

# A Model for Heterogeneous Fixed Fleet Vehicle Routing Problem

[Alireza Naseri, Masoud Naseri]

**Abstract**—Transportation is one of the important infrastructures constituting the base and requisite for different levels of people's access and movement to various locations. Transportation systems are one of the indices of the development of a region. Optimizing these systems contributes to improving performance and reducing costs of offering services to users. Hence, methods should be adopted with which this problem can be efficiently modeled. The final purpose of the vehicle routing problem is fulfilling all needs optimally and with minimum cost. Heterogeneous fixed fleet vehicle routing problem is an applied problem in routing and transportation planning. Its approach is to service customers with a limited number of vehicles through specifying certain routes with minimum cost for transportation fleet that will meet the demands of customers. The composition of vehicle fleet includes several types of vehicles with different capacities. In this research, a practical model and algorithm is proposed and a sample problem is solved by the algorithm and the numerical results are presented.

**Keywords**—vehicle routing problem, heterogeneous fixed fleet, optimization, transportation planning

## I. Introduction

Urban transportation planning is one of the fundamental problems of countries. This importance arises from the fact that this problem involves three essential factors, namely cost, time, and citizens' safety. Another reason why this problem is important is that transportation planning has a direct relationship with citizens' satisfaction [1]. Moreover, the traffic congestion phenomenon is one of the urban transportation problems in metropolises and large networks of urban transportation. If traffic load is not distributed optimally throughout the network, environmental pollution, acoustic pollution, and network users' waste of time increase. Therefore, presenting efficient managerial tools plays a decisive role and can decrease the undesirable factors existing in the network to some extent [2].

---

Alireza Naseri  
Department of Civil Engineering, Tabriz Branch, Islamic Azad University,  
Tabriz, Iran  
Iran

Masoud Naseri  
MBA in International Marketing, Central Tehran Branch, Islamic Azad  
University, Tehran, Iran  
Iran

One of the ways of managing networks scientifically is to study the routing problem. Vehicle routing problem is of considerable importance from a practical and theoretical perspective. From a practical viewpoint, with vehicle routing improvement as a part of transportation planning, respective costs can be reduced markedly and additionally, other goals such as minimizing time, vehicle number, etc. may also be achieved. Numerous studies have been conducted so far in this regard. For example, Descrochers used a definite mathematical method to obtain an answer; however, the time spent for arriving at the answer was too long. This makes the method impossible to use in many instances where the bulk of data is huge [3]. Nevertheless, as new innovative methods were developed, indefinite methods were used to solve this sort of problems in a short time. Since these methods yielded proper results, they became more popular [4, 5].

## II. Method

Single-objective routing problem is investigated as the study methodology and an approach for optimization [6]. In this section, model assumptions are initially presented and then, the structure of the problem mathematical model is examined and in the end, numerical results are studied.

### A. Model Assumptions and Characteristics

The following assumptions are made in the problem under study in this research.

- There is only one center offering and servicing travel and several travel demands.
- All of the travels take place between the offering center and demand points.
- Number of people willing to travel is known.
- There are several types of vehicles for passenger transportation. The capacity and number of each of them is known. In case the number of vehicles is unknown, the model determines the number.
- There are mutual complete relationships in the network between demand points and the service center.
- The distance unit cost is different for each type of vehicle and the total cost equals the sum of the distance traveled by each vehicle multiplied by its distance unit cost.

- The problem aims at allocating each of demand points to a route, in a way that the total movement cost is minimized.

In order to model the problem, the required parameters are defined at first. Constraints and the objective function are investigated subsequently. Further, necessary explanations on the details of the mathematical model are presented [7, 8].

### B. Defining Parameters

n: number of demand points

m: number of vehicles

$d_{ij}$ : distance between nodes i and j  $d_{ii}=0$   $i, j=0,1,2,\dots,n$

$m_k$ : unit cost of the  $k^{\text{th}}$  vehicle's distance  $k=1, 2,\dots,m$

$D_k$ : capacity of the  $k^{\text{th}}$  vehicle

$d_i$ : demand of  $i^{\text{th}}$  point  $i=1, 2,\dots,n$

TC: total cost

### C. Defining Variables

$$X_{ijk} = \begin{cases} 1 & \text{If vehicle } k \text{ travels the distance between } i \text{ and } j \\ 0 & \text{Otherwise} \end{cases}$$

$$y_{ik} = \begin{cases} 1 & \text{If vehicle } k \text{ provides service for demand point } i \\ 0 & \text{Otherwise} \end{cases}$$

### D. Mathematical Model of the Problem

In view of the defined parameters and variables, the mathematical model is comprised of an objective function and 6 constraint nodes as follows:

$$\text{MinTc} = \sum_{i=0}^n \sum_{j \neq i}^n \sum_{k=1}^m m_k .d_{jk} \quad (1)$$

Subject to:

$$\sum_{k=1}^m y_{ik} = 1, i=1,2,\dots,n \quad (2)$$

$$\sum_{k=1}^m y_{ok} = m \quad (3)$$

$$\sum_{j=0}^n x_{ijk} = \sum_{j=0}^n x_{jik} = y_{ik} \quad (4)$$

$$i=0,1,2,\dots,n \quad K=1,2,\dots,m$$

$$\sum_{i=1}^n d_i .y_{ik} \leq D_k, K=1,2,\dots,m \quad (5)$$

$$\sum_{ijk} x_{ijk} \leq |S| - 1, S=[2,3,\dots,n], K=1,2,\dots,m \quad (6)$$

$$X_{ijk}, Y_{ik} \in \{0,1\} \quad (7)$$

### E. Objective Function

Since the problem aims at minimizing total cost and on the other hand, total cost depends on the distance traveled by each vehicle and the unit cost of the distance traveled by that vehicle, the objective function is as presented [9].

### F. Problem Constraints

The constraint regarding offering service by one vehicle to each demand is called the first constraint. The second constraint demonstrates that all of the routes start from and end in the service center. Each vehicle entering into a center should leave it. That is why the third constraint is introduced. The constraint of vehicles capacity is expressed as the fourth constraint.

Sub route elimination constraint, which omits incomplete travels, is fulfilled by the fifth constraint. This constraint states that if the distance from i to j is traveled by vehicle k, the distance from j to i should not be traveled by vehicle k.

$$x_{ijk} + x_{jil} \leq 1, i, j = 1,2,\dots,n, i \neq j, k = 1,2,\dots,m$$

Furthermore, if vehicle k travels from center i to center j and then to center k, it should not return from center h to center i.

$$X_{ijk} + x_{jhl} + x_{hil} \leq 2, i, j, h = 1,2,\dots,n$$

$$i \neq j \neq h, k = 1,2,\dots,m$$

On the whole, denoting service center by 0 and each of demand points by 1 to n, if vehicle k travels through each of the points (except point 0), it should not return to that point.

The number of constraints concerning the elimination of L-fold sub routes ( $L=2,3,\dots,n$ ) is obtained via the following relation:

Number of constraints of L-fold sub routes elimination =

$$\binom{N}{L} \times (L-1)!M = \frac{N!M}{(N-L)!L} \quad (8)$$

For instance, if there are 4 demand centers ( $n=4$ ), the number of 2-fold, 3-fold, and 4-fold route elimination constraints are 6m, 8m, and 6m, respectively. There are a total of 20m constraints.

Similarly, for  $n=5$ , the number of sub route elimination constraints are 10m, 20m, and 30m, and 24m respectively. There are a total of 84m constraints. For  $n=6$ , the number of sub route elimination constraints are 409m.

### III. Describing Model Status

The model of the respective problem is of zero and one type and with an increase in  $n$ , the number of sub route elimination constraints increases considerably, in a way that it is impossible to use precise methods in practical large-scale problems and an approximate algorithm has to be used for solving the problem [10].

### IV. Generalizing Optimum Algorithm for the Problem

According to previous discussions, precise algorithms are not applicable to practical vehicle routing problems. Meanwhile, none of the existing innovative algorithms is in complete agreement with the problem structure. Among the examined algorithms, the structure of C&W optimum algorithm is closer to that of the problem.

In the optimum algorithm, the optimization level resulting from the combination of node  $(i,j)$  is as follows:

$$S_{ij} = d_{io} + d_{oj} - d_{ij} \quad (9)$$

According to the above relation, optimization level is independent of the route length and the points in it. It merely depends on arches connected to the center and the distance between the selected points for combination from each other. In the problem under study, it is assumed that at the beginning of the problem, each center is receiving service with the most economical vehicle given the demand level. That is, the closest vehicle (capable of transporting the respective demand) is allocated to it given the demand level.

At the beginning of the problem where one vehicle is allocated to each demand center, total cost is as follows:

$$Tc = 2 \sum_{j=1}^n m_j . d_{0j} \quad (10)$$

Given the demand level of each point  $(d_j)$ , the required vehicle capacity is determined and thereby,  $m_k$  is calculated which is equivalent to  $m_j$  (unit cost of the distance of a route to which center  $j$  belongs).

Now, if nodes  $i$  and  $j$  are combined,  $(i,0)$  and  $(0,j)$  distances are eliminated and  $(i,j)$  distance is added. However,

combination of  $i$  and  $j$  may change vehicle type given the capacity intended. As a result, unit cost of the distance of the route encompassing centers  $i$  and  $j$  ( $M_{ij}$ ) may also change. The economization level of  $i$  and  $j$  combination is as follows:

$$S_{ij} = 2(M_i . d_{oi} + M_j . d_{oj}) - M_{ij} (d_{oi} + d_{jo} + d_{ij}) \quad (11)$$

Similarly, optimization resulting from the combination of this route with a new point such as  $k$  can be calculated. In this mode, it is also possible that vehicle type changes given the intended capacity leading to change in the unit cost of route distance, i.e.  $M_{ijk}$ .

Thus, in the required algorithm, optimization level resulting from the combination of two points depends on the following factors:

- Distance between demand points and service centers
- Relative distance between demand points
- Unit transportation cost of the distance given the required vehicle type

It is clear that  $S_{ij}$  changes, as routes change during calculations. Its value may be calculated from the following relation:

$S_{ij} = (\text{old cost}) - (\text{new cost}) = (\text{cost of old route } i) + (\text{cost of old route } j) - (\text{cost of } i \text{ and } j \text{ route combination}) =$

$$L_i, L_i + L_j . M_j - L_{ij} . M_{ij} \quad (12)$$

$L_i$  = Length of the route to which demand point  $i$  belongs

$L_j$  = Length of the route to which demand point  $j$  belongs

$L_{ij}$  = Length of the route resulting from  $i$  and  $j$  combination

$R_j$  = Demands of the entire route in which demand point  $j$  exists

At the beginning of the problem, each demand point is allocated to one vehicle.

$$R_j = d_j \quad L_j = 2d_j$$

Therefore, the algorithm is generalized as follows.

### V. Presenting Numerical Results

In order to evaluate algorithm efficiency, 10 problems with  $n=5$  and  $m=2$  and with the assumption of the distance unit transportation cost identicalness were selected randomly and solved by the precise method and the algorithm. Results indicate that the algorithm response reduces costs as much as 49% compared with allocating each vehicle to one demand point. In these problems, distances between demand centers are considered to be random numbers between 0 and 100, and demand level values are considered to be random numbers between 0 and 400 (distances and demand levels are selected in a way to have the most accordance with real data).

**Step 1:** Allocate the most economical vehicle to each of demand points given their demand level.

**Step 2:** Calculate length and demand level of each route according to the following relations:

$$L_j = 2d_{0j} \quad R_j = d_j \quad j = 1, 2, \dots, n$$

**Step 3:** Calculate the productivity level resulting from i and j route combination as follows, and sort  $S_{ij}$  values in descending order.

- Unit cost of i and j distance ( $M_j, M_i$ ) is calculated using  $R_i$  and  $R_j$ .

$$R_{ij} = R_i + R_j \quad i < j$$

$$L_{ij} = L_i + L_j + d_{ij} - d_{oi} - d_{oj} \quad i < j$$

$$S_{ij} = L_i.M_i + L_j.M_j - L_{ij}.M_{ij} \quad i < j$$

**Step 4:** Start from the top of the list and in case the following conditions are satisfied:

- i and j were not previously on the same route,

-  $S_{ij} > 0$ ,

- At least one end of i and j is connected to the origin,

- Capacity constraint is observed (maximum capacity is not exceeded),  $R_i + R_j \leq \max\{D_k\}$

- Other constraints (such as maximum route length, etc.) are added, if needed.

Then:

a- Combine route i and j.

b- Length and demand level of each route should be modified for each point on the formed route according to the following relations:

$f =$  [any f on the new route]

$$L_f = L_{ij}$$

$$R_f = R_{ij}$$

c- Go to Step 3.

In case conditions are not met (combination is impossible), move on to the next option in the list.

**Step 5:** Continue Step 4 until no further combination is possible.

**Step 6:** In case the answer is unacceptable (given the number of each type of vehicles, etc.), continue performing the algorithm with negative  $S_{ij}$  until arriving at an acceptable answer.

In addition, the algorithm answer has a 4% difference compared with the optimum answer obtained by Lindo software. This is a relatively acceptable answer. Respective results are presented in Table (1). Results are reflected in Figure (1) for better understanding.

Meanwhile, the algorithm was performed using real data of four time periods and by assuming 1, 1.2, 1.4, and 1.7 as distance unit costs and 3, 4, 7, and 10 as vehicle capacity. Results indicate 29% economization compared with allocating each of demand centers to one vehicle.

TABLE I. COMPARING THE RESULTS OF ALGORITHM CALCULATIONS WITH THE OPTIMUM ANSWER

No.	d <sub>01</sub>	d <sub>02</sub>	d <sub>03</sub>	d <sub>04</sub>	d <sub>05</sub>	d <sub>12</sub>	d <sub>13</sub>	d <sub>14</sub>	d <sub>15</sub>	d <sub>23</sub>	d <sub>24</sub>	d <sub>25</sub>	d <sub>34</sub>	d <sub>35</sub>	d <sub>45</sub>	d <sub>1</sub>	d <sub>2</sub>	d <sub>3</sub>	d <sub>4</sub>	d <sub>5</sub>	Optimal Solution	Algorithm Solution	Upper Limit
1	20	88	55	83	45	12	10	22	77	46	81	87	31	78	20	165	132	296	54	188	270	270	582
2	24	27	56	97	3	82	7	78	46	14	54	25	11	37	3	70	268	370	222	69	174	206	414
3	75	97	30	87	5	21	11	55	61	96	3	44	46	1	73	370	230	95	79	124	222	222	588
4	99	33	43	45	55	15	21	46	58	81	78	12	50	96	83	189	87	163	66	377	255	274	550
5	40	31	78	3	26	2	4	70	2	20	80	48	63	68	19	268	370	22	68	172	185	224	356
6	87	63	67	31	70	38	1	66	51	92	36	47	80	95	32	43	357	110	70	51	216	216	636
7	12	62	23	86	42	65	51	65	87	5	87	100	92	73	22	352	33	188	277	322	255	255	450
8	55	15	21	46	58	81	78	12	50	96	83	36	82	63	3	290	162	64	295	328	248	302	390
9	14	54	25	11	37	4	29	95	34	19	38	82	8	70	58	290	162	95	328	248	168	168	282
10	36	20	88	55	83	45	13	11	22	77	46	81	87	31	78	342	100	160	165	240	313	373	564

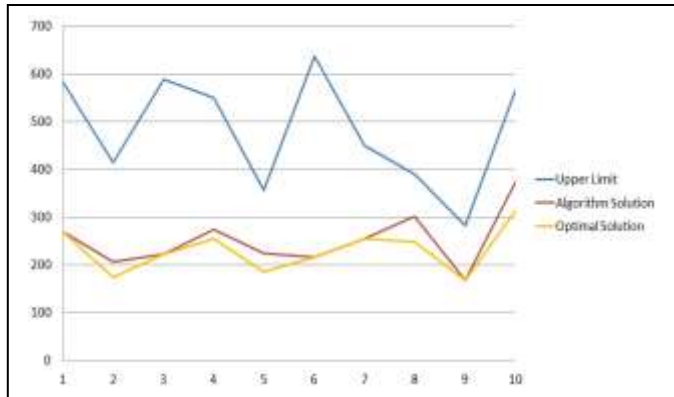


Figure 1. Comparing of results with the optimum answer

## VI. Conclusion

As stated earlier, urban transportation planning is one of the most significant sections of urban development and solving traffic and pollution problems. In this research, a method was presented for the public transportation system. This method can be an appropriate solution for minimizing route length, satisfying time constraints, optimum number of vehicles, algorithm responsiveness in a short time, and algorithm simplicity and applicability as well as its efficiency. Vehicle routing problem is applicable to a great number of distribution problems. Solving this problem in the single-objective mode is appropriate and practical. Results are indicative of the necessity of conducting scientific studies on the distribution problem. If these methods are properly employed, unnecessary costs are avoided to a great extent, the ultimate result of which is increase in productivity in the system, which is the final result of every project.

## References

- [1] J. Jasbi, P. Mackvandi, "Travel forecasting process modeling in transportation planning based on the fuzzy inference approach," *Management Future*, Tehran, 2011, vol. 17, pp. 7–32.
- [2] M. Asgharpour, S. Ebrahimnejad, "Traffic assignment model in urban transportation network and solve it using a genetic algorithm," *Journal of the College of Engineering*, Tehran, 2001, vol. 35, No. 4, pp. 587–602.
- [3] M. Descrochers, J. Desrosiers, and M. Solomon, "A new optimization algorithm for the vehicle routing problem with time windows," *Operations Research*, 1992, pp. 342–352.
- [4] O. Braysy, M. Gendreau, "Route construction and local search algorithms. In vehicle routing problem with time windows," *Transportation Science*, 2002.
- [5] L. Cordeau, G. Desaulniers, J. Desrosiers, M. Solomon, and F. Soumis, "The VRP with time windows," In Toth, P. and Vigo, D, editors. *The vehicle routing problem*, SIAM monographs on discrete mathematics and applications. Philadelphia: SIAL, 2001, pp. 94–157.
- [6] L. Bodin, B. Golden, "Classification in vehicle routing and scheduling," *Journal of Networks*, 1981, vol. 11, pp.9 7–108.
- [7] E. Lawler, J. Lenstra, A. Rinooy, and D. Shmoys, "The traveling salesman problem," 1<sup>st</sup> Ed. Chapter 12, John Wiley and Sons Pub. New York, 1985.

- [8] G. Laporte, "The vehicle routing problem: An overview of exact and approximate algorithm," *European Journal of Operation Research*, 2005, vol. 162, pp. 126–141.
- [9] M. Chao, B. Golden, and E. Wasil, "Acomputational study of a new heuristic for the site dependent vehicle routing problem," *Information Ottawa*, 1999, vol. 37, pp. 319–336.
- [10] G. Nagy, S. Salhi, "Heuristic algorithm for single and multiple depot vehicle routing problem with pickups and deliveries," *European Journal of Operation Research*, 1992, vol. 59, pp. 345–358.

About Author:



**Alireza Naseri** was born in Tehran, Iran 1980. He works as a lecture at Islamic Azad University, Tabriz branch, Iran. He is working on Transportation Planning, urban transit and Intelligent Transportation Systems (ITS). Alireza is a member of Iranian Association of Rail Transport Engineering. Also, he is a member of Iranian Society of Transportation and Traffic Engineers.