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A Study of the Stress Ratio Effects on Fatigue Crack Growth using LOWESS Regression

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***Abstract*— In this paper, seven fatigue crack growth models, namely Priddle, McEvily, Weertman, Collipriest, Broek, Walker and Forman have been examined. The mean stress effect on fatigue crack growth rate is commonly introduced into fatigue crack growth (FCG) relation through the stress ratio, *R*. Therefore, the ability to correlate and predict the fatigue crack growth rate, FCGr, for different R values is of significant importance for damage tolerant design. Performance of the crack driving force of these models in accounting stress ratio effects in fatigue crack growth rate is evaluated by fitting a lowess curve on transformed FCG data. Experimental fatigue crack growth data of a typical Al 2024 T351 obtained under constant amplitude loading tests for six load ratios has been used in the present work. From the studies carried out, it is observed that Walker and Collipriest models are found to be in good agreement with the experimental FCG data.**

***Index Terms*— Fatigue crack growth; Stress ratio effects; Crack driving force; Lowess Curve.**

I. Introduction

Most of the load bearing components and structures experience an alternating load which comprises a mean load in form of dead load during their service application. The mean stress effect on fatigue crack growth rate is commonly introduced into fatigue crack growth (FCG) relation through the stress ratio, *R*. Therefore, the ability to correlate and predict the fatigue crack growth rate, FCGr, for different R values is of significant importance for damage tolerant design. During the last five decades, a lot of research effort has been focused on fatigue crack growth prediction models. The most successful and popular has been Paris' relation [1] which is based on the applied stress intensity factor range, ΔK_{appl}, as the only governing parameter for FCG. One of the fundamental problems concerning the Paris expression is its ineffectiveness for quantification of the mean stress effect (or stress ratio, R effects).The Paris equation prompted widespread research aiming at possible improvements of its original form and at the analytical modeling of fatigue crack growth and its various aspects. Several models have been suggested in literature to account for the R effect, namely, crack closure [2,3], residual compressive stresses [4,5]. Kujawski [6] showed that crack closure models give an approximate account of stress ratio effect on fatigue crack growth. Closure models use $K_{op,}$ the stress intensity factor for crack opening load as one of the parameters to depict crack growth which has to be determined from experiments. The phenomena is also attempt to explain in terms of the maximum stress intensity factor, K_{max}, driving force and these models have been quite popular [7-8]. A crack growth model should be able to portray R effect on fatigue crack growth rate, (FCGr).

II. Background

The first fatigue crack propagation expression formulated in terms of the stress intensity factor was proposed by Paris et al. The relation states that in the log-log scale the fatigue crack growth rate (FCGr), *da/dN* depends linearly on the applied stress intensity factor range, $\Delta K_{appl.}$ in the region II of fatigue rate curve. Paris has proposed the following equation:

$$\frac{da}{dN} = C\Delta K^m \quad (1)$$

Since Paris' work, many variations of the power law equation have been postulated to take into account the stress ratio dependence of FCGr. Forman et al. [9] proposed a relation, Eq. (2), that explain the stress ratio effect on FCGr and it is also effective in Region III of fatigue growth curve using fracture toughness, K_c and two curve fitting constants C and m. The model requires the prior knowledge of fracture toughness, K_c

$$\frac{da}{dN} = \frac{C(\Delta K_{appl})^m}{(1-R)K_c - \Delta K_{appl}} \quad (2)$$

Broek, Schijve, and Erdogan proposed a relation, Eq. (3) which accounts for the mean stress effect in region II of fatigue rate curve [10] with C as only curve fitting constants.

$$\frac{da}{dN} = C K_{max}^2 \Delta K_{appl} \quad (3)$$

Another relation, Eq. (4), between applied loading parameters and fatigue crack growth rate was proposed by Weertman [11]. Weertman model is applicable only in region II and III (intermediate and high propagation rate) regions of fatigue rate curve and it uses only one curve fitting constant.

$$\frac{da}{dN} = \frac{C\Delta K_{appl}^4}{K_c^2 - K_{max}^2} \quad (4)$$

Priddle [12] proposed the equation which can describe the fatigue rate curve in all three regimes by introducing fracture toughness, K_c and threshold stress intensity range, ΔK_{th} in the fatigue growth model. The Priddle's relation, Eq. (*5*), is based on two assumptions. First, the fatigue crack growth rate has to tend to zero while the applied stress intensity factor range approaches the threshold stress intensity factor range and, second, the fatigue crack growth should tend to infinity when the maximum applied stress intensity factor approaches the fracture toughness. The model requires the prior knowledge of two material constants, ΔK_{th} and K_c, and two additional curve fitting constants. C and m are curve fitting constants and have to be obtained from experimental fatigue crack growth data.

$$\frac{da}{dN} = C\left(\frac{\Delta K_{appl} - \Delta K_{th}}{K_C - K_{max}}\right)^m \quad (5)$$

McEvily [13] proposed another empirical relation based on the same logic as Priddle's relation and which could describe the entire fatigue crack growth curve in the form of:

$$\frac{da}{dN} = C\left(\Delta K_{appl} - \Delta K_{th}\right)^2 \left(1 + \frac{\Delta K_{appl}}{K_C - K_{max}}\right) \quad (6)$$

Walker [14] proposed a model to improve the Paris model by including three curve fitting constant, C, m and γ. The effect of stress ratio is taken care solely by the constant γ no other parameter like fracture toughness or threshold stress intensity range is required. Walker proposed a parameter, $\overline{\Delta K}$ which is expressed by relation

$$\overline{\Delta K} = K_{max}(1 - R)^{\gamma}$$
$$where, K_{max} = \frac{\Delta K_{appl}}{(1 - R)} \quad (7)$$

The significance of this equation is that on a log-log plot of *da/dN* versus $\overline{\Delta K}$ should result in a single curve regardless of the stress ratio for which the data is obtained. It has been observed that Walker model is able to collapse FCG data to one curve for stress ratio -2<R<1 [15]. The curve fitting constant γ is determined by hit and trial, its value is the one that best consolidates the data along a single straight line on the log-log plot of *da/dN* versus $\overline{\Delta K}$. It is possible that no value of γ can be found, and in this situation the Walker equation cannot be used. If the value of γ happens to be one then $\overline{\Delta K}$ equals ΔK_{appl} which indicates that the stress ratio has no effect on the data. Walker's model is represented by

$$\frac{da}{dN} = C(\overline{\Delta K})^m$$

Or,

$$\frac{da}{dN} = C\left(\Delta K_{appl}^{\gamma} K_{max}^{(1-\gamma)}\right)^m \quad (8)$$

Collipriest [16] proposed a crack growth model capable of describing all three regions of fatigue rate curve and includes the stress ratio effect. The model is given by mathematical relation

$$\frac{da}{dN} = C\left(K_c \Delta K_{appl}\right)^{\frac{m}{2}} EXP\left[ln\left(\frac{K_c}{\Delta K_{th}}\right)^{\frac{m}{2}} \tanh^{-1}\left\{\frac{ln\left(\frac{\Delta K_{appl}^2}{(1-R)K_c \Delta K_{th}}\right)}{ln\left(\frac{(1-R)K_c}{\Delta K_{th}}\right)}\right\}\right] \quad (9)$$

The model requires the knowledge of two material constants, ΔK_{th} and K_C, and two additional curve fitting constants, C and m, which are to beobtained from experimental fatigue crack growth data.

III. Influence of Stress ratio on FCG

Most load bearing components and structures experience an alternating load which comprises a mean load viz. dead load during their service application. The mean stress effect on fatigue crack growth rate is commonly introduced into fatigue crack growth relation through the stress ratio *R.*

A. Crack Driving Force

In order to check the performance of an FCG model in its effectiveness to account for stress ratio effects on FCG one has to calculate corresponding crack driving force, D for the FCG model and plot it against FCGr. Various FCG models with their respective crack driving force are given in Table 1. The crack driving force, D which accounts the mean stress effect should collapse all the experimental data for different stress ratio, R into one curve when it is plotted against fatigue crack growth rate, FCGr. The ability of crack driving force to collapse FCG data onto a single fatigue is criteria for selection of it as basis for the proposed model.

To estimate the performance of the Forman FCG expression, Eq.(2) in its ability to account for stress ratio effect one has to calculate the new parameter, Forman's parameter, F and plot it against the applied stress intensity factor range, ΔK_{appl} . Forman's parameter, F is expressed in the form of:

$$F = \frac{da}{dN}\left[(1 - R)K_c - \Delta K_{appl}\right] \quad (10)$$

The experimental FCG data for Al 2024 T351 material obtained at six different stress ratios, R is taken from [17]. Al 2024 T351 is used for aircraft fittings, gears and shafts, bolts, and various other structures because of its good machinability and surface finish capabilities. The physical properties of Al 2024 T351 for 0.2% proof stress is 379 MPa, the tensile strength is 480 MPa, elongation is 19.6% and reduction of area is 17.0%. The fracture properties for Al 2024 T351 are obtained from reference [18]. The fatigue crack growth data sets obtained at various stress ratios, R are shown in Fig. 1 as a function of the applied stress intensity range, ΔK_{appl}. All six FCG curve for different stress ratio, R are represented on a single transformed data, i.e. FCGr, *da/dN* versus Crack driving force, D for all above mentioned models (see Fig.2–Fig.4) except for Forman FCG model where the transformed data is represented as Forman's parameter, F versus applied stress intensity range, ΔK_{appl}. None of the models other than Walker model are able to collapse FCG data for negative stress ratio onto a single curve (see Fig.2–Fig.3). Walker model is able to collapse all FCG data even with high negative stress ratio onto a single curve (see Fig.4) because of fitting constant, γ, which is very sensitive to FCG data and has to be found out by hit and trial method. Subsequent investigations are carried out for the case of positive and zero stress ratio as most of the models are unable to collapse FCG data for negative stress ratio appropriately.

The Curve fitting constants for different models are found out using least square regression analysis on transformed data. Non-dimensional forms of the root mean square error (RMSE) which are generally referred as normalized root mean square (NRMSE) are often used to compare RMSE with different units. In this case, Forman has $MPa\sqrt{m} \times mm/cycle$ as unit for RMSE while other models have *mm/cycles* as unit for RMSE.

$$NRMSE = \frac{RMSE}{X_{obs,max} - X_{obs,min}} \quad (11)$$

where, $X_{obs,max}$ and $X_{obs,min}$ are the maximum and minimum of the observed values.

The fitting constants, domain of applicability of the model and measures of goodness of fit for different models in the regions of their applicability are listed in Table 2.

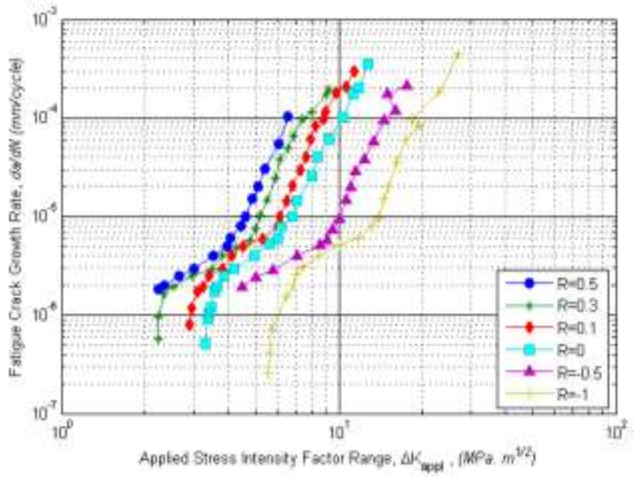

Fig. 1. FCG data for Al 2024 T351 aluminum alloy obtained at stress ratios -1≦R≦0.5

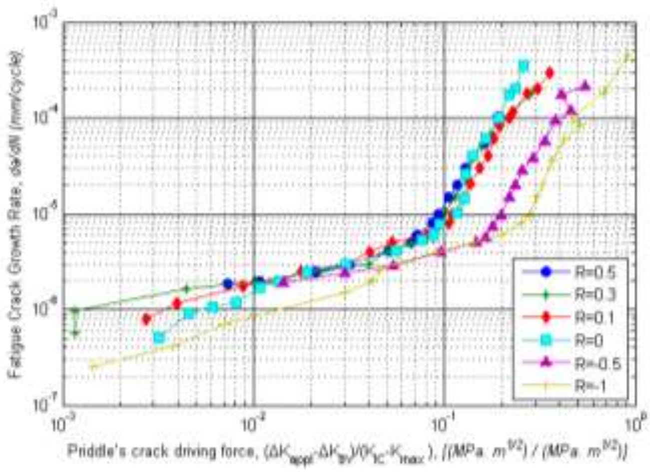

Fig. 2. FCG data in terms of Priddle's crack driving force

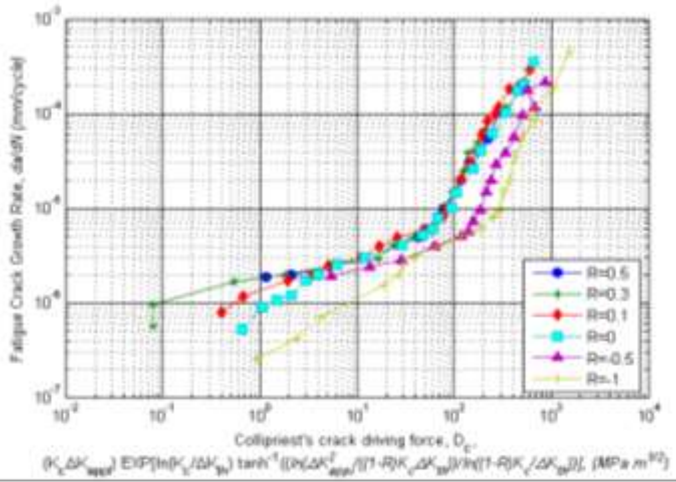

Fig. 3. FCG data in terms of Collipriest's crack driving force

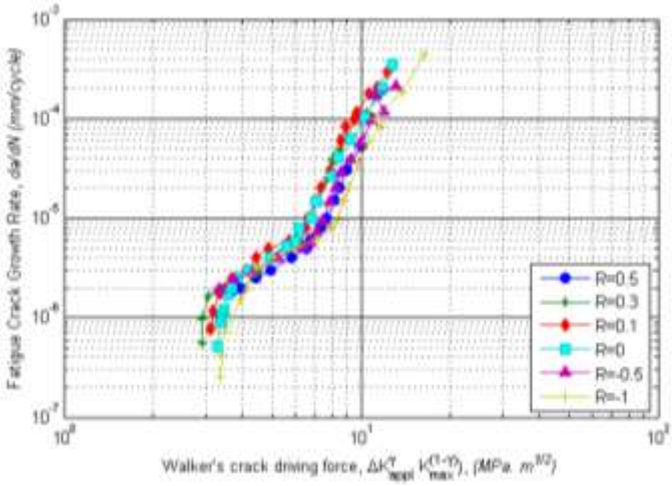

Fig. 4. FCG data in terms of Walker's crack driving force

B. LOWESS Regression fit

The model which collapses all FCG data for different stress ratio to a single curve is an ideal model for depicting stress ratio effect on FCG. As the exact fatigue rate curve is not known and to judge the ability of different fatigue crack growth model to collapse in single curve one need to carry out nonparametric regression analysis. LOESS and LOWESS (locally weighted scatterplot smoothing) are two strongly related nonparametric regression methods used for plotting a smooth curve through a set of data points [19]. When each smoothed value is given by a weighted quadratic least squares regression over the span of values of the y-axis scattergram criterion variable than the curve is called loess curve, while smoothed value is given by a weighted linear least squares regression over the span then the curve is known as a lowess curve.

TABLE I. FCG Models and their respective Crack Driving Force

Model	***Governing Equation***	***Crack Driving Force, D***
Priddle	$\frac{da}{dN}=C\left(\frac{\Delta K_{appl}-\Delta K_{th}}{K_{IC}-K_{max}}\right)^{m}$	$\left(\frac{\Delta K_{appl}-\Delta K_{th}}{K_{IC}-K_{max}}\right)$
Collipriest	$\frac{da}{dN}=C(K_c\Delta K_{appl})^{m/2}EXP\left[ln\left(\frac{K_c}{\Delta K_{th}}\right)^{m/2}\tanh^{-1}\left\{\frac{ln\left(\frac{\Delta K_{appl}^2}{(1-R)K_c\Delta K_{th}}\right)}{ln\left(\frac{(1-R)K_c}{\Delta K_{th}}\right)}\right\}\right]$	$(K_c\Delta K_{appl})EXP\left[ln\left(\frac{K_c}{\Delta K_{th}}\right)\tanh^{-1}\left\{\frac{ln\left(\frac{\Delta K_{appl}^2}{(1-R)K_c\Delta K_{th}}\right)}{ln\left(\frac{(1-R)K_c}{\Delta K_{th}}\right)}\right\}\right]$
McEvily	$\frac{da}{dN}=C(\Delta K_{appl}-\Delta K_{th})^2\left(1+\frac{\Delta K_{appl}}{K_{IC}-K_{max}}\right)$	$(\Delta K_{appl}-\Delta K_{th})^2\left(1+\frac{\Delta K_{appl}}{K_{IC}-K_{max}}\right)$
Weertman	$\frac{da}{dN}=\frac{C\Delta K_{appl}^4}{K_c^2-K_{max}^2}$	$\frac{\Delta K_{appl}^4}{K_c^2-K_{max}^2}$
Broek .	$\frac{da}{dN}=CK_{max}^2\Delta K_{appl}$	$K_{max}^2\Delta K_{appl}$
Walker	$\frac{da}{dN}=C(\Delta K_{appl}^{\gamma}K_{max}^{(1-\gamma)})^m$	$(\Delta K_{appl}^{\gamma}K_{max}^{(1-\gamma)})$

TABLE II FCG Models and their fitting constants with respective errors for Al 2024 T351

FCG Model	Number of Fitting Constants	Fitting Constants	Applicable in Fatigue Rate Curve Region	Measures of goodness of fit		
				R-Square	RMSE	NRMSE
Priddle	2	$C=2.606\times10^{-4}$; m=1.102;	I, II & III	0.8001	5.8025×10^{-05}	0.1633
Collipriest	2	$C=8.629\times10^{-7}$; m=0.3579;	I, II & III	0.8327	5.3771×10^{-05}	0.1513
McEvily	1	$C=1.811\times10^{-6}$	I, II & III	0.7374	2.4694×10^{-05}	0.0695
Weertman	1	$C=1.968\times10^{-5}$	II & III	0.7517	3.9398×10^{-05}	0.1109
Broek .	1	$C=5.117\times10^{-8}$	II	0.6463	2.0365×10^{-05}	0.0573
Walker	3	$C=1.361\times10^{-10}$; m=5.857 ; γ=0.73	II	0.8125	1.5830×10^{-05}	0.0445
Forman	2	$C=2.864\times10^{-7}$; m=4.067	II & III	0.8984	5.5657×10^{-04}	0.0647

TABLE III Statistical analysis results of various crack driving forces with a LOWESS fit

CDF	SSE	RMSE	MAE	Error	R-Square
Priddle's	4.4419×10^{-08}	2.4500×10^{-05}	8.7668×10^{-06}	4.4321×10^{-05}	0.8844
Collipriest's	1.4792×10^{-08}	1.4138×10^{-05}	6.0226×10^{-06}	2.3968×10^{-05}	0.9615
McEvily's	6.5380×10^{-08}	2.9525×10^{-05}	1.3236×10^{-05}	1.4275×10^{-04}	0.8300
Weertman's	5.4200×10^{-08}	2.7064×10^{-05}	1.4010×10^{-05}	7.9835×10^{-05}	0.8591
Broek's	1.9839×10^{-08}	1.6157×10^{-05}	7.0530×10^{-06}	3.9218×10^{-05}	0.9486
Walker's	3.0193×10^{-08}	2.0200×10^{-05}	8.6008×10^{-06}	6.1580×10^{-05}	0.9261
Forman's	5.5672×10^{-07}	1.8122×10^{-05}	4.600×10^{-02}	6.5864×10^{-05}	0.9310

C. Comparison among different Crack Driving Forces

Lowess curve is fitted in transformed FCG data for all FCG models (see Fig 5). Table 3 shows various measures of goodness of fit for lowess curve with respect to experimental FCG data. The crack driving force with best measures for goodness of fit would be a right choice for handling stress ratio effect in fatigue crack growth phenomena. Error measures of goodness of fit for Forman Model are obtained after multiplying it with normalization factor, NF which is ratio of range of fatigue growth rate, FCGr and Range of Forman's parameter, F.

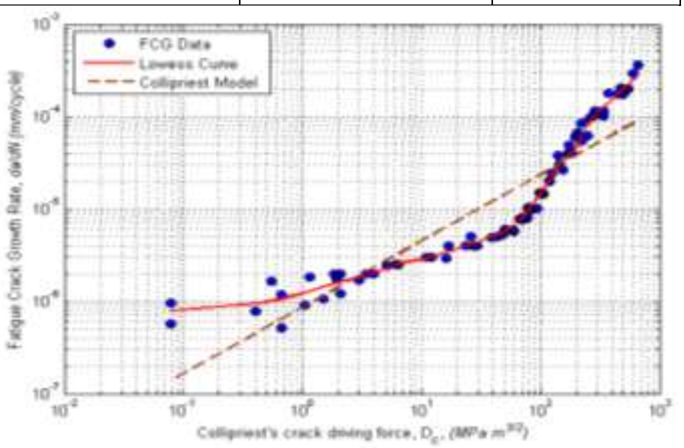

Fig. 5. Lowess curve fit on FCG data in terms of Collipriest crack driving force for Al 2024 T351

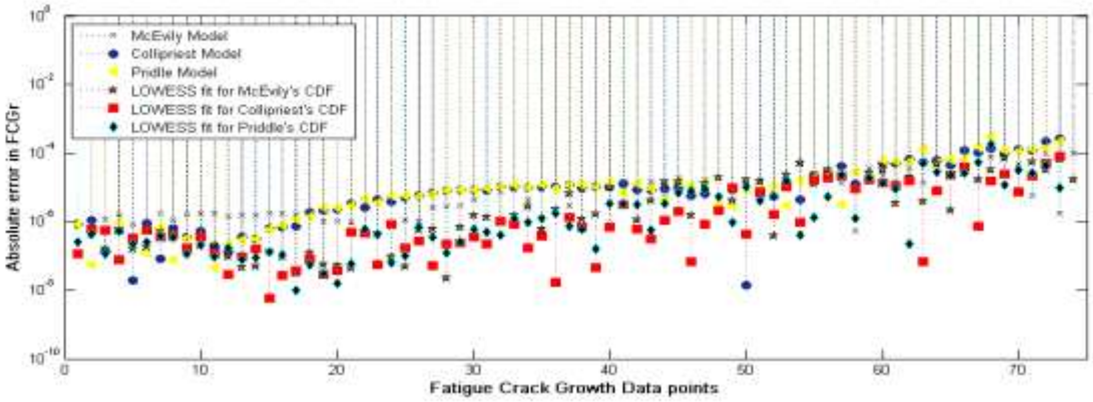

Fig. 6. Comparison among different Crack Growth Models and their CDF

IV. Summary and Conclusions

Performance of the crack driving force of various FCG models in accounting stress ratio effects in fatigue crack growth rate is evaluated by fitting a lowess curve on transformed FCG data. Experimental fatigue crack growth data of a typical Al 2024 T351 obtained under constant amplitude loading tests for six load ratios has been used in the present work. From the studies, it is observed that Walker and Collipriest models are found to be in good agreement with the experimental FCG data. It can be noted that Collipriest model gives highest value of R^2 among models which are applicable in all three regions of fatigue rate curve viz. Priddle and McEvily model.

Further, it has been observed that Walker model is applicable for stress ratio -2 <R<1. The curve fitting constant γ in Walker model is determined by hit and trial, its value is the one that best consolidates the FCG data. Collipriest's crack driving force, D_c, gives highest R^2 and least value for error measures with respect to lowess curve. It also has ability to collapse FCG data in all three regions of fatigue rate curve onto a curve very less scatter. From stem plot of crack growth models and their CDF (see Fig. 6), it is evident that LOWESS fit for Collipriest CDF gives least error among all models and LOWESS fit of their CFD.

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