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LAMINAR MIXED CONVECTION FROM A SEMI-CIRCULAR CYLINDER IN AN AIDING BUOYANCY CONFIGURATION: A VALIDATION STUDY

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ABSTRACT

The laminar Newtonian flow and heat transfer phenomena across a semi-circular cylinder have been investigated in the steady regime under aiding buoyancy. A through validation study has been carried out for Reynolds number (Re) =1 - 30 and Richardson number (Ri)=0 - 2 for a Prandtl number of unity. The numerical calculations are completed by using Ansys solver. The total drag coefficient shows a maximum percentage difference of about only 0.4% with literature. However, the corresponding maximum percentage difference in the values of average Nusselt numbers is found about 2.5%. Moreover, the maximum enhancement in the heat transfer is found approximately 28% with forced convection (Ri=0).

Keywords: Semi-circular cylinder, Reynolds number, Richardson number, Prandtl number, Nusselt number and Total drag coefficient.

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1. INTRODUCTION

Flow over a cylinder of semi-circular cross section is the area of interest for many researchers due to variety of its engineering applications. Flow over a semi-circular cylinder is encountered in tubular and pin type heat exchange systems, processing of fibrous suspensions, screens to dewater coal-water slurries, filtration of sewage sludge and polymer melts, removal of oversized particles from coating suspensions, polymer and food processing applications, flow metering devices, electronic cooling, probe and sensors. On the contrary, mixed convection from a semi-circular cylinder has received very less attention compared to flow over a circular cylinder.

Recently, Chandra and Chhabra [1] carried out extensive numerical investigations on a semi-circular cylinder in an unconfined configuration for the varying ranges of Reynolds numbers (Re) and Prandtl numbers (Pr) in the steady regime. They reported that the size of the wake region grows almost linearly with Reynolds number. They also reported that the total drag is dominated by the pressure contribution even at low Reynolds number. Similarly, Gode et al. [2] did the numerical investigations on a semi-circular cylinder at low Re for the maximum value of Peclet number (i.e., the product of Re and Pr) being 4,000. The result shows that at low Reynolds numbers the drag force is dominated by its shearing components, but this contribution decreases with increase in Reynolds number. In another study, Chandra and Chhabra [3] investigated Newtonian/non-Newtonian flow over a semi-circular cylinder to find out the influence of power-law index (n) on the flow transitions for the ranges of settings: Re = 0.01 - 40 and n = 0.2 - 1.8. Irrespective of the type of fluid behaviour, the flow transitions are reported to occur at the value of the Reynolds number which is lower than that for a circular cylinder. Along the same line, Chandra and Chhabra [4] simulated the flow and heat transfer of power-law fluids over a semi-circular cylinder in the 2-D steady flow regime. They reported that the curved surface of semi-circular cylinder contributes up to 80 - 90% to overall heat transfer. Similarly, Chandra and Chhabra [5] reported and discussed the flow and heat transfer of power-law fluids over a semi-circular cylinder in an aiding buoyancy configuration in the steady flow regime. The result shows that the average Nusselt number increases with an increase in the values of Reynolds number, Prandtl number and Richardson number.

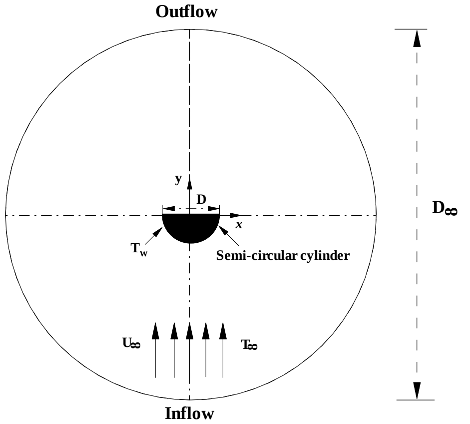

Fig. 1: Schematic of the flow for the problem under consideration.

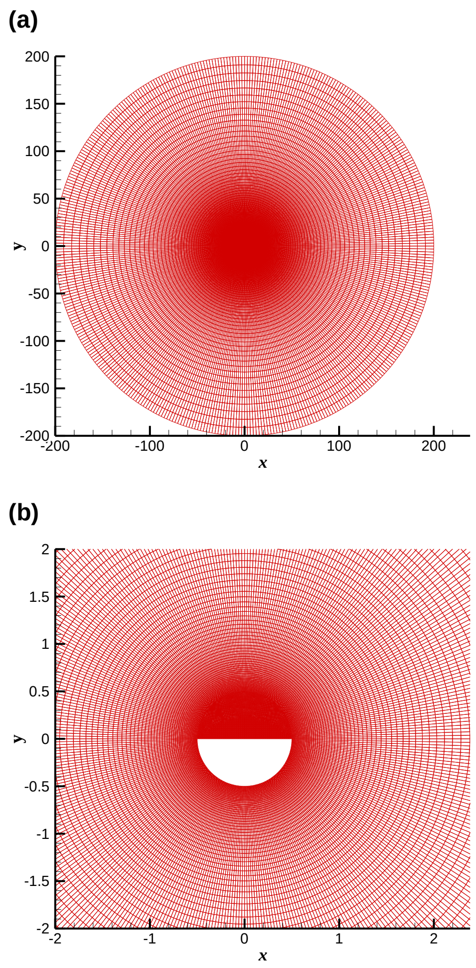

Fig. 2: Computational grid (a) Full grid and (b) Grid near a cylinder.

2. PROBLEM DESCRIPTION

The 2-D, steady and incompressible flow of Newtonian fluids (at a temperature T_∞) over a semi-circular cylinder of diameter D (heated to a constant wall temperature of T_w) with its curved surface facing upstream is considered as shown in Figure 1. The free stream of the fluid approaches the cylinder with a uniform velocity U_∞. Both the imposed flow and gravity induced flow are in the upward direction thereby resulting in the so called aiding flow configuration. The computational domain size (D_∞/D) is taken as 400 based on the extensive studies carried out in ref. [5].

The continuity, *x*- and *y*- components of Navier-Stokes and thermal energy equations are given as follows

Continuity equation

$$\frac{\partial U_x}{\partial x}+\frac{\partial U_y}{\partial y}=0 \tag{1}$$

Momentum equations

$$\frac{\partial U_x}{\partial t}+\frac{\partial U_x U_x}{\partial x}+\frac{\partial U_y U_x}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\nu\left(\frac{\partial^2 U_x}{\partial x^2}+\frac{\partial^2 U_x}{\partial y^2}\right) \tag{2a}$$

$$\frac{\partial U_y}{\partial t}+\frac{\partial U_x U_y}{\partial y}+\frac{\partial U_y U_y}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial y}+\nu\left(\frac{\partial^2 U_y}{\partial x^2}+\frac{\partial^2 U_y}{\partial y^2}\right)+RiT \tag{2b}$$

Energy equation

$$\frac{\partial T}{\partial t}+\frac{\partial U_x T}{\partial x}+\frac{\partial U_y T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{3}$$

where U_x, U_y, P, T are the x and y components of velocity, pressure and temperature respectively. ν is the kinematic viscosity $(\frac{\mu}{\rho})$ and α is the thermal diffusivity $(\frac{k}{(\rho C_P)})$. Here, flow and heat transfer phenomena are governed by three dimensionless groups namely Re, Ri and Pr, which are defined as Reynolds number $(\mathrm{Re})=\frac{\rho U_\infty D}{\mu}$, Richardson number $(Ri)=\frac{Gr}{\mathrm{Re}^2}$ and Prandtl number $(\mathrm{Pr})=\frac{\mu C_P}{k}$ respectively.

The following boundary conditions may be written for the present system.

- *At the inlet*: U_x=0, $U_y=U_\infty$ and $T=T_\infty$
- *At the surface of a semi-circular cylinder:* $U_x=0$, $U_y=0$ and $T=T_w$
- *At the outlet*: It is located sufficiently far downstream from the semi-circular body, $\partial U_x/\partial y=0$, $\partial U_y/\partial y=0$ and $\partial T/\partial y=0$.

A commercial solver Ansys Fluent is being used to solve governing equations along with the above noted boundary conditions. The 2-D, steady, laminar, coupled solver was used to solve the incompressible flow on the collocated grid arrangement. To discretize convective terms of momentum and energy equations, the second order upwind scheme is being utilized. The Semi-Implicit Method for the Pressure Linked Equations (SIMPLE) algorithm is used to circumvent pressure-velocity decoupling. The algebraic equations resulting after linearization are solved by using the Gauss-Seidel iterative method in conjunction with an Algebraic Multi Grid solver (AMG). The relative convergence criteria of 10^{-08} each and 10^{-15} each are used for the residuals of continuity, *x*-momentum, *y*-momentum and energy equations in steady regime.

The non-uniform quadrilateral grid has also been generated in Ansys. In order to capture the steep gradients near the surface of the semi-circular cylinder, sufficiently fine grid was chosen near the vicinity of the cylinder (Fig. 2). There are 67137 cells in the optimized computational domain compared to 70065 cells in ref. [5], but the smallest cell size is taken as same as ref. [5] (e.g. 0.01D) and is shown in Fig. 2.

3. RESULT AND DISCUSSION

3.1 VALIDATION

The present study used the standard literature to establish the validity of the present numerical simulation approach. The validation is done with Chandra and Chhabra's [5] observations and the present results are showing an excellent agreement. The maximum percentage deviation of average Nusselt number is found to be about 2.5%; however, the total drag coefficient shows a variation of only about 0.4% with ref. [5].

3.2 VARIATION OF DRAG COEFFICIENT WITH Re AND Ri

The overall drag coefficients for different Reynolds numbers are calculated and plotted in Fig. 3. The overall drag coefficient is showing the decremental behaviour with increase in the value of Reynolds number. The overall drag coefficient increases with increase in the value of the Richardson number for the constant value of the Reynolds number. The validation results show a maximum relative percentage difference of only about 0.37% for a Reynolds number of 30 and Richardson number of 2.

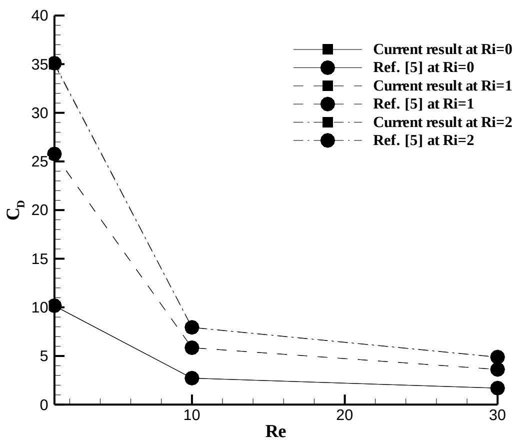

Fig. 3: The variation of total drag coefficient with Reynolds numbersat different Richardson number.

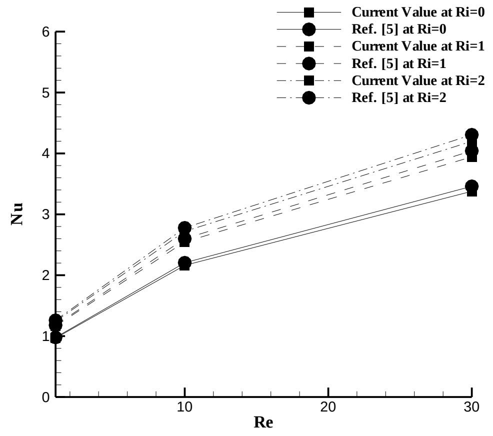

Fig. 4: The variation of average Nusselt number with Reynolds number and Richardson number.

3.3 VARIATION OF AVERAGE NUSSELT NUMBER WITH Re AND Ri

The variation of the average Nusselt number with Reynolds number for different Richardson number is shown in Fig. 4. It is found that the average Nusselt number increases with increase in Reynolds number. Similarly, the average Nusselt number increases with increase in Richardson number for a fixed value of Reynolds number. The present results are showing a maximum relative difference of about 2.5% for a Reynolds number of 30 and a Richardson number of 2. Analogous to the unbounded semi-circular cylinder [1], the value of the average Nusselt number is found to be higher on the curved portion of the semi-circular cylinder compared to the flat surface of the semi-circular cylinder. The average Nusselt number is showing a maximum relative percentage enhancement of approximately 28% with forced convection (Ri=0) for the Reynolds number of 1 and the Richardson number of 2.

4. CONCLUSIONS

In this study, the effects of Richardson numbers and Reynolds numbers on the aiding-buoyancy mixed convection around an unconfined semi-circular cylinder are investigated and validated for the Prandtl number of unity. The drag coefficient is showing the decremental behaviour with increase in Reynolds number for the constant value of the Richardson number. The drag coefficient increases with increase in Richardson number for the constant Reynolds number. As the value of Richardson number increases, the average Nusselt number increases for the fixed Reynolds number. The present drag results show the maximum percentage deviation of only about 0.37% for a Reynolds number of 30 and Richardson number of 2. However, the present heat transfer results are showing a maximum deviation of about 2.5% for the preceding ranges of settings. The maximum enhancement in the heat transfer is found approximately 28% with forced convection (Ri=0).

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NOMENCLATURE

β_V Coefficient of volumetric expansion,1/K

C_D Total drag coefficient (=F_D/(0.5 U^2_∞D))

C_P Specific heat, J/(kg.K)

D Diameter of a semi-circular cylinder, m

F_D Drag force per unit length of the cylinder, N/m

g Acceleration due to gravity, m/s^2

Gr Grashof number ($= \dfrac{g\beta_V (T_W - T_\infty)\rho^2 D^3}{\mu^2}$)

$\bar{h}$ Average heat transfer coefficient, $W/(m^2.K)$

k Thermal conductivity, W/(m.K)

n Power-law index

Nu Average Nusselt number ($= \dfrac{\bar{h}D}{k}$)

P Pressure (Pa)

Pr Prandtl number ($= \dfrac{\mu C_P}{k}$)

Re Reynolds number ($= \dfrac{D\rho U_\infty}{\mu}$)

Ri Richardson number ($= Gr/\mathrm{Re}^2$)

t Time (s)

T_∞ Free stream temperature, K

T_w Cylinder surface temperature, K

T Temperature, K

U_∞ Average velocity at inlet, m/s

U_x Stream velocity in x direction, m/s

U_y Stream velocity in y direction, m/s

α Thermal diffusivity

ρ Density of fluid, kg/m^3

μ Dynamic viscosity, $N.s/m^2$

ν Kinematic viscosity, m^2/s

Subscripts

w Surface of the cylinder

∞ Inlet condition

x Stream wise coordinate, m

y Transverse coordinate, m