# The Proposed Method for the Estimation of the Pile Critical Length in Clayey Soil for Offshore Structures

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**Abstract**: The goal of this study is to prepare a new proposed method for the estimation of pile critical length in clayey soil for offshore structures. For this purpose the dolphin of Khor Al-Amaya berth No. 8 (in the Arabian Gulf, south of Iraq) is taking as a case study which subjected to dynamic load using the interface finite element method. A computer program has been written by Fortran 90 language to get the required solution. The (p-y), (t-z), and (q-z) curves which are adopted by American Petroleum Institute (API) are used to find normal, and tangential interface moduli and end bearing modulus. To get the required results, the Subspace Method is carried out for free vibration analysis and the Newmark method is adopted in direct integration time domain.

The obtained results are compared with both elastic and Matlock and Reese solutions proved that the interface solution gives higher response values of vertical pile other than the two solutions because of the nonlinear effects of the interface solution. Also, the effect of the variation of soil strength is investigated in the problem. The proposed equation of pile critical length depends only on two parameters given as the bending stiffness of the pile and the soil cohesion.

Key words: Piles, pile length, soil interaction, offshore structures.

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#### INTRODUCTION

Offshore structures are structures that are placed in specific locations in the sea or ocean for specific purpose. The jacket, or template, structures are still the most common offshore structures used for drilling and production which are subjected to various kinds of forces which can be classified into two primary types; static and dynamic loads so that these loads translate to the foundation, usually pile foundation (piles are commonly adopted as a foundation for a wide variety of marine structures because of their ability to develop the required resistance for bearing, uplift, and lateral loads) which must be adequate embedded into the soil to translate these loads safely into the different soil layers. Therefore, the choice of a correct pile length is considered as a key element in the economic view of the design of these structures because the most critical job in the construction of offshore structures is not placing the tower but the many and different operations related to the completion of the required pile foundation.

According the nature of the loads applied to offshore structures (wind, wave, ship berthing, earthquake, vibration of equipment, etc...) [1] which occurs in horizontal direction leading to the design of such piles are controlled by lateral and vertical loads. One important feature in the analysis and design of laterally loaded piles is to establish the critical pile length. At this length, only one point of zero deflection will occur in the computations [2].

Fig. 1.a shows a pile with applied loads, it is assumed that the design is controlled by lateral load and moment; the pile has a constant cross section with an initial length. Fig.1.b shows ground pile deflection as a function of pile length and this figure shows an important fact that the ground line deflection is unaffected until the critical pile length is approached. Therefore, the designer should select



the safe and economic pile length noting that the accuracy of any obtained solution will depend on how the soil response curves reflect the actual situation in the field [2].

## **METHODS OF SOLUTION**

The first step in finding the critical length of pile is to determine a suitable solution of the problem. Therefore, many methods have been developed by many researches to find an accurate solution for the analysis of laterally loaded pile using the analytical and experimental methods. In these methods, the selected behavior is different from one approach to another such as linear elastic, non-linear elastic, elastic-plastic, and et... The Analytical methods are limited in practical applications because it can be used only for simple cases; therefore more attention is given to the numerical methods. But those researches and their studies have not yielded any simple design method which can be universally applied to any type of soil [3]. Various necessary simplifications have been made in order to provide simple solutions to complex problems for the soil-pile interaction. Those various methods are briefly illustrated as follows.

Hansen (1961) [4] proposed a method of predicting the ultimate lateral load of rigid piles based on the theory of plasticity. Later, Broms (1964) presented a method to determine the ultimate lateral capacity of loaded (short, intermediate and long) pile within different soil types and for two pile cap cases; free head and fixed head by assuming soil pressure distribution at failure condition. This is basically the procedure suggested by Hansen (1961) with a few modifications.

Matlock et. al. (1970) [2,4,5] who was the first to develop curves for soft clays, then followed by Reese et. al. (1974) for sands, and again Reese et. al. (1975) for stiff clay. Those curves were derived from actual field measurements, full scale test, which represent non-linear stiffness of soil springs which later adopting by API [6]. Another approach to investigate the pile response that was achieved by various experimental researches for vertical piles subjected to cyclic lateral loads [7]. Good comparisons of the above methods and others are given by details in reference [3].

In present paper only the elastic and Reese solution are used in comparison of the obtained results from this study. The elastic solution for long pile in clayey soil is given below as [2]:

$$u_{x} = \frac{2\alpha^{2} \cdot e^{-\alpha \cdot x}}{K} \left[ \frac{H}{\alpha} \cdot \cos(\alpha x) + M \cdot (\cos(\alpha x) - \sin(\alpha x)) \right] - 1$$

$$M_x = e^{-\alpha \cdot x} \left[ \frac{H}{\alpha} \cdot \sin(\alpha x) + M(\sin(\alpha x) + \cos(\alpha x)) \right] - - - 2$$

Also Reese and Matlock proposed maximum pile deflection according the following expression [3]:

$$u_{\text{max}} = \sqrt{2} \left( \frac{H}{K} \right) \left( \frac{4}{L_c} \right) + \left( \frac{M}{K} \right) \left( \frac{4}{L_c} \right)^2 - \dots 3$$

Where

$$\alpha = \sqrt[4]{\frac{K \times D}{4 \times EI}} - - - - 4$$

K: coefficient of sub grade reaction (assumed to be a constant with depth) calculating from [8]:

$$K = \frac{E_s}{D(1-v^2)}$$
 -----5

 $E_s$  = Young' modulus of soil, N/m<sup>2</sup>

*v*= Poisson's ratio of soil,

D= pile diameter, m

H= applied lateral load on the pile, N

*M*= moment induced due to lateral force, N.m

 $L_c$  = critical pile length which is defined as the length measured from the pile head to the corresponding point of zero deflection will occurs along pile length in which below this length the pile is acting as an infinitely long pile, and it is obtained using the following equation [3]:

$$L_c = 4\sqrt[4]{\frac{EI}{KD}} - 6$$

E= Young's modulus of pile material, N/m<sup>2</sup>

*I*= moment of inertia of the pile, m<sup>4</sup> *EI*= bending stiffness of the pile, N.m<sup>2</sup>

# FORMULATION OF THE PROBLEM

The domain of the problem include four different types of materials which are given as the jacked structure, the soil, the interface element, and the end bearing element which illustrate as follows:.

1-The soil is represented by brick element that has eight nodes. For each node, there is three degree of freedom considered which perform the translation motion in three global coordinates.

2-The structure is represented by flexure frame element that has prismatic cross section (thin tubular circle, wide flange) and it connect to two nodes, one at each end. Also, the element is connected to other



elements only at the nodes. Each node has six degree of freedom, three translation and three rotations, in which the positive direction of rotation is based on right hand role.

3-A typical interface zero thickness element for 3D applications that is based and developed from the 2D joint element proposed by Goodman [9]. The element formulation is derived on the basis of relative nodal displacements of the pile elements and the surrounding of the interface elements (soil elements). Each interface element is represented by four nodes i.e. two for the pile and two for the soil. The resulting degree of freedom for each node is three representing a degree of freedom for translation movement only for both the pile and the soil and for one element is twelve degrees of freedom. Before loading process, each corresponded two nodes (one for pile and other for soil) have the same coordinates. Therefore, the strain at any point in the element may be defined by tangential and normal displacements between the pile and the soil that is given by:

$$\{\varepsilon\} = [B] \times \{\delta\} - 7$$
Where

Where

$$\begin{split} \left\{ \mathcal{E} \right\} &= \left\{ \mathcal{E}_t \quad \mathcal{E}_{n1} \quad \mathcal{E}_{n2} \right\}^T \\ \left[ B \right] &= \begin{bmatrix} N_1 & 0 & 0 & N_2 & 0 & 0 & -N_1 & 0 & 0 & -N_2 & 0 & 0 \\ 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & -N_1 & 0 & 0 & -N_2 & 0 \\ 0 & 0 & N_1 & 0 & 0 & N_2 & 0 & 0 & -N_1 & 0 & 0 & -N_2 \end{bmatrix} \end{split}$$

$$\{\delta\}^T = \begin{bmatrix} u\mathbf{1}_p & v\mathbf{1}_p & w\mathbf{1}_p & u\mathbf{2}_p & v\mathbf{2}_p & w\mathbf{2}_p & u\mathbf{1}_s & v\mathbf{1}_s & w\mathbf{1}_s & u\mathbf{2}_s & v\mathbf{2}_s \end{bmatrix}$$

$$N_1 = (1 - \xi)/2$$
------8.a  
 $N_2 = (1 + \xi)/2$  -----8.b

 $\varepsilon_{\scriptscriptstyle t}$ : Tangential strain;  $\varepsilon_{\scriptscriptstyle n1}$ :first normal strain; and  $\mathcal{E}_{n2}$ :second normal strain

[B]: Shape function matrix;  $\delta$ : local nodal displacements of the pile and the soil at two nodes,  $N_1$  and  $N_2$  are shape functions,

 $\xi$ : Local coordinates changing between -1 and 1 to represent the location of any point within the interface element.

The stress–strain matrix is given as:

$$\{\sigma\} = [D] \times \{\varepsilon\} \tag{3}$$

Where

where
$$\{\sigma\} = \{\sigma_{t} \quad \sigma_{n1} \quad \sigma_{n2}\}^{T}$$

$$[D] = \begin{bmatrix} k_{t} & 0 & 0\\ 0 & k_{n} & 0\\ 0 & 0 & k_{n} \end{bmatrix}$$

Where

[D]: Constitutive stress relative displacements

 $\sigma_t$ : Tangential stress;  $\sigma_{nl}$ : first normal stress; and  $\sigma_{n2}$ :Second normal stress

 $k_t$  and  $k_n$  tangential and normal modulus. In the present paper, these moduli are derived from the (p-y) and (t-z) curves. The (p-y) curves are given by H. Matlock in 1970 for soft to firm clay, L. C. Reese and W. R. Cox in 1975 for stiff clay.

4-The end bearing element is introduced to take into account the interaction between the end pile and the soil. It consists of two nodes, one represents the pile tip point and the other indicates to the soil, and its alignment is the same as for the pile. It takes translation deformations only. The stiffness modulus is based on (q-z) curve for both clay and sand soils. This stiffness is only compression in other word for pullout pile its value takes zero [6].

## METHOD OF SOLUTION

Global stiffness and mass domain (soil +structure + interface+ end bearing) matrix are obtained by the assemblage of individual element types. For present problem, the subspace iteration method is used to find solution for free vibration problem while Newmark's beta method, also referred to as the constant acceleration method, is used for solving the forced vibration problem [11] noting that the conjugate gradient method [12] is used in these two problems as a solver to find the unknown variables. A computer program has been written in FORTRAN language to get the required results of the present study.

## **RESULTS and DISCUSSION**

For the current study, the dolphin of khor Al-Amaya berth no. 8 [13] is analyzed to impact loads that is given by Al-Jasim [14] for three cases which represent the same energy but different berthing velocities. From previous studies [13, 14], the maximum dynamic response is associated with velocity 1. Therefore, the dynamic analysis is carried for this velocity only. The response of vertical pile, pile 4 as shown in Fig. 2, is considered only because the other available methods are applicable to this case only rather than batter piles. The response considered here involves only pile lateral deflection and bending moment and the other



methods include the solution of elastic theory and Matlock and Reese solution for clayey soils. The obtained results are summarized in tables 1-2 and figures 3-6.

Table 1 shows the maximum deflection and the pile critical length for clayey soil using elastic, interface, and Reese solution. This table declares that the elastic and Reese solution are lesser than the interface solution by significant differences. For example, the deflection ratio between them is ranging (2.43-2.84) for elastic solution while these ratios become (1.84-2.13) for Reese solution. For the pile critical length, the interface solution gives higher values and a good agreement compared with Reese solution while elastic solution gives lesser values. This result can be explained from the stiffness of elastic solution which is more than the other solution, while interface solution is the least one since it includes both elastic and plastic deformations in the analysis.

Figures 3-6 demonstrate the deflection and bending moment for vertical pile along the pile length for clayey soils obtained using the elastic solution and interface solution for berthing velocity one, respectively. From these figures, a number of interesting points can be seen as follows:

1- The nonlinear response of pile to lateral loads using interface solution is more surmised compared with elastic solution that gives higher values (approximately 20%) in negative moment while the relative ratio between their positive bending moments is approximately ranging about 8 times for clay. There are many reasons can be attributed to these results given as follows:

A- The gap effect is ignored in the elastic solution but it is considered in the interface solution.

- B- The pile end rotation is ignored in the elastic solution but its effect is considered in the interface solution.
- 2- The trend of elastic solution is not changed as the soil strength increasing.
- 3-The trend of pile deflection gutted by interface solution is approximately similar to the trend of elastic solution, rapidly lowered with increasing soil depth, and remains unaltered as soil strength.
- 4- For two solutions, elastic and interface, the bending moment reaches approximately zero value at depth equal to 16 pile diameter from seabed for all clayey soils.

## THE PROPOSED CRITICAL LENGTH

The equation is proposed to estimate the critical length value for clay soils by modifying the Reese solution. The proposed equation is based on the following:

- 1- In proposed equation, the critical length is expressed by the undrained shear strength of the clay soils, soil cohesion, while in Reese equation this length depends on a modulus of sub grade reaction, given by equation 5 in unit N/m³, that depends on the elastic modulus of soil, Poisson's ratio of soil, and pile diameter.
- 2- The elastic modulus of pile and second moment area of pile cross section remain unchanged in proposed solution as in Reese equation.

According to the above assumptions, the proposed critical length becomes:

$$L_{cp} = Co \times \sqrt[4]{\frac{EI}{C_u}} - ----8$$

Where

 $L_{co}$  = proposed critical length,

*Co*= factor is found from obtained results and it is based on average values,

C= undrained shear strength of the clay soils, ranging from 24 kN/m<sup>2</sup> to 96 kN/m<sup>2</sup>,

From the obtained results, it is found that the average value of factor Co equals to 0.91, therefore the proposed critical length is calculated from:

$$L_{cp} = 0.91 \times \sqrt[4]{\frac{EI}{C_u}} - 9$$

Table 1 shows the critical length obtained from FORTRAN program and the critical length obtained from proposed solution in addition to the percentage error based on FORTRAN program. It is found from this table that a very good agreement is obtained for the proposed critical length which may be used to introduce a new solution for bending and deflection of pile foundation.

## **CONCLUSIONS**

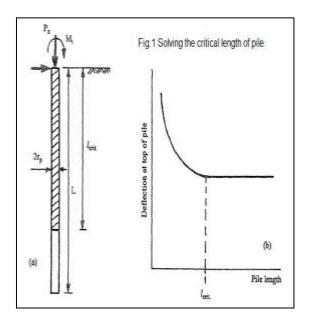
This paper focuses on finding a new approach to estimate the pile critical length in clayey soil using the application of interface element method to imitate the soil-structure interaction including normal and slip mode. A practical case study, an actual case study is taken to illustrate this method. The main conclusions are shown as follows:

1-The formulation of soil-structure interaction using the developed zero thickness interface model proposed by Goodman gives an appropriate results



without the appearance of the round off errors and singularity problems during different operations in the analysis.

- 2- The use of (p-y), and (t-z) curves in the deriving the nonlinear normal and tangential interface moduli seems suitable to represent the nonlinear behavior of pile-soil interaction, and a similar sequence is obtained for (q-z) curve that was used for deriving the stiffness modulus for the tip pile element.
- 4- The pile response is increased many times (2 to 3) times by using interface solution other than elastic or Reese method.
- 5-The critical pile length that defined as the required length to transfer lateral load and bending moment from the pile to the soil is found related to soil strength and pile properties (diameter and bending stiffness) . Therefore, the proposed solution of pile critical length (modified from Reese solution) for clayey soils is expressed in terms of soil cohesion and pile properties, and a proposed formula to estimate this length is given.



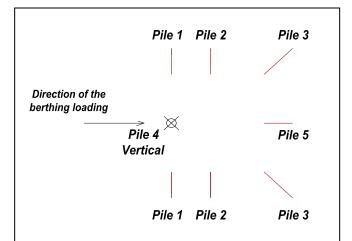


Fig. 2 Top view for the pile foundation of the dolphin of khor Al-Amaya berth no. 8

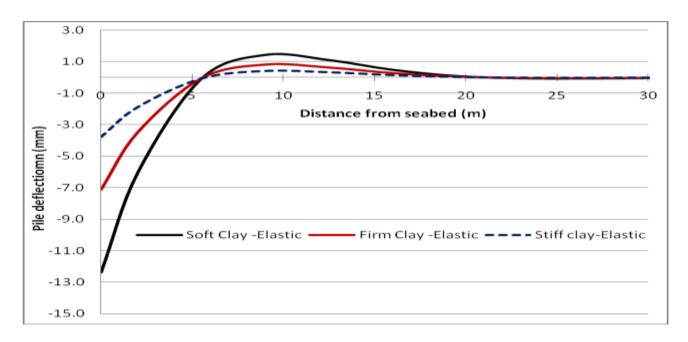


**Table 1** The pile maximum deflection and critical length for different clay soils using elastic, interface, and Reese solution

Method of Solution	Maximum deflection (mm)			Critical Length (m)		
	Soft Clay	Firm Clay	Stiff Clay	Soft Clay	Firm Clay	Stiff Clay
Elastic Solution	-12.39	-7.13	-3.76	5.66	5.66	6.01
Program Fortran	-30.09	-19.90	-10.60	15.71	13.80	11.15
Matlock and Reese	-16.37	-9.42	-4.97	14.15	11.89	10.00
Fortran/Elastic Ratio	2.43	2.79	2.82	2.78	2.44	1.86
Fortran/Reese Ratio	1.84	2.11	2.13	1.11	1.16	1.11

**Table 2** The critical length values obtained from Fortran program and from Proposed solution

Cohesion of Soil N/m <sup>2</sup>	24	48	72	96
Critical length Program	15. 70	13.80	12.01	11.15
Critical length Proposed	15.89	13.35	12.08	11.23
Error %	1.2	-3.2	0.6	0.8





**Fig. 3** The variation of maximum pile deflection along pile length for clay soils using elastic solution.

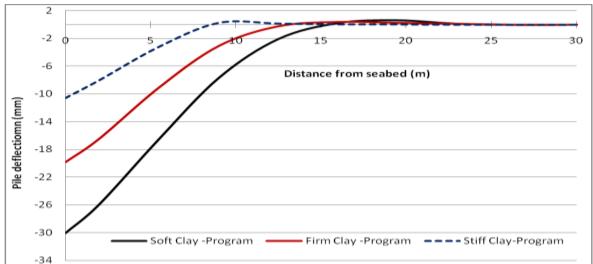
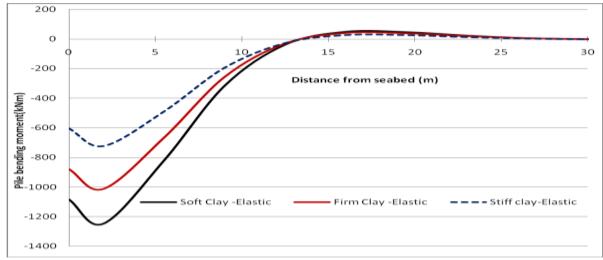
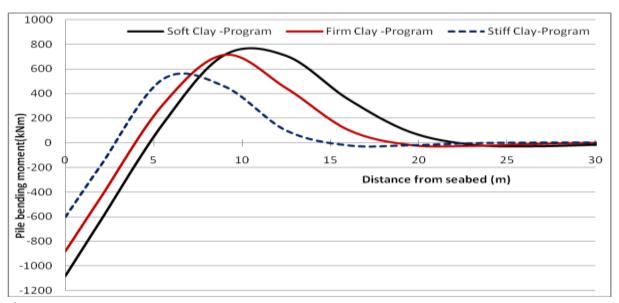


Fig. 4 The variation of maximum pile deflection along pile length for clay soils using interface solution.



**Fig. 5** The variation of maximum pile bending moment along pile length for clay soils using elastic solution.





**Fig. 6** The variation of maximum pile bending moment along pile length for clay soils using interface solution.

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