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Modified Weibull Approach to Deterioration Modelling: An Application to Bridge Elements

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Abstract— **State-based Markov deterioration models (SMDM) sometimes fail to find accurate transition probability matrix (TPM) values, and hence lead to invalid future condition prediction or incorrect average deterioration rates mainly due to drawbacks of existing nonlinear optimization-based algorithms and/or subjective function types used for regression analysis. Furthermore, a set of separate functions for each condition state with age cannot be easily derived by using Markov model for a given bridge element group, which however is of interest to industrial partners. This paper presents a new approach for deterioration modelling that follows homogeneous Markov models, namely, the Modified Weibull approach, which consists of a set of appropriate functions to describe the percentage condition prediction of bridge elements in each state. These functions are combined with Bayesian approach and Metropolis Hasting Algorithm (MHA) based Markov Chain Monte Carlo (MCMC) simulation technique for quantifying the uncertainty in model parameter estimates. In this study, the inspection data for 1,000 Australian railway bridges over 15 years were reviewed and filtered accordingly based on the real operational experience. Network level deterioration model for a typical bridge element group was developed using the proposed Modified Weibull approach. The condition state predictions obtained from this method were validated using statistical hypothesis tests with a test data set. Results show that the proposed model is not only able to predict the conditions in network-level accurately but also capture the model uncertainties with given confidence interval.**

Keywords—*Deterioration modelling, Modified-Weibull approach, MCMC simulation, Markov model, MHA Algorithm.*

I. Introduction

Infrastructure deterioration is a serious problem across many countries in the world. Use of mechanistic and numerical models for evaluation of the service lives of infrastructure components/elements is quite complex due to higher degree of uncertainties involving in the deterioration process and the inspection data have been popularly and widely used for presence of multiple-deterioration process that often occurs. Therefore, facility deterioration models are used for estimating future conditions [1], [2], [3] and [4] especially for network level deterioration modelling. Discrete conditions are normally used for rating infrastructure facilities by inspectors, based on visual condition of a given component according to guideline provided by their agencies. For example, one of the main Australian rail bridge agencies uses condition scale of 1 to 6; where condition rating one indicates the best condition without any visual defect, whereas rating 6 represents the unserviceable condition state of the given bridge or component. Inspection actions are normally performed annually or biannually [1].

Infrastructure components/elements gradually deteriorate with time and the conditions of those do not improve with time unless some kind of repair or rehabilitation work was done. Deterioration patterns of these components depend on potential contribution factors. For example, deterioration patterns and rates of rail bridge elements may vary with different potential determinants such as age, rail-traffic volume, environmental categories, etc [1]. By keeping other potential factors constant except the age, it is possible to make homogeneous bridge element group in terms of other contribution factors. If historical condition rating data are available for adequate period of time for these groups, those can be utilized to develop Homogeneous Markov Deterioration models.

State-based Markov Chain approach is the most popular infrastructure stochastic deterioration modelling technique that has been extensively used for especially network level deterioration modelling of bridges and other infrastructure facilities [1], [3], [4] and [5]. For example, in advanced Bridge Management Systems (BMS) in the world such as PONTIS and NYSDOT in the United States, KUBA in Switzerland, OBMS in Canada, etc. In [1] and [4], State-based Markov Bridge Deterioration Models (SMBDM) is employed. The most challengeable task of developing state-based Markov deterioration models is to obtain the Transition Probability Matrixes (TPM), which is also known as calibrating the Markov model. Morcous and Hatami (2011) [6] applied Regression-based Nonlinear Optimisation (RNO) method to calibrating homogeneous Markov deterioration models of bridges belonging to Nebraska Department of Roads since the time when they decided to adopt PONTIS. Although there are other methods available for estimating TPMs with their own advantages and limitations [1], RNO has been commonly used for estimating TPMs of SMBDM in previous studies [2], [3] and [6]. Wellalage et al. [1] pointed out the drawbacks of RNO for TPM estimation and proposed a MCMC simulation-based model for estimating TPM of Homogeneous SMDM of railway bridge elements by overcoming the above limitations.

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In [1], it was noticed that an important feature of the Markov model output which the percentage of each condition state resulted from Homogeneous Markov models could be obtained through one type of Weibull related functions. This paper presents an alternative approach to deterioration modelling. In this approach, a set of analytical formulas for percentage condition distributions is proposed in which Bayesian and Metropolis Hasting Algorithm (MHA) based MCMC methods are used to estimate model parameters. The proposed modified Weibull model approach is applied to a case study to investigate accuracy and suitability of the model.

II. Markov Approach

State-based Homogeneous Markov Deterioration Modelling (SHMDM) is only considered in this study. The main task to use the state based Markov deterioration models is to estimate Transition Probability Matrix/matrixes (TPM). According to five possible condition states considered by main Australian rail bridge authorities, it is 5×5 in dimension. Without a rehabilitation or repair work, the bridge component condition ratings either increase to a higher number or remain unchanged in one inspection period [1], [3] and [6]. Assuming no multi-transition events occur within one year [1], [2] and [3], one year TPM can be simplified and expressed as given in Eq.1. If TPM and initial condition state matrix ($C_{(0)}$) are known, condition state matrix after time *t* can be obtained by Chapman-Kolmogorov formula as stated in Eq.2.

$$P = \begin{bmatrix} p_{11} & 1-p_{11} & 0 & 0 & 0 \\ 0 & p_{22} & 1-p_{22} & 0 & 0 \\ 0 & 0 & p_{33} & 1-p_{33} & 0 \\ 0 & 0 & 0 & p_{44} & 1-p_{44} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

$$[C_{1(t)}\ C_{2(t)}\ C_{3(t)}\ C_{4(t)}\ C_{5(t)}] = [C_{1(0)}\ C_{2(0)}\ C_{3(0)}\ C_{4(0)}\ C_{5(0)}] \times \boldsymbol{P}^t \quad (2)$$

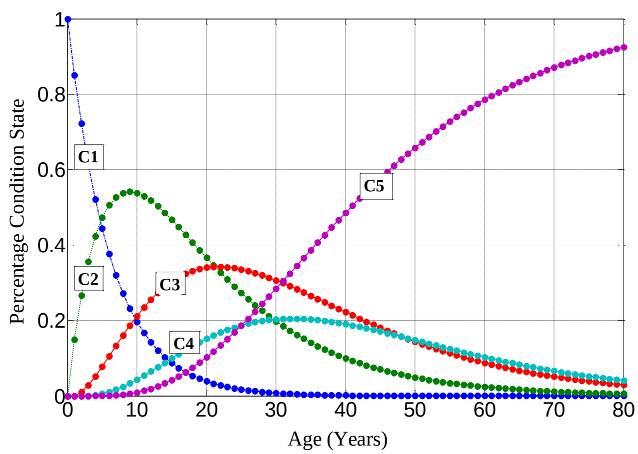

Figure 1. Typical condition state distribution patterns of SHMDM

III. Methodology: Modified Weibull Model

A. *Modelling of percentage condition states*

It was observed that the network level deterioration models of similar characteristic infrastructure component group that follows Homogeneous Markov deterioration models can be modelled by using this proposed approach. In this study, percentage condition distribution of such component group is assumed to follow a set of analytical functions with unknown model parameters. Percentage condition distributions for all condition sates except for the first (Best condition) and the last (unserviceable) condition states are modelled by using following form of modified Weibull function:

$$C_i(t,\theta_i) = a_i(t-r)\exp\left(b_i^{(t-r)}\right)\cdot wbl\left((t-r), c_i\ d_i\right) \quad (3)$$

where, $C(t,\ \theta_i)$ is percentage distribution function of bridge elements in condition *i* at age *t* , for $n > i > 1$; $\theta = (a_i, b_i, c_i, d_i)$ is of unknown model parameters at state *i*, (i = 2, 3, 4, …., n–1 with State *n* is the unserviceable condition sate); and a_i is scale parameter ($1 \geq a_i > 0$); b_i is the exponential model parameter ($1 > b_i > 0$); c_i, and d_i are Weibull parameters ($c_i > 0$, $d_i > 0$); *t* is the bridge element age in years ($t \geq 0$); *r* is location parameter ($r \geq 0$ and $r = i-2$ for all $i > 1$, $r \in \mathbb{Z}^+$); and $wbl((t-r),\ c,\ d)$ is the Weibull distribution with location parameter *r*, expressed as

$$wbl\left((t-r), c, d\right) = \frac{c}{d}\left(\frac{t-r}{d}\right)^{c-1}\exp\left[-\left(\frac{t-r}{d}\right)^{c}\right]. \quad (4)$$

Percentage condition curve of last condition state in Homogeneous Markov deterioration models was noticed as an S-shaped curve. Thus, the best sigmoid function form for modelling final, which is modelled as

$$C_{(n,\theta_n)} = \frac{1}{\left(1+(a_n/t)^{b_n}\right)^{c_n}}. \quad (5)$$

Although in Markov model, $C_{(1,\theta_n)} = p_{11}^2$ according to Equation 1 and 2, it was not found as the optimal function type and hence was not considered. If other condition proportions are known, $C_{(1,\theta_n)}$ can be easily calculated by using the total probability theorem as the sum of all percentages for a given time is equal to 1.

B. *Modelling uncertainty in parameter estimation*

Like all models, an assumption is made that the function $C_{i(t,\theta_n)}$ relating a set of causal input variables θ_i, to the dependent variable of interest, *y*, as follows:

$$y = C_{i(t,\theta_n)} + \varepsilon. \quad (6)$$

Where, θ is unknown and ε is a random noise component with zero mean and constant variance (known as Gaussian noise model). The problem is to estimate the unknown parameters θ based on the measurement, y. The objective of this work is to find an optimal estimate of θ such that the best fit between the observed percentage condition rating data for each state and the relevant proposed Modified Weibull curve for desired condition state (modelled response) is obtained.

C. *Bayesian approach*

Let consider a set of data (percentage of components) available for a bridge element group as Y = {y_1, y_2, y_3, ..., y_n} and θ represents unknown model parameter vector (in here unknown elements in C_{it} equation). Joint probability distribution P(Y/θ) is known as the sampling distribution or likelihood function which should be a known parameter to perform any inference [7]. P(θ/Y) is the posterior distribution or target distribution and P(θ) is called prior distribution of unknown model parameter. According to Bayes' rule for known value of data y, posterior density is proportional to multiple of prior density into likelihood function [1] as given in Eq. 7.

$$P(\theta/Y) \propto P(\theta)\, P(Y/\theta). \tag{7}$$

Primary task of specific application is to develop a model for P(θ/Y) and perform required computation to estimate the target density P (θ /Y) [1]. Non-informative prior density is a common case and can be assumed as a uniform distribution based on Bayes-Laplace ''principle of insufficient reason'' [8]. As a result, the posterior density is proportional to the likelihood function.

Assuming that the model residuals are independently and normally distributed with zero mean and constant variance σ^2, the likelihood function can be described by [10]:

$$P_{(y_{ti}/\theta)} = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{1}{2}\left(\frac{y_{ti} - c_{(t,\theta_i)}}{\sigma}\right)^2\right) \tag{8}$$

where, y_{ti} = Percentage of components in data set for age t in condition i.

With assumption of model residuals are normally distributed with zero mean and same constant variance σ^2 for $c_{(t,\ \theta i)}$ for i = 2, 3, 4, 5, overall likelihood function can be obtained and deduced into the following format

$$P(Y/\theta) = \prod_{t=1}^{T}\prod_{i=1}^{5} P_{(y_{ti}/\theta)} = \frac{1}{\left(\sqrt{2\pi\sigma^2}\right)^n} \exp\left(-\frac{SS_\theta}{2\sigma^2}\right) \tag{9}$$

where

$$SS_\theta = \sum_{t=1}^{T}\sum_{i=1}^{5}\left(y_{ti} - c_{(t,\theta_i)}\right)^2 ,$$

t is the bridge element service life in years, T is the largest age found in the data set, $c_{(t,\ \theta i)}$ is percentage distribution function value of bridge elements in condition i at age t and y_{ti} is percentage of condition state i bridge elements at age t in the data set.

D. *MCMC Simulation with Metropolis Hasting Algorithm (MHA)*

Posterior density of given model parameters is proportional to multiple of prior density into likelihood function from Eq. 7. This property is used in this analysis to allow Metropolis Hasting Algorithm (MHA)-based MCMC simulation to generate samples from posterior distribution [2, 6]. In MHA, it's required to choose a proposal density q(x, y) where total integral of q(x, y) is equal to 1 for sampling from the target distribution [7]. The proposal density q(x, y) is often selected from a symmetric and multivariate distribution, which is also known as symmetric random-walk metropolis algorithm (RWM). With assumption of a standard nonlinear model (in Eq. (6) with Gaussian noise and a non-informative prior, acceptance probability of Metropolis algorithm can be rewritten as follows [9]:

$$\alpha(x,y) = \min\left\{1, \frac{\pi(y)}{\pi(x)}\right\} = \min\left(1, e^{-0.5\sigma^{-2}(SS_{\theta_{new}} - SS_{\theta_{old}})}\right) \tag{10}$$

Initial values of model parameters for MHA were randomly chosen within the boundary conditions because they do not affect the convergence to the target distribution of the chain according to the theory [10]. Initial values for the Variance Covariance Matrix (VCM) is arbitrarily chosen and tuned until the acceptance rate becomes near to the optimum value of acceptance, 0.234 [11]. This MCMC simulation model is run with historical condition rating percentage data until the model parameters converge to stationary distributions.

IV. Verifying the Model for Given TPM

According to Australian rail bridge rating system, n = 5. Hence, 15 parameters were optimised as they represent Markov model output. Here, $\theta = [\theta_1, \theta_2, \theta_3, \theta_4]$.

A Transition Probability Matrix estimated for timber deck elements in previous study [1] was considered for this analysis,

$$P = \begin{bmatrix} 0.85 & 0.15 & 0 & 0 & 0 \\ 0 & 0.93 & 0.07 & 0 & 0 \\ 0 & 0 & 0.93 & 0.07 & 0 \\ 0 & 0 & 0 & 0.90 & 0.10 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{11}$$

For r = 0, 1, 2 and i = 2, 3 and 4;

$$C_i(t,\theta_i) = a_i(t-r)\exp\left(b_i^{(t-r)}\right) \cdot wbl\left((t-r), c_i\ d_i\right). \tag{3}$$

For i = 5,

$$C_{(5,\theta_5)} = \frac{1}{\left(1 + (a_5/t)^{b_5}\right)^{c_5}}.$$

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Therefore, $C_{1(t)}$ can be calculated by

$$C_{1(t)} = 1 - \sum_{i=2}^{5} C_{i(t)} \, . \tag{12}$$

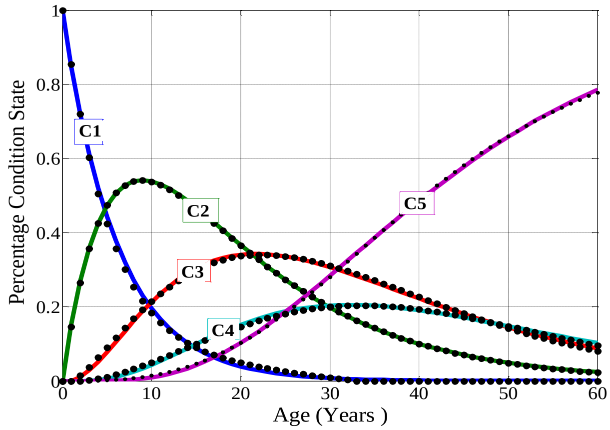

Figure 2. Modified Weibull fit for given TPM in SHMDM

In Fig. 2, the dotted points were obtained from homogeneous SMDM for given TPM and solid curves represent the relevant fitted curves by Modified Weibull functions obtained by MCMC with MHA. Total error was found to be less than 1% and R^2 values were greater than 0.99 for each condition state.

TABLE I. Parameter values for Markov model fitting

	a	b	c	d	r
θ_2	0.799	0.946	12.210	1.05	0
θ_3	0.318	0.988	24.052	1.361	1
θ_4	0.330	0.286	31.507	1.680	2
θ_5	42.97	3.300	0.8754	-	-

V. Case Study

According to data availability, past condition rating data of 50 transverse timber bridge decks that are homogeneous in terms of contribution factors (such as average tonnage passes per week**,** Environmental Categories**,** etc.) except age, in 3 major inland railway lines, over past 15 years were selected to do this analysis. Due to insignificant number of bridges in each line, analysis was done by combining bridges in different railway line with similar characteristics based on assumption that the observed bridge condition ratings are randomly distributed about their true values.

Please refer to [1] for detailed description of the data filtration process. 387 total records were obtained after filtration process for this data set. For statistical validation and comparison purposes, data set was split randomly such that 75% as calibrated data set and the rest as the test data set. Calibrated data set was used for analysis first and then validated with test data set by using Chi-square test [2].

A. *Analysis results*

MATLAB program codes were developed with MHA algorithm according for MCMC. The MHA ran with 150,000 iterations for the calibration data set until the condition state functions' variable values converge to stationary distributions by avoiding first 100000 warm-up runs. Variance covariance matrix was adjusted until acceptance rate becomes near to optimum acceptance rate of 0.234. Variance of the random noise component ε was kept as 1×10^{-3} throughout the analysis. It was noticed that the mean values of these parameters converge to almost constants. By using estimated parameter values, the proposed formulas given in Eqs. 3 and 5 were applied to obtain the percentage of bridge elements in each state *vs* time as given in Fig. 3.

TABLE II. Estimated parameter values from modified Weibull approach

	a	b	c	d	r
θ_1	0.436	1.000	12.223	1.215	0
θ_2	0.258	0.992	23.349	1.542	1
θ_3	0.424	0.058	31.410	1.607	2
θ_4	42.589	3.738	0.806	-	-

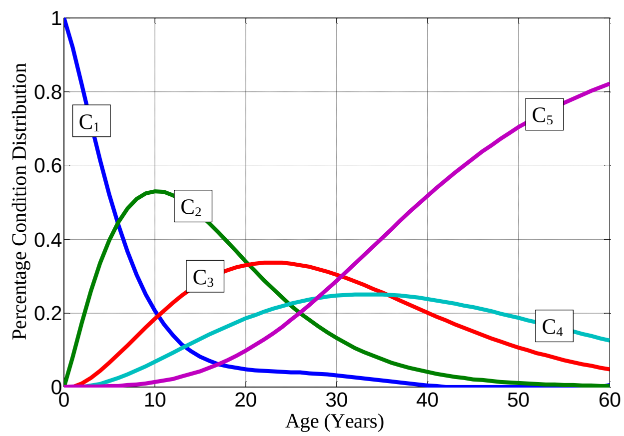

Figure 3. Modified Weibull Model for case study

VI. Verification of Results

Test results are validated by using Chi-square test for calibrated and test data sets. According to Table III, the Modified Weibull approach results passed the goodness-of-fit test since Chi-Square values for both calibration and test data set are well below the Chi-Square critical value. Therefore, it can be argued that Modified Weibull approach presents in this paper is suitable for network-level deterioration modelling of selected bridge elements in this study. The estimated model parameters (Mean in Table IV), standard deviations and 95% confidence intervals are given on Table IV. It is clear that the standard deviation is very small comparing to the mean value of each model parameter.

TABLE III. CHI-SQUARE VALUES OF MCMC METHOD RESULTS

Method	Chi-Square values with 4 degrees of freedom (≤ 9.49)	
	Calibrated data set	*Test data set*
MW approach with MCMC	0.4237	1.812

TABLE IV. MODEL PARAMETERS AND 95% CONFIDENCE INTERVALS

Model parameter	Mean	STD	Upper limit	Lower limit
a_2	0.436	0.00282	0.4364	0.4356
b_2	1.000	0.00225	1.0003	0.9997
c_2	10.223	0.00349	10.2234	10.2226
d_2	1.215	0.00482	1.2156	1.2144
a_3	0.258	0.00454	0.2586	0.2574
b_3	0.992	0.00079	0.9921	0.9919
c_3	23.349	0.00670	23.3499	23.3481
d_3	1.542	0.00696	1.5429	1.5411
a_4	0.424	0.00276	0.4244	0.4236
b_4	0.058	0.00670	0.0589	0.0571
c_4	31.410	0.01220	31.4115	31.4085
d_4	1.607	0.00097	1.6071	1.6069
a_5	42.589	0.01173	42.5905	42.5875
b_5	3.738	0.01277	3.7396	3.7364
c_5	0.806	0.00201	0.8063	0.8057

VII. Conclusion

This paper proposed an alternative approach for network level Homogeneous Markov bridge deterioration modeling, named Modified Weibull approach. In this approach, a set of analytical formulas for percentage condition prediction is obtained in which Bayesian and Metropolis Hasting Algorithm (MHA) based on MCMC methods are used to estimate model parameters. The model obtained from this approach is applied to a case study to investigate the accuracy and suitability of the model.

Outputs of the deterioration models are validated by using goodness-of-fit test. Results show that Chi-square values of the model with the parameter estimates for calibrated and test data set are well below the critical Chi-square value. Obtaining very small Chi-square values compared to critical value convinced the superiority of the proposed approach with powerful MHA for bridge deterioration modeling. Furthermore, the proposed approach can capture the uncertainties of model parameters with given confidence intervals.

Further, it is promising that the proposal approach is able to be applied to network-level deterioration modelling for other bridge components and other infrastructure facilities that follow homogeneous Markov process.

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