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A comparison of metaheuristics in structural optimization

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***Abstract*— In the recent decades metaheuristic search techniques have been widely employed for developing structural design optimization algorithms. Amongst these techniques are genetic algorithms, simulated annealing, evolution strategies, particle swarm optimization, tabu search, ant colony optimization, harmony search, big bang-big crunch, and bat-inspired search. The main concern of the study is to evaluate the performance of aforementioned nine techniques in discrete sizing optimization of structural systems. The optimization algorithms, which are implemented in an unbiased coding platform, are evaluated and compared in terms of their solution accuracies and reliabilities using a real size structural design instance. Here, the design criteria imposed by AISC-ASD (Allowable Stress Design Code of American Institute of Steel Construction) are considered in the course of optimization. The study provides general guidelines about the efficiency of investigated algorithms in practical structural optimization applications.**

***Keywords*—structural optimization, discrete optimization, optimum sizing, metaheuristics**

I. Introduction

The optimization has always been an inseparable component of structural design. This fact has resulted in development of numerous optimization techniques in the past few decades to achieve robust and reliable design tools for dealing with complicated structural optimization problems. One main category of structural optimization problems is referred to as optimum design of steel skeletal structures.

Despite the fact that numerous optimization techniques in the literature have been implemented until now to optimize steel structures, the research on the subject matter is still on the rise, where the ongoing studies are mostly centered around finding algorithms ideal for practical applications. In general, the optimum design of a steel skeletal structure is an attempt to find the best combination of design variables that results in a minimum weight or cost design of the structure. Meanwhile, for practical applications the optimum design should satisfy a set of predefined constraints imposed according to a given code of practice.

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Mathematical programming [1] and optimality criteria [2] techniques that have found a vast amount of applications in the past are now conceived as traditional techniques of structural optimization. The drawbacks of these techniques (such as their gradient based formulations and inefficiency in handling discrete design variables) have led to an increasing tendency towards non-traditional stochastic search algorithms or the so-called metaheuristics. Typically, metaheuristic techniques borrow their working principles from natural phenomena [3], and follow non-deterministic search strategies in locating the optimum solutions. A sound reputation of metaheuristics in structural optimization can be attributed to their competitive solutions, robust performances, ease of implementation/use, independency to gradient information, and capability of handling both continuous and discrete design variables. The state-of-the-art reviews of metaheuristic algorithms considering their applications in structural optimization instances are outlined in [4, 5].

The present study covers the performance evaluation of nine metaheuristic search techniques; namely simulated annealing (SA) [6] , evolution strategies (ESs) [7], particle swarm optimization (PSO) [8], tabu search method (TS) [9], ant colony optimization (ACO) [10], harmony search method (HS) [11], genetic algorithm (GA) [12], big bang-big crunch (BB-BC) [13], and bat-inspired search (BI) [14] in structural design optimization. In the paper the discrete sizing optimization problem is formulated, where design limitations are imposed according to AISC-ASD [15]. A 354-bar steel braced dome is considered as a practical design optimization instance, where the structural members are sized for minimum weight using 37 standard circular hollow sections. Through conducting three independent runs with each of the abovementioned nine optimization techniques efficiency of the techniques in locating the optimum solution are compared, and the obtained results are discussed in details.

II. Statement of the design optimization problem

Typically in practical design optimization of truss structures the aim is to find a minimum cost or weight design by selecting the cross-sectional areas of structural members from a table of available sections such that the final design satisfies strength and serviceability requirements determined by technical standards. For a given truss structure composed of N_m members grouped into N_d sizing design variables, the optimization problem can be formulated as follows. The objective is to find a vector of integer values **I** (Eq. 1) representing the sequence numbers of standard sections in a given section table,

$$\mathbf{I}^T = [I_1, I_2, ..., I_{N_d}] \quad (1)$$

to generate a vector of cross-sectional areas **A** (Eq. 2) for N_m members of the structure,

$$\mathbf{A}^T = [A_1, A_2, ..., A_{N_m}] \quad (2)$$

such that **A** minimizes the following weight objective function:

$$W = \sum_{m=1}^{N_m} \rho_m L_m A_m \quad (3)$$

where W is the weight of the structure, ρ_m, L_m, A_m are unit weight, length, and cross-sectional area of the *m-th* member, respectively. The design constraints consist of the limitations imposed on overall structural response and behavior of individual members according to AISC-ASD [15].

III. Metaheuristic search techniques: an overview

Metaheuristic algorithms are stochastic search techniques capable of providing acceptable solutions for complicated optimization problems in a computationally acceptable time. In general, these techniques employ particular metaheuristic search procedures developed based on a natural phenomenon. In the recent decades there has been a great tendency towards practical application of metaheuristics in the field of structural optimization. A short overview of abovementioned metaheuristics is provided here as outlined in [16].

SA optimization algorithm which is a well-known variant of metaheuristic search techniques, seeks for minimum energy states through an analogy based upon the physical annealing process. In this process, a solid initially at a high energy level is cooled down gradually to reach its minimum energy and thus to regain proper crystal structure with perfect lattices. The idea that this process can be simulated to solve optimization problems was pioneered independently by Kirkpatrick et al. [6] and Cerny [17], establishing a direct analogy between minimizing the energy level of a physical system and lowering the cost of an objective function. Successful applications of SA in discrete structural optimization problems have been reported in a number of early works in the literature, such as Refs. [18-20]. The enhancement of the technique is accomplished in some recent publications, such as Refs. [21, 22] for enhancing its search capability in complex design applications.

ESs are another promising representative of evolutionary algorithms. The fundamentals of the technique were originally laid in the pioneering studies of Rechenberg [23]. They were first developed in a rather simple form known as $(1+1)-\mathrm{ES}$ that implements on the basis of two designs; a parent and an offspring individual. Today, the modern variants of ESs are accepted as $(\mu+\lambda)-\mathrm{ES}$ and $(\mu,\lambda)-\mathrm{ES}$, which were developed by Schwefel [24]. Both variants employ design populations consisting of μ parent and λ offspring individuals, and are intended to carry out a self-adaptive search in continuous design spaces. The extensions of these variants to solve discrete optimization problems were put forward in Refs. [25-27]. A literature survey turns up several publications reporting a very successful use of this method in discrete optimum design of structural systems [28, 29].

PSO is a population based metaheuristic search technique inspired by social behavior of bird flocking or fish schooling. This behavior is concerned with grouping by social forces that depend on both the memory of each individual as well as the knowledge gained by the swarm [8, 30]. The procedure involves a number of particles which represent the swarm being initialized randomly in the search space of an objective function. Each particle in the swarm represents a candidate solution of the optimum design problem. The particles fly through the search space and their positions are updated using the current position, a velocity vector and a time step. The successful applications of this technique have also been reported in the field of structural optimization, especially in size/shape optimum design of skeletal structures. Amongst some recent applications are Perez and Behdinan [31], He et al. [32] and Fourie and Groenwold [33].

TS technique is another metaheuristic method, which was first developed by Glover [9]. The method implements a simple yet an efficient iterative based local search strategy for solving combinatorial optimization problems. At each step a number of candidate solutions are sampled in the close vicinity of the current design by perturbing a single design variable called a move. The best candidate is chosen and replaced with the current design even if it offers a non-improving solution, and the move leading to this candidate is recognized as a successful move. To protect the search against cycling within the same subset of solutions, information regarding most recently visited solutions is collected in a list referred to as tabu list. A candidate is allowed to replace the current design provided that its move is not in tabu list; otherwise the search is preceded with the current solution. The method has been mostly employed for weight minimization of structural systems in the literature, such as Bland [34].

ACO algorithm is inspired from the way that ant colonies find the shortest route between the food source and their nest. Ants being completely blind individuals can successfully discover as a colony the shortest path between their nest and the food source. They manage this through their characteristic of employing a volatile substance called pheromone. When finding food, the ants deposit pheromones on the ground while traveling, which is used by other ants in the colony as a guide to find the food sources. Ant colony optimization was developed by Colorni et al. [10] and Dorigo [35] and used in the solution of traveling salesman problem. The optimum structural design applications of the technique have been presented in Camp et al. [36], Aydoğdu and Saka [37].

HS method is based on natural musical performance processes that occur when a musician searches for a better state of harmony. The resemblance, for example between jazz improvisation that seeks to find musically pleasing harmony and the optimization is that the optimum design process seeks to find the optimum solution as determined by the objective function. The pitch of each musical instrument determines the aesthetic quality just as the objective function is determined by the set of values assigned to each design variable. The recent applications of HS algorithm in structural optimization reveal that it is a very powerful technique for relatively small-to-medium scale discrete optimization problems [38, 39]. An enhancement of the technique is proposed in Hasançebi et al. [40] for larger scale problems, where an adaptive change of its parameters is facilitated for establishing the most advantageous search automatically by the algorithm.

The most well known stream of evolutionary algorithms is GA, which have been initially pioneered by Holland [41]. These algorithms are based on the evolutionary ideas of natural selection and genetics mechanism. The first application of the technique in optimum structural design is presented by Goldberg and Samtani [42], where the weight minimization of the classical 10-bar truss is accomplished with GAs. Today, many variations and extensions of the technique have been proposed, and successful applications of the technique are available in a vast amount of discrete and continuous optimization literature [43-45]. In the present study, a genetic algorithm with standard components referred to as simple genetic algorithm (SGA) is implemented due to its generality and wide acceptability.

BB-BC optimization method has first appeared in Erol and Eksin's study [13]. It is emerged from the big bang and big crunch theories of the universe evolution. As its name implies, the method is based on the continuous application of two successive stages, namely big bang and big crunch phases. During big bang phase, new solution candidates are randomly generated around a point called center of mass. This point is recalculated and updated every time in the big crunch phase with respect to the solution candidates generated. The first application of the BB-BC for optimum design of skeletal structures was carried out by Camp [46]. There are many applications of the BB-BC method in the field of structural optimization. A recent performance evaluation of the BB-BC algorithm was carried out by Kazemzadeh Azad et al. [47], where efficiency of the method in benchmark engineering optimization problems is investigated.

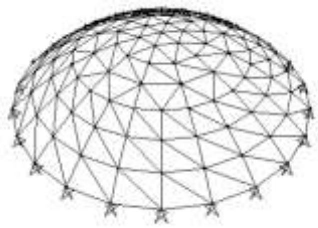
One latest addition to metaheuristic algorithms is the BI search, which was recently proposed in Yang [14]. The bath-inspired search makes use of echolocation behavior of bats in searching for the optimum. The efficiency of the BI algorithm was validated and compared with other existing algorithms using some single and multi-objective standard functions of unconstrained optimization in Yang [14] and Yang [48], respectively. Besides, the performance of the technique in benchmark problems of constrained engineering optimization was investigated in Yang and Gandomi [49] and Gandomi et al.[50]. The results obtained in these studies have clearly documented the superiority of the bat-inspired search over other techniques. Recently, the method is refined and adapted for structural optimization in Hasançebi et al. [51].

It should be underlined that there is no a unique formulation or a standardized algorithm used to implement any of the metaheuristic search techniques mentioned above. Rather, each technique has been devised in various algorithmic forms and has numerous extensions and modifications. In this study the algorithms to implement the techniques are selected on the basis of their generalities and reported performances in the published literature. The implementation specifics and detailed outlines of these algorithms can be found in Hasançebi et al. [51, 52] with complete parameter settings and enhancements proposed to accelerate their performances. The performance evaluation of the considered optimization techniques is carried out in the following section.

IV. Design example

The design example considered in this study is a 354-bar braced dome truss (Figure 1) with a 40 m (131.23 ft) diameter designed for covering the top of an auditorium at an elevation of 10 m (32.8 ft). The structure has a height of 8.28 m (27.17 ft), and consists of 127 joints and 354 members. The 354 members are grouped into 22 independent sizing variables (Figure 1), which are selected from the entire set of 37 standard circular hollow sections. The dome is subjected to the following three load cases considering various combinations of dead (D), snow (S) and wind (W) loads computed according to the specifications of ASCE (1998): (i) D + S, (ii) D + S +W (with negative internal pressure), and (iii) D + S +W (with positive internal pressure). It is important to note that the load cases resulting from unbalanced snow loads are disregarded in the study to avoid excessive computational burden. The complete details of load cases and related calculations can be found in Hasançebi et al. [52]. The stress and stability constraints of the members are computed according to the specifications of AISC-ASD [15]. The displacements of all nodes are limited to 11.1 cm (4.37 in.) in any direction.

The minimum weights obtained by each technique for 354-member braced dome is given in Table 1. For this structure, SA, ESs, PSO, improved BB-BC, and BI techniques give the least weight, which is 32574.9 lb.



(a)

(b)

(c)

Figure 1. 354-member braced truss dome (a) 3-D view (b) top view (c) side view

TABLE I. COMPARISON OF SOLUTIONS FOR 354-BAR BRACED DOME

Sizing variables	Optimal cross sectional areas (in^2)									
	SA	ESs	PSO	ACO	TS	HS	SGA	BB-BC		BI
								Improved	Standard	
1	1.07	1.07	1.07	1.07	1.07	1.07	1.48	1.07	1.07	1.07
2	3.17	3.17	3.17	3.17	3.17	3.17	2.68	3.17	3.17	3.17
3	2.23	2.23	2.23	2.23	2.68	2.23	4.3	2.23	2.68	2.23
4	2.68	2.68	2.68	2.68	2.68	2.68	2.68	2.68	4.3	2.68
5	2.23	2.23	2.23	2.23	2.68	2.23	2.23	2.23	2.23	2.23
6	2.23	2.23	2.23	2.23	2.23	2.25	2.23	2.23	2.68	2.23
7	2.23	2.23	2.23	2.23	2.68	2.23	2.23	2.23	2.68	2.23
8	2.23	2.23	2.23	2.23	2.68	2.68	2.23	2.23	4.3	2.23
9	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7	1.7
10	2.23	2.23	2.23	2.23	2.23	2.25	2.25	2.23	2.68	2.23
11	1.7	1.7	1.7	2.66	1.7	2.25	2.66	1.7	1.7	1.7
12	1.7	1.7	1.7	2.23	1.7	1.7	2.23	1.7	2.68	1.7
13	1.7	1.7	1.7	1.7	1.7	1.7	2.23	1.7	2.68	1.7
14	1.7	1.7	1.7	1.7	1.7	2.23	1.7	1.7	1.7	1.7
15	1.7	1.7	1.7	1.7	1.7	1.7	2.25	1.7	1.7	1.7
16	1.7	1.7	1.7	1.7	2.68	1.7	1.7	1.7	2.68	1.7
17	1.48	1.48	1.48	1.48	1.7	2.66	1.7	1.48	1.48	1.48
18	1.48	1.48	1.48	2.68	1.48	3.02	3.17	1.48	1.48	1.48
19	1.07	1.07	1.07	1.07	1.07	1.7	1.48	1.07	2.68	1.07
20	1.07	1.07	1.07	1.07	1.07	1.7	1.48	1.07	2.68	1.07
21	1.07	1.07	1.07	1.07	1.07	1.7	1.48	1.07	1.48	1.07
22	1.07	1.07	1.07	1.48	1.07	1.07	1.7	1.07	1.48	1.07
Weight, lb (kg)	32574.9 (14775.7)	32574.9 (14775.7)	32574.9 (14775.7)	33557.5 (15221.4)	35370.1 (16043.6)	34944.3 (15850.5)	36343.3 (16485)	32574.9 (14775.7)	41413.5 (18784.8)	32574.9 (14775.7)

This design is considered to be the optimum solution of the problem. However, relatively higher design weights have been attained for the structure with other metaheuristic algorithms; namely 33557.5 lb by ACO, 34944.3 lb by HS, 35370.1 lb by TS, 36343.3 lb by SGA, and 41413.5 lb by standard BB-BC algorithm. Regarding the attained results, SA, ESs, PSO, improved BB-BC, and BI techniques can be considered as the most successful algorithms in locating the optimum. However, the standard BB-BC algorithm demonstrates a poor performance, and is not a suitable technique for tackling discrete sizing optimization instances. Relatively better performances are observed using SGA, TS, HS and ACO algorithms, however, none of these techniques were capable of locating the optimum solution.

V. Conclusion

Metaheuristic based design optimization is quite sensitive to a large set of issues that result from decisions and assumptions made when an optimization model is established for a problem. Amongst these parameters are (i) the number and nature of design variables used, (ii) discrete sets for design variables, (iii) the choice of starting solutions, (iv) the number of design cycles, (v) the method of handling infeasible solutions, and (vi) the number of independent runs performed. An objective performance evaluation of stochastic optimization techniques requires that all these parameters are kept identical from one method to another. This fact is observed in the application of the nine different metaheuristic techniques considered in this study. Regarding the obtained results it can be deduced that amongst all the metaheuristic search techniques investigated here, SA, ESs, PSO, improved BB-BC, and BI techniques are the most powerful ones for structural optimization applications. However, the standard BB-BC algorithm demonstrates a poor performance, and was not capable of locating a reasonable solution for the investigated discrete sizing optimization instance. Relatively better performances are observed using SGA, TS, HS and ACO algorithms, however, none of these techniques were capable of locating the optimum solution. It is worth mentioning that all the investigated optimization algorithms are indeed problem dependent, and no strict conclusion can be reached with regard to their performances using only a single design instance. However, the results provide some level of general guidelines for users of the metaheuristic algorithms.

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