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Numerical Modeling of Wave Propagation using RBF-based Meshless Method

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***Abstract*— Radial basis functions (RBFs) are the functions whose values depend only on the distance from the point called center. These functions are employing in the approximation theory and they can be utilized in the point interpolation method as a type of meshless approach for constructing the shape function. This paper discusses on the results of RBF-based meshless numerical modeling of wave propagation for additional mass detection in isotropic rod which it can be assumed as damage. Number of nodes, position and magnitude of additional mass and the variation of multiauadric RBF parameters have been studied in this paper.**

***Keywords*— Radial basis function, Radial point interpolation, Meshless method, Wave propagation, Damage detection**

I. *Introduction*

In the field of structural health monitoring, since the presence of crack or other small size defects leads to low changes in the modal data, therefore the employing modal-based methods for this type of damage assessment are not appropriate. Furthermore, these methods are suitable for discrete lumped-parameter systems. Among various non-destructive techniques, elastic wave-based methods are effective approach which can be overcome to the restrictions of traditional methods. These methods are based on the well-known fact that the traveling wave with high frequency content scatters from defect and discontinuity in solids.

A variety of numerical techniques have been applied in order to modeling the wave propagation and scattering in the time or frequency domain. Most frequent method is conventional FEM. Conventional mesh-based methods are not well suited to treat problems with strong inhomogeneity, large deformations, mesh distortion and discontinuities that do not align with element edges. The existence of meshes may cause strong mesh dependency of the calculation for dynamic problems. One strategy for dealing with moving discontinuities in mesh-based methods is re-meshing or discontinuous enrichment. However, re-meshing is costly, still difficult in three dimensions and requires projection of quantities between successive meshes and leads to largely decreasing of efficiency and introducing large errors into the calculation [1].

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In the last decades, meshless methods have been developed for solving differential equations as an alternative to the mesh-based methods which do not pose the mesh-related difficulties. Many meshless methods were proposed in [2-4]. Wen [5] utilized a meshless local Petrov–Galerkin (MLPG) method which is applied to solve wave propagation problems of three-dimensional poroelastic solids. Gao et al [6] proposed a new MLPG Method to analyze stress-wave propagation and dynamic fracture problems in anisotropic and cracked media. Zhang and Batra [7] applied a modification to the smoothed particle hydrodynamics (SPH) method for improvement of the accuracy of the approximation especially at points near the boundary of the domain. Also they [8] used the modified smoothed particle hydrodynamics (MSPH) method to study the propagation of elastic waves in functionally graded materials. Li et al [9] formulates a meshless method based on the RBF collocation technique and generalized trapezoidal method for the numerical simulation of the wave propagation problems.

The point interpolation method (PIM) [10, 11] is a meshless method can be categorized as a series representation interpolation. PIM employs Galerkin weak form and shape functions that are constructed based on nodes distributed in the support domain. A background cell is required to evaluate the integration in the Galerkin weak-form. The major advantage of PIM is that the shape functions created possess the Kronecker delta function property, which allows simple enforcement of essential boundary conditions. There are two types of PIM shape functions with different forms of basis functions: polynomial basis functions and radial basis functions (RBFs) [12, 13]. RBFs are the functions whose values depend only on the distance from the point called center. In order to avoid the singularity problem in the polynomial PIM, RBFs are used to develop the radial point interpolation method (RPIM) shape functions for weak-form methods. It has been proved that the moment matrix of the RBF interpolation is usually invertible for arbitrary scattered nodes. The RPIM has been successfully applied to 1-, 2- and 3D solid mechanics [12, 14], plate structures [15], geometrically nonlinear problems [16] and material non-linear problems [17].

In this paper, the numerical modeling of wave propagation are investigated based on RPIM in rods and the capability of this method for detection of additional mass which is located in the length of rod are studied. Multiquadric type of RBF in this study is considered as basis function and the effects of its parameters are investigated and the results are presented in detail.

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II. Basic Formulation

A. Radial Point Interpolation Method

Consider a continuous function $u(\mathbf{x})$ (i.e. displacement) in a domain Ω. $u(\mathbf{x})$ can be approximated in an influence domain that has a set of arbitrarily distributed nodes using radial basis function $R_i(x)$ as follows:

$$u(\mathbf{x}) = \sum_{i=1}^{n} R_i(\mathbf{x}) a_i = \mathbf{R}^T(x)\mathbf{a} \quad (1)$$

Where n is the number of nodes in the influence domain of $\mathbf{x} = [x, y, z]^T$. a_i is the corresponding coefficient of the basis functions and can be determined by enforcing $u(\mathbf{x})$ to be the nodal displacement at n nodes in the influence domain:

$$\mathbf{d}_s = \mathbf{R}_Q \mathbf{a} \quad (2)$$

Where ds is the vector of nodal displacements:

$$\mathbf{d}_s = \{u_1 \;\; u_2 \;\; u_3 \;\; \ldots u_n\}^T \quad (3)$$

a is the vector of unknown coefficients:

$$\mathbf{a} = \{a_1 \;\; a_2 \;\; a_3 \;\; \ldots a_n\}^T \quad (4)$$

and RQ is the moment matrix of RBF:

$$\mathbf{R}_Q = \begin{bmatrix} R_1(r_1) & R_2(r_1) & \cdots & R_n(r_1) \\ R_1(r_2) & R_2(r_2) & \cdots & R_n(r_2) \\ \vdots & \vdots & \ddots & \vdots \\ R_1(r_n) & R_2(r_n) & \cdots & R_n(r_n) \end{bmatrix} \quad (5)$$

since in the radial basis function, the variable is only the distance between the point of interest (x, y, z) and a node at (x_k, y_k, z_k):

$$r = [(x - x_k)^2 + (y - y_k)^2 + (z - z_k)^2]^{1/2} \quad (6)$$

Because the distance is directionless:

$$R_i(r_j) = R_j(r_i) \quad (7)$$

Therefore the moment matrix RQ is always symmetric and invertible. Then vector of unknown coefficients a can be obtained as:

$$\mathbf{a} = \mathbf{R}_Q^{-1} \mathbf{d}_s \quad (8)$$

Substituting (8) into (1) yields:

$$u(\mathbf{x}) = \mathbf{R}^\mathbf{T}(\mathbf{x}) \mathbf{R}_Q^{-1} \mathbf{d}_s = \varphi(x) \mathbf{d}_s \quad (9)$$

$$\boldsymbol{\varphi}(\mathbf{x}) = \{\phi_1(\mathbf{x}) \;\; \phi_2(\mathbf{x}) \;\; \ldots \;\; \phi_n(\mathbf{x})\} \quad (10)$$

$\varphi(x)$ is the vector of RPIM shape functions. Constructed RPIM shape functions are, in general, incompatible, not consistent, and they have the Kronecker delta function property which allows essential boundary conditions to be easily treated in the same way as in the standard FEM. There are different RBFs such as multi-quadratic, Gaussian, and Logarithmic. In this paper the multi-quadratic form has been used as follows:

$$R_i(\mathbf{x}) = [r_i + (\alpha_c d_c)^2]^q \qquad \alpha_c \geq 0 \quad (11)$$

α_c and q in (11) are shape parameters and their values depend on the problems and d_c is the characteristic length that is usually the average nodal spacing for all the n nodes in the influence domain.

B. Wave propagation problems

A wave propagation modeling problem is well-known ordinary differential equations, which can be written in a matrix form:

$$\mathbf{M}\ddot{\mathbf{U}} + \mathbf{C}\dot{\mathbf{U}} + \mathbf{K}\mathbf{U} = \mathbf{F} \quad (12)$$

where M is the global mass matrix, K is the global stiffness matrix, and F is a vector of the time dependent excitation signal with the following components expressed as:

$$\mathbf{k}_{ij} = \int_\Omega \mathrm{B}_i^T \mathrm{E} \mathrm{B}_j d\Omega \quad (13)$$

$$\mathbf{m}_{ij} = \int_\Omega \phi_i^T \rho \phi_j d\Omega \quad (14)$$

$$\mathbf{f}_i = \int_\Omega \phi_i^T \mathbf{b} d\Omega + \int_\Gamma \phi_i^T \mathbf{T} d\Gamma \quad (15)$$

In above expressions, E, ρ, b and T are material matrix, mass density, body force density and prescribed surface traction, respectively. Γ is the boundary along which the surface traction is imposed and B is the strain matrix as the following in one-dimensional problems:

$$B_i = [\phi_{i,x}] \quad (16)$$

$C = \alpha M + \beta K$ is the global damping matrix with damping parameter α, β.

For the numerical integrations of (13-15), a background mesh is necessary which is independent of nodes for interpolations. Gauss-quadrature is used for this numerical integration in this paper.

C. *Time integration scheme*

For solving (12), the central difference time integration scheme is used. Zero initial conditions, $U = 0$ and $\dot{U} = 0$ at $t = 0$ are assumed to be implemented as initial displacement and velocity in the central difference time integration scheme [2]:

$$\left(\frac{1}{\Delta t^2}\mathbf{M} + \frac{1}{2\Delta t}\mathbf{C}\right)\mathbf{U}_{t+\Delta t} = \mathbf{F}_t - \left(\mathbf{K} - \frac{2}{\Delta t^2}\mathbf{M}\right)\mathbf{U}_t - \left(\frac{1}{\Delta t^2}\mathbf{M} - \frac{1}{2\Delta t}\mathbf{C}\right)\mathbf{U}_{t-\Delta t} \quad (17)$$

Where t and Δt denotes time and time step of integration. The central difference time integration scheme is stable if $\Delta t \le \Delta t_{cr} = 2/\omega_{\max}$, where $\omega_{\max}$ is the largest frequency of the n degrees of freedom system.

III. Numerical analysis

A. *Characteristics of model*

In this study, a rod with an additional concentrated mass has been considered as a one-dimensional model for investigation of the wave propagation based on RPIM as presented in Fig. 1. The capability of this approach and the effects of each parameter in multiquadric RBF have been studied to detection of additional mass. It has been assumed that the material is homogeneous and isotropic.

The rod has the cross-section 0.0025 m2, Young's modulus 200GPa and the mass density 7850 kg/m3. Damping matrix is applied as $C = \alpha M + \beta K$ such a way $\alpha, \beta = 0.001$. The length of the rod is 0.5m and is denoted by L and the position of additional mass is denoted by l from the left end. A Hanning-windowed sinusoidal wave with the frequency of 2 kHz applies at the free end as an excitation signal which is shown in Fig. 2 in the time domain.

The effects of number of nodes (*Nnodes*), position ($\theta = l/L$) and magnitude of additional mass (M) and the variation of multiauadric parameters (α_c, q) are parameters that are investigated in this study.

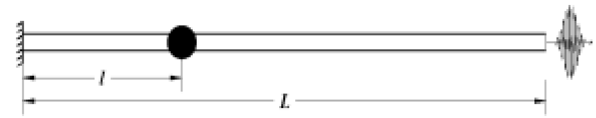


Figure 1. A rod with an additional mass

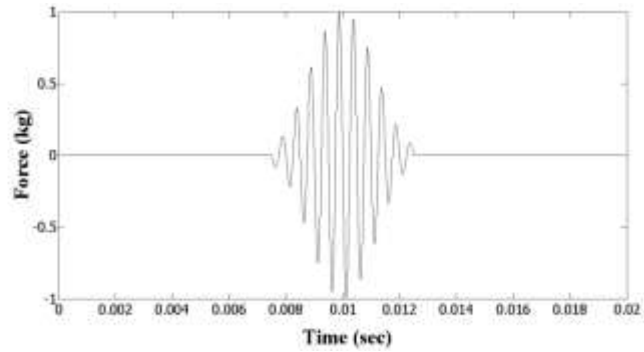


Figure 2. An excitation signal in time domain

B. *Study on number of nodes*

Since wave propagation is a multi-modal phenomenon involving vibrational high frequencies therefore considering high degrees of freedom is necessary. In present research, the rod has been modeled in four pattern of number of nodes (Nnodes): 50, 100, 150 and 200. In these pattern other parameters regarded as: $\theta = 0.25$, $M = 30\%$, $\alpha_C = 2$ and $q = 2$. In view of the fact that damage detection is an inverse problem, therefore, θ and M can be regarded as assigned damage. The obtained results in Fig. 3 represent the variation of longitudinal acceleration relative to rod length at $t = 0.015s$. In this figure, the location of mass $\theta = 0.25$ are detected in distance 0.125m. It is obvious $Nnodes = 50$ yield no mass detection however by increasing the Nnodes, location of damage detected precisely.

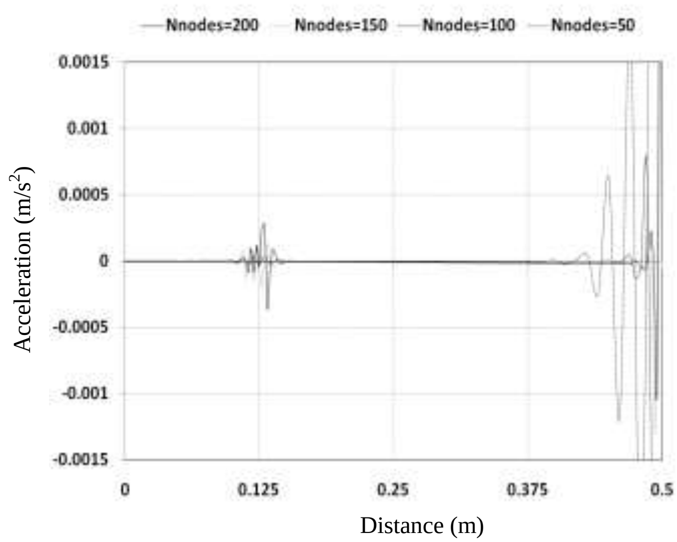
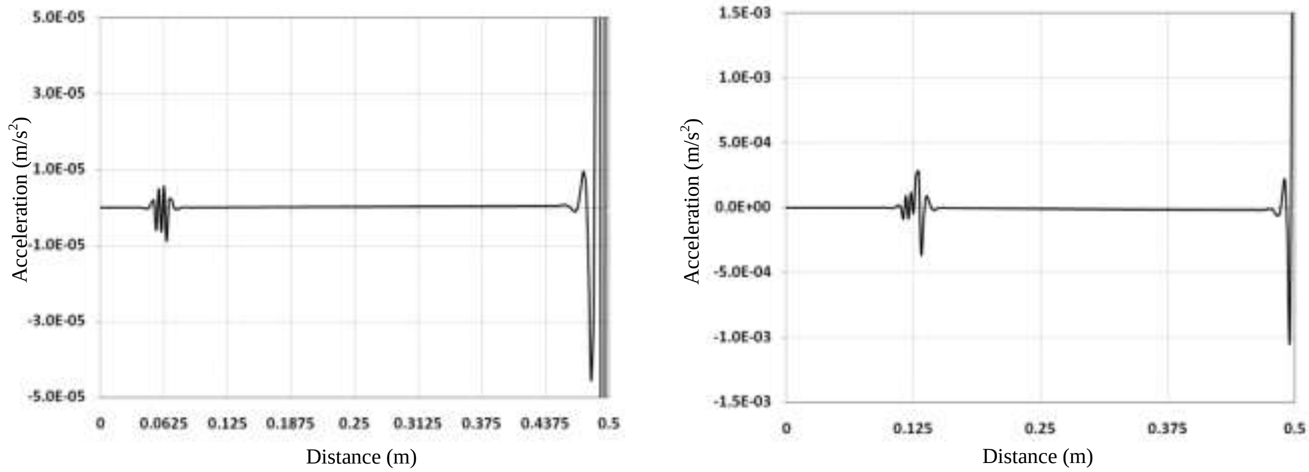


Figure 3. Effect of number of nodes on acceleration at t=0.015 s.



(a) (b)

Figure 4. Effect of number of nodes on acceleration at t = 0.015 s. a) $\theta = 0.125$ b) $\theta = 0.25$

C. *Study on additional mass position*

The influence of the position of additional mass on wave propagation in the rod is obvious in Fig. 4a,b at $t = 0.015s$. These figures represent the satisfactory capability of RPIM in additional mass detection distinctly. Constant parameters are $Nnodes = 200$, $M = 30\%$, $\alpha_C = 2$ and $q = 2$.

D. *Study on magnitude of additional mass*

Determination of damage quantity is one of the main parts in the structural health monitoring and the algorithm of damage assessment should be able to achieve damage quantity. In this paper, results are obtained based on additional mass magnitude equal to 1, 2 and 3% of the total rod mass. Fig. 5 shows the results in the distance 0.1 m because of the clearness of detection. Constant parameters are $Nnodes = 200$, $\theta = 0.1$, $\alpha_C = 2$ and $q = 2$.

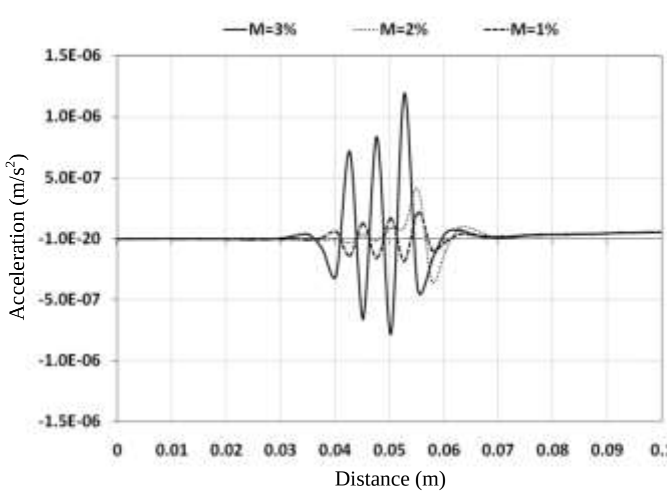


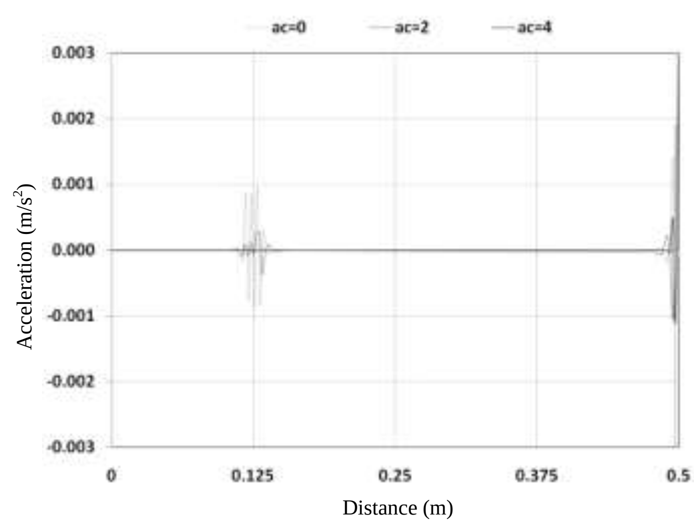
Figure 5. Effect of additional mass magnitude on acceleration at t=0.015 s.

E. *Study on shape parameter α_c, q*

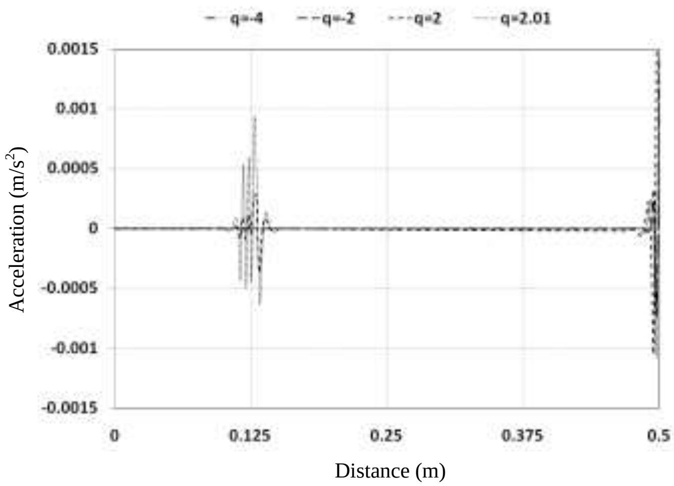
With respect to the presence of various RBFs and their parameters, implementation of appropriate basis function requires supplementary investigation and it cannot be decided definitely based on preliminary results. However, these results can be as primary options.

Fig. 6 represents the effects of shape parameters on the wave propagation. It is concluded that by increasing the value of $\alpha_c \geq 0$, the authority of mass detection decreases such that $\alpha_c = 0$ leads to distinguished detection.

The analysis illustrates increasing of positive values for q causes notable detection but its value cannot be greater than 2.01 because the analysis will be unstable and the interpolation cannot be work satisfactory. The other significant point is that the negative values for q which refer to the inverse multiquadric RBFs are not suitable for mass detection using wave propagation and no variation in wave is visible. It is to be noted $Nnodes = 200$, $\theta = 0.25$, $M = 30\%$.



(a)



(b)

Figure 6. Effect of shape parameters on longitudinal acceleration at t=0.015s. a) α_C b) q

IV. Conclusion

A variety of numerical methods had been proposed for numerical modeling of wave propagation. This paper discussed about meshless modeling of wave propagation for additional mass detection in isotropic rods. RPIM as a RBF-based meshless method represents appropriate application for shape function construction which does not have disadvantages of polynomial basis functions. In this study, multiquadric RBF adopted as a basis function of interpolation. The quantities including number of nodes, position, magnitude of additional mass and the variation of RBF parameters were studied. It has been demonstrated that this method is capable for wave propagation-based damage detection modeling however obtained results show that the selection of suitable RBF parameters requires supplementary investigation.

References

[1] V.P. Nguyen, T. Rabczuk, S. Bordas, M. Duflot, Meshless methods: A review and computer implementation aspects, Mathematics and Computers in Simulation, 79 (2008) 763-813.

[2] Y. Chen, J.D. Lee, A. Eskandarian, Meshless Methods in Solid Mechanics, Springer New York, 2006.

[3] G.R. Liu, Mesh Free Methods Moving Beyond the Finite Element Method, CRC Press, United States of America, 2010.

[4] G.R. Liu, Y.T. Gu, An introduction to meshfree methods and their programming, in, Springer, Netherlands, 2005.

[5] P.H. Wen, Meshless local Petrov–Galerkin (MLPG) method for wave propagation in 3D poroelastic solids, Engineering Analysis with Boundary Elements, 34 (2010) 315-323.

[6] L. Gao, K. Liu, Y. Liu, A meshless method for stress-wave propagation in anisotropic and cracked media, International Journal of Engineering Science, 45 (2007) 601-616.

[7] G.M. Zhang, R.C. Batra, Modified smoothed particle hydrodynamics method and its application to transient problems, Computational Mechanics, 34 (2004).

[8] G.M. Zhang, R.C. Batra, Wave propagation in functionally graded materials by modified smoothed particle hydrodynamics (MSPH) method, Journal of Computational Physics, 222 (2007) 374-390.

[9] K. Li, Q. Huang, L. Lin, A Meshless Method for 3-D Wave Propagation Problems, in: Xvth International Seminar/Workshop on Direct and Inverse Problems of Electromagnetic and Acoustic Wave Theory (DIPED), Tbilisi, 2010.

[10] G.R. Liu, K.Y. Dai, K.M. Lim, Y.T. Gu, A point interpolation mesh free method for static and frequency analysis of two-dimensional piezoelectric structures, Computational Mechanics, 29 (2002) 510-519.

[11] G.R. Liu, Y.T. Gu, A point interpolation method for two dimensional solids, Int J Numer Methods Eng, 50 (2001) 937–951.

[12] G.R. Liu, G.Y. Zhang, Y.T. Gu, Y.Y. Wang, A meshfree radial point interpolation method (RPIM) for three-dimensional solids, Computational Mechanics, 36 (2005) 421-430.

[13] J.G. Wang, G.R. Liu, A point interpolation meshless method based on radial basis functions, International Journal for Numerical Methods in Engineering, 54 (2002) 1623-1648.

[14] G.R. Liu, Y.T. Gu, A Local Radial Point Interpolation Method (Lrpim) for Free Vibration Analyses of 2-D Solids, Journal of Sound and Vibration, 246 (2001) 29-46.

[15] G.R. Liu, X.L. Chen, A mesh-free method for static and free vibration analyses of thin plates of complicated shape, Journal of Sound and Vibration, 241 (2001) 839–855.

[16] K.Y. Dai, G.R. Liu, X. Han, Geometrically nonlinear analysis using meshfree RPIM, in: G.R. Liu, V.B.C. Tan, X. Han (Eds.) Computational Methods, Springer Netherlands, 2006.

[17] J.G. Wang, G.R. Liu, P. Lin, Numerical analysis of Biot's consolidation process by radial point interpolation method, International journal of solids and structures, 39 (2002) 1557–1573.

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