$F_x = F_n \cos\psi + F_t \sin\psi$$
$$F_y = F_n \sin\psi - F_t \cos\psi \qquad (2)$$

where

$$F_n = \begin{cases} k_c e & e > 0 \\ 0 & e \leq 0 \end{cases}$$

$$F_n = \begin{cases} \delta\mu k_c e & e > 0 \\ 0 & e \leq 0 \end{cases}$$

$$e = \sqrt{x^2 + y^2} - c_r$$

$$\psi = \tan^{-1}\frac{y}{x}$$

The direction of the friction force F_t is determined from the value of δ by the following relationship

$$\delta = \begin{cases} -1 & if\ \vartheta > 0 \\ 0 & if\ \vartheta = 0 \\ 1 & if\ \vartheta < 0 \end{cases} \qquad (3)$$

ϑ, which is the velocity of the contact point between the rotor and stator, is given in Equation (4). It is obtained by adding the velocity of the center of the disk to the velocity of the contact point relative to the disk center. *R* in Equation (4) is the radius of the disk.

$$\vartheta = \frac{x\dot{y} - y\dot{x}}{\sqrt{x^2 + y^2}} + R\omega \qquad (4)$$

III. Invariant Measures of Chaos

There are several qualitative and quantitative invariant measures of chaos. A brief description of the quantitative measures used in this work, namely the correlation dimension, largest Lyapunov exponent and maximum approximate entropy, are presented herein. A short exposition of the method of time delay used in the reconstruction of attractor, which is essential for the computation of these invariant measures, is also outlined.

A. Attractor Reconstruction in Pseudo-Space

The reconstruction of an attractor utilizes the time delay method, which requires only one variable in space to unveil the true dynamics of the system provided it is embedded in an equivalently large dimension where the attractor can potentially unfold itself to reveal the components of all other variables. Since knowledge of a system's dynamics in real-world applications are not readily available, the dynamics of the original system cannot be easily constructed. The equation below proves that an attractor could be simply reconstructed with the acquaintance of the proper time delay τ and embedding dimension *m*.

$$\mathbf{x}_k = [x_k, x_{k-\tau}, \ldots, x_{k-(m-1)\tau}]^T \qquad (5)$$

$\mathbf{x}_k$ is the reconstructed state space attractor where the choice of τ and *m* plays a very crucial role and is often the most important reason for errors ref.[11]. Thus, with the appropriate selection of both these quantities, the theorem above guarantees that the *m*-dimensional attractor would unravel itself without any intersections.

B. Proper Selection of m and τ

At low dimensions, the percentage of having self-interaction with spatially nearby points on the attractor is rather high and therefore should be avoided. Sufficiently higher embedding dimensions exhibit real neighbors only due to the system's dynamics. Consequently, the threshold value *m* is chosen when the false nearest neighbor percentage drops to zero as shown in Fig. 2(a). This method is also known as the false nearest neighbor method ref.[12].

One of the methods for the selection of τ is the average mutual information that was first described by Fraser and Swinney ref.[13]. The selection of the time delay τ with too small of a delay would produce an attractor that is stretched along its diagonal which displays a phenomenon called redundancy. On the other hand, if the chosen τ is too large, it would present irrelevance due to successively delay coordinates being causally unrelated ref.[14]. Consequently, the threshold value for the proper choice of τ should give the coordinates more independence from each other but nevertheless still preserving the systems dynamic information.

A good estimate of τ would be the first minimum point of the average mutual information graph as plotted in Fig. 2(b).

C. Correlation Dimension, D_2

Grassberger and Procaccia [15] proposed the correlation integral as a new way to estimate the dimensions of an attractor, Equation (6).

$$C(r) = \lim_{N \to \infty} \frac{1}{N(N-1)} \sum_{i,j=1, i \neq j}^{N} \Theta(r - |X_i - X_j|) \quad (6)$$

N is the number of data points used, θ is the Heaviside function for which $\theta = 1$ for $x > 0$ and $\theta = 0$ for $x \leq 0$, X_i and X_j are points between two reconstructed vectors and *r* is the distance parameter. It is assumed that for small *r*, *C(r)* behaves as follows,

$$C(r) \propto r^{D_2} \quad (7)$$

D_2 is referred to as the correlation dimension. In order to compute the correlation dimension, a plot of $\log C(r)$ against $\log r$ must first be obtained. The slope of a suitable scaling region or interval in this plot represents the correlation dimension D_2.

D. Largest Lyapunov Exponent, λ_1

Another widely used qualitative measure of chaos is the Lyapunov exponent. It has been proven that a system is chaotic if at least one of its Lyapunov exponents is positive; there are as many Lyapunov exponents as there are dimension governing the system. The algorithm for the computation of the largest Lyapunov exponent used in this work was proposed by Rosenstein et al. ref.[16]. This algorithm is conceptually slightly different from the well know Wolf algorithm ref.[17]. Rosenstein's method is primarily based on Sato's ref.[18] work and is shown in a simplified equation below.

$$y(i) = \frac{1}{\Delta t} < \ln d_j(i) > \quad (8)$$

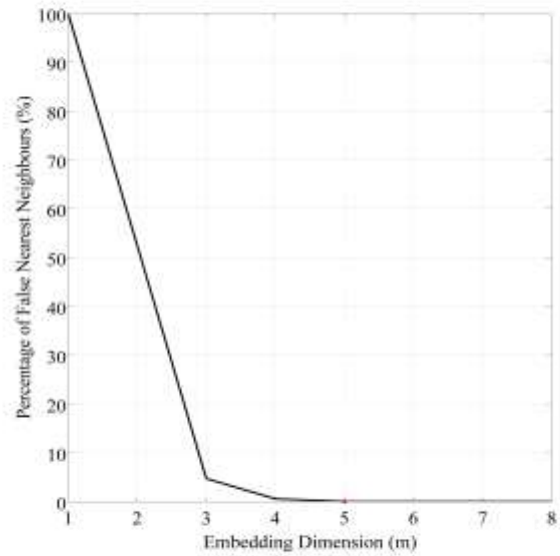
$d_j(i)$ is the current separation distance after each time step. The $\ln d_j(i)$ is equal to $\ln(divergence)$ that represent the mean logarithmic divergence for all pairs of nearest neighbor over time. Equation (8) characterizes an approximate set of parallel lines whose slope is that of λ_1.

E. Maximum Approximate Entropy, $ApEn_{max}$

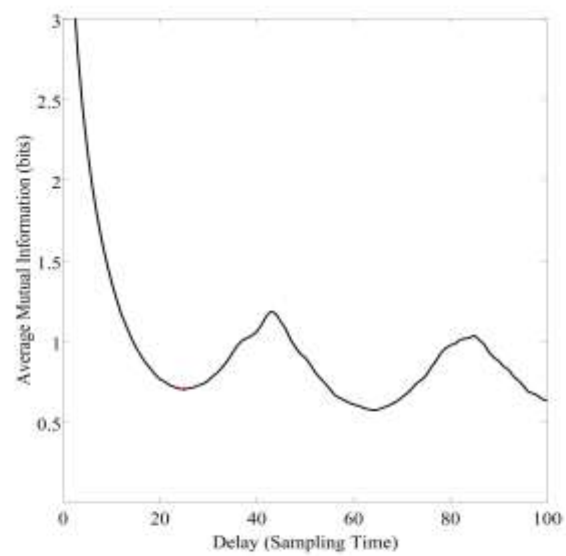
The approximate entropy, *ApEn*, was first proposed in the early nineties by Pincus ref.[19]. It measures the system's complexity by quantifying how regular the measured time series is. The idea of this measure is simple, given a time-series with *N* data points sampled at a constant rate, a successive number of $N - m + 1$ vectors are produced where each vector comprise of *m* sequential vectors; *m* here being the embedding dimension. Each vector created in turn acts as a reference or a template vector for comparison with all other vectors in the time series including itself. Equation (9) was used to compute the *ApEn* value.

$$ApEn(m, r, N) = \Phi^m(r) - \Phi^{m+1}(r) \quad (9)$$

Input parameters *m* and *r* must be known before hand to compute *ApEn*. Embedding dimension *m* can be obtained from the false nearest neighbor method ref.[12, 20, 21] and *r* from the maxima of the plotted *r* vs. *ApEn* graph ref.[20]. In order to accurately compute the system's dynamic complexity, the maximum value of *r* at which $ApEn_{max}$ is maximum is selected, as opposed to just choosing a specific value from a certain range, for example $0.1 \leq r \leq 0.25$, as recommended in some previous work ref.[19,21].



(a)



(b)

International Journal of Fluid Mechanics & Machinery– IJFMM
Volume 1 : Issue 2 *[ISSN 2374-1481]*
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Fig. 2. (a) False nearest neighbor first zero crossing (indicated by the red mark), m = 5, and (b) average mutual information first minimum (indicated by the red mark), $\tau = 25$ for a response at $u/c_r = 0.5$ in the X-direction

IV. Numerical Results and Discussion

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Numerical simulation of the rotor system was undertaken using MATLAB software package, which employed the continuous solver based on the Runge-Kutta of order 4 method with a constant time step of 0.0001. The parameters chosen for the numerical simulation were m = 1.715 kg, ω = 750 rad/s, c = 268 Ns/m, c_r = 0.0002 m, R = 0.06 m, k = 115969 N/m, k_c = 2319380 N/m, μ = *0.3*, and g = 9.81 m/s^2. The severity of the rotor-rub is based on the rotor's imbalance, u (m) where it was increased from 0.00008 to 0.00011 with 0.00001 increments, divided by the rotor-to-stator clearance of c_r = 0.0002m. Therefore, the non-dimensional values of rub severity (u/c_r) in increasing order were 0.40, 0.45, 0.50 and 0.55 as presented in Table 1.

Sets of 8192 data points were obtained from the numerical simulation for each rub severity parameter. These data points, which represented the response of the rotor in both X- and Y-directions, were obtained after removing the transients. The reconstruction of the attractor was independently undertaken for the data in both the both X- and Y-directions. The time delay and embedding dimension of each set of data were computed using the average mutual information and false nearest neighbor methods, respectively. Examples of the time delay and embedding dimension obtained using these methods for the data in the X-direction with u/c_r=0.50 are shown in Fig. 2. The values of the time delay and embedding dimension were then use in subsequent computation of correlation dimension, largest Lyapunov exponent and approximate entropy. Using the values of time delay and embedding dimension of Fig. 2, the correlation dimension, largest Lyapunov exponent and approximate entropy were computed and the results are presented in Fig. 3.

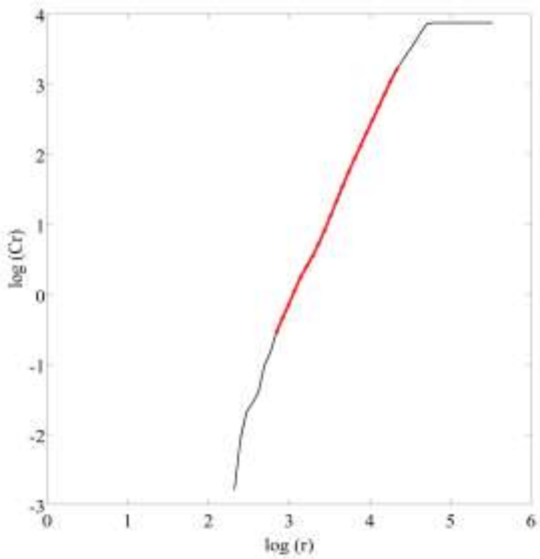
The results for all values of rub severity investigated in this work are presented in Table 1. It is seen that when rub severity increased from 0.45 to 0.55, the correlation dimensions computed based on the vibration data in the X-direction increased significantly from value of D_2 1.813, corresponding to u / c_r =0.45, to value of D_2 3.187, corresponding to u/c_r=0.55. The same trend was observed for the correlation dimensions computed based on the vibration data in the Y-direction; the correlation dimension for the case of u/c_r=0.40, D_2 1.903, was seen to increase to D_2 2.389 as the rub severity increased to u / c_r =0.55. These results showed that the correlation dimension could be used as a quantitative measure to assess the severity of rotor-to-stator rub in rotating machinery.

Table 1 also presents the largest Lyapunov exponent for each time-series data. Similar trend that was seen for the correlation dimension with the increase in the severity of rotor-to-stator rub was also observed for the largest Lyapunov exponent λ_1. For the vibration data in the X-direction, as the severity of the rotor-to-stator rub increased from u/c_r = 0.40 to u/c_r = 0.55, the largest Lyapunov exponent was also found to increase from λ_1 0.172 to λ_1 0.6033. The same trend was generally observed for the vibration data in the Y-direction where the largest Lyaounov exponent was seen to increase from λ_1 0.2075 to λ_1 0.4284 as the severity of the rotor-to-stator rub increased from u/c_r = 0.45 to u/c_r = 0.55. The positive values of the largest Lyapunov exponents also ascertained that the response of the rotor subjected to rub was indeed chaotic. These values provided further insight into the rotor system's dynamic properties where nearby trajectories diverge exponentially even after only several iterations. This consequently conforms to the fact that chaotic systems like the one examined in this work is indeed sensitive to initial conditions.

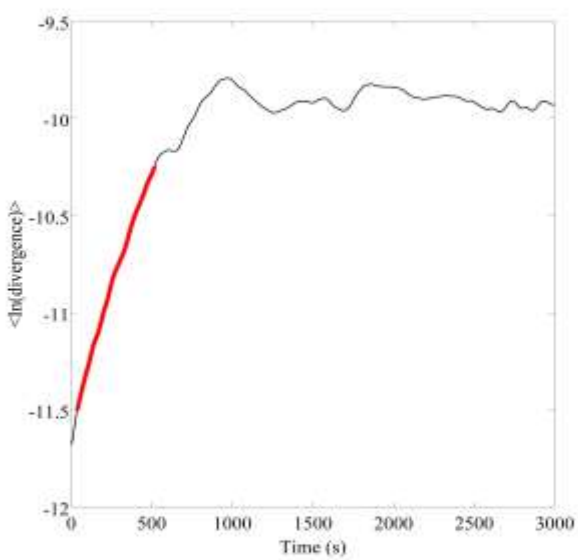
The maximum approximate entropy $ApEn_{max}$ values computed for all values of rub severity are shown in Table 1. The $ApEn_{max}$ was seen to increase with the increase in the severity of rub. For the vibration data in the X-direction, $ApEn_{max}$ = 0.1443, corresponding to u/c_r = 0.40 increased to $ApEn_{max}$ = 0.3615 when u/c_r was increased to 0.55. Similar trend of linear increments were also seen in the Y-direction vibration data where the approximate entropy values increased from $ApEn_{max}$ = 0.2932 to $ApEn_{max}$ = 0.3561 as rub severity was increased from u/c_r = 0.40 to u/c_r = 0.55. Since a larger value of the maximum approximate entropy implies more irregularity or disorder in the time-series data, the results for the $ApEn_{max}$ presented in Table 1 point to the fact that the response of the rotor system subjected to rub was getting increasingly complex as the severity of rub increased. The clear trend of increasing values of approximate entropy with the increase in the severity of rub in the rotor system demonstrates the potential of this invariant measure as a diagnostic tool for rub related faults in rotating machinery.

TABLE 1
COMPARISON OF THE VALUES OF CORRELATION DIMENSION, LARGEST LYAPUNOV EXPONENT AND APPROXIMATE ENTROPY WITH INCREASING RATIO OF u/c_r.

Axis	Rub Severity (u/c_r)	Correlation Dimension (D_2)	Largest Lyapunov Exponent (λ_1)	Approximate Entropy ($ApEn_{max}$)
X	0.40	1.866	0.1721	0.1443
	0.45	1.813	0.2285	0.2275
	0.50	2.586	0.2667	0.2852
	0.55	3.187	0.6033	0.3615
Y	0.40	1.903	0.2855	0.2932
	0.45	2.021	0.2075	0.3082
	0.50	2.203	0.3254	0.3280
	0.55	2.389	0.4284	0.3561



(a)



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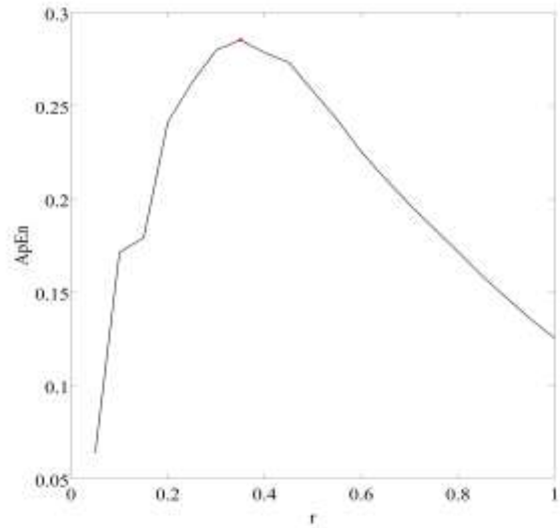

(c)

Fig. 3. (a) Correlation dimension with the outline indicating the scaling region of $D_2 = 2.586$, (b) largest Lyapunov Exponent with the outline indicating the scaling region of $\lambda_1 = 0.2667$, (c) approximate entropy for maximum *r* at $ApEn_{max} = 0.2852$ for a response at $u/c_r = 0.5$.

V. Conclusions

Using vibration response data obtained from the numerical simulation of the Jeffcott rotor subjected to rotor-to-stator rub, this work has shown the effectiveness of employing some quantitative invariant measures of chaos to diagnose the severity of rub in rotating machinery. The specific invariant measures utilized in this work were correlation dimension D_2, largest Lyapunov exponent λ_1 and approximate entropy $ApEn_{max}$. Due to the chaotic nature of the rotor response when subjected to rotor-to-stator rub, the invariant measures used in this work were able to capture the increasing complexity of the rotor response as the severity of rub increased. Such strong correlation between rub severity and the invariant measures of chaos seen in this work would not have been attained if linear quantitative measures such as root-mean-square or peak amplitudes had been used instead. This work has demonstrated the potential of using some invariant measures of chaos for the diagnosis of rub related faults in rotating machinery. Implementation of these methods in practical rotating machinery is expected to reduce unplanned outages and their associated economic repercussions.

Acknowledgment

The authors would like to acknowledge the financial support for this work provided by the Ministry of Education, Malaysia in the form of a research grant (FRGS/FASA1-2012/TK01/UNITEN/02/5).

References

[1] A. Muszynska, “Rotor-to-Stationary Element Rub Related Phenomena in Rotating Machinery-Literature Survey”, The Shock and Vibration Digest 1989;21:3-11.
[2] F. Lin, M.P. Schoen, U.A Korde, “Numerical Investigation with Rub-Related Vibration in Rotating Machinery”, Journal of Vibration and Control 2001;7:833-848.
[3] Z. Sun, J. Xu, T. Zhou, “Analysis on Complicated Characteristics of a High-Speed Rotor System with Rub-Impact”, Mechanism and Machine Theory 2002;37:659-672.
[4] N.Q. Hu, X.S. Wen, “Chaotic Behavior Identification of a Rub-Impact Rotor Using Short-Term Predictability of Measured Data”, Proceedings of the Institution of Mechanical Engineers Part C – Journal of Mechanical Engineering Science 2002;216:675-681.
[5] F. Chu, Z. Zhang, “Periodic Quasi-Periodic and Chaotic Vibrations of a Rub-Impact Rotor System Supported on Oil Film Bearings”, International Journal of Engineering Sciences 1997;35:963-973.
[6] F. Chu, Z. Zhang, “Bifurcation and Chaos in a Rub-Impact Jeffcott Rotor System”, Journal of Sound and Vibration 1998;210(1):1-18.
[7] X. Li, T. Shi, S. Yang, “Nonlinear Dynamics of a Rotor Rubbing against Rotor-Stator on Flexible Supports”, Journal of Vibration Engineering 2001;14(3):303-308.
[8] Y.S. Choi, “Experimental Investigation of Partial Rotor Rub”, KSME International Journal 2000: 14(11);1250-1256.
[9] T.H Patel, A.K. Darpe, “Study of Coast-Up Vibration Response for Rub Detection”, Mechanical Machine Theory 2009;44:1570-1579.
[10] Nehmzow, “Robot Behavior-Design, Description, Analysis and Modelling” Springer 2009:99-133.
[11] W. Wang, J. Chen, Z. Wu, “The Application of a Correlation Dimension in Large Rotating Machinery Fault Diagnosis”, Proceedings of the Institution of Mechanical Engineers 2000;214 Part C:921-930.
[12] M.B. Kennel, R. Brown, H.D.I Abarbanel, “Determining Embedding Dimension for Phase-Space Reconstruction Using a Geometrical Construction”, Physical Review A 1992;45(6):3403-3411.
[13] A.M. Fraser, H.L. Swinney, “Independent Coordinates for Strange Attractor from Mutual Information” Physical Review A 1986;33(2):1134-1140.
[14] M. Casdagli, S. Eubank, J.D. Farmer, J. Gibson, “State Space Reconstruction in The Presence of Noise”, Physica D 1991;51:25-98.
[15] P. Grassberger, I Procaccia, “Characterization of Strange Attractor”, Physical Review Letters 1983;50(5):346-349.
[16] M.T. Rosenstein, J.J. Collins, C.J. De Luca, “A Practical Method for Calculating Largest Lyapunov Exponents from Small Data Sets”, Physica D 1993;65:117-134.
[17] A. Wolf, J.B. Swift, H.L. Swinney, J.A. Vastano, “Determining Lyapunov Exponents from a Time Series”, Physica D 1985;16:285-317.
[18] S. Sato, M. Sano, Y. Sawada, “Practical Methods of Measuring the Generalized Dimension and the Largest Lyapunov Exponent in High Dimensional Chaotic Systems”, Progress of Theoretical Physics 1987;77:1-5.
[19] S.M. Pincus, “Approximate Entropy as a Measure of System Complexity”, Proceedings of the National Academy of Sciences 1991;88:2297-2301.
[20] S. Lu, X. Chen, J.K. Kanters, I.C. Solomon, K.H. Chon, “Automatic Selection of the Threshold Value *r* for Approximate Entropy”, IEEE Transactions on Biomedical Engineering 2008;55(8):1966-1972.
[21] X. Chen, I.C Solomon, K.H. Chon, “Comparison of the Use of Approximate Entropy and Sample Entropy: Application to Neural Respiratory Signal”, Proceedings of the IEEE, Engineering in Medicine and Biology 27th Annual Conference 2005;27:4212-4215.

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