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THEORETICAL APPROACH TO STUDY DISTURBANCES DUE TO MECHANICAL SOURCE IN A GENERALIZED THERMOELASTIC DIFFUSIVE HALF SPACE

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Abstract— **Disturbances caused by impulsive mechanical source in a homogeneous, isotropic half-space are studied within the context of generalized thermoelastic diffusion with one relaxation time. A two dimensional half space with a permeating substance in contact with the bounding plane is considered in axisymmetric distribution. The chemical potential is assumed to be a known function of time. Integral transform technique is used to find the analytic solution in the transform domain by using direct approach. Inversion of transforms is done employing a numerical scheme. Mathematical model is prepared for Copper material and numerical results for temperature, stress, displacement, chemical potential and concentration are obtained and illustrated graphically.**

Keywords—*: impulsive, half space, diffusion, generalized, thermoelastic, relaxation time.*

Introduction

Generalized thermoelasticity theories are successful in removing the paradox of infinite speed of propagation of thermal signals inherent in the classical coupled thermoelasticity introduced by Biot [1]. Lord and Shulman [2] developed a theory modifying the Fourier law of heat conduction by introducing the heat flux rate and a relaxation time for the special case of an isotropic body. The heat equation associated with this theory is of wave type.

Diffusion in thermoelastic solids is a transport phenomena governed by Fick's law which states that the passive movement of molecules or particles is along the concentration gradient. Thermoelastic diffusion involves the coupling of the fields of temperature, mass diffusion and strain. It has a wide range of applications in geophysics and industries. In particular, diffusion is used to form the base and emitter in bipolar transistors, form integrated resistors, form the source/drain regions in Metal oxide semiconductors (MOS) transistors and dope poly-silicon gates in MOS transistors.

Study of phenomenon of diffusion is used to improve the conditions of oil extractions and is of great deal of interest for oil extraction companies. Nowacki [3-6] developed the theory of thermoelastic diffusion. The theory of Nowacki uses Fick's law. Sherief et al. [7] introduced the theory of thermoelastic diffusion in the framework of Lord-Shulman theory by introducing thermal relaxation time parameter and diffusion relaxation parameters governing the field equations. Many researchers [8-13] studied various types of problems in thermoelastic diffusion. Tripathi et al. [14, 15] studied problems on generalized thermoelasticity in a semi-infinite solid circular cylinder with one relaxation time and discussed a problem of generalized thermoelastic diffusion in a thick circular plate with axisymmetric heat supply. Elhagary [16] solved a two dimensional generalized thermoelastic diffusion problem for a half-space subjected to harmonically varying heating.

The objective of this work is to study the effects of impulsive mechanical source on thermoelastic diffusion interactions in a half space under axisymmetric distribution within the context of Lord-Shulman theory of generalized thermoelastic diffusion (TEDLS). The Classical coupled thermoelastic diffusion theory (TEDCT) is recovered as a special case. Analytic solutions for temperature, concentration, chemical potential, displacement and stresses are obtained in the Laplace transform domain using direct approach. Numerical inversion of Laplace transforms are performed using Gaver-Stehfast Algorithm [17-19] and all integrals were evaluated using Romberg's integration technique [20] with variable step size. A mathematical model is prepared for Copper material and results are discussed along with the graphical representation.

I. Formulation of the Problem

We shall consider a homogeneous isotropic thermoelastic solid occupying the region $z \geq 0$. The z-axis is taken perpendicular to the bounding plane pointing inwards. The problem is considered within the context of the theory of generalized thermoelastic diffusion with one relaxation time. We shall assume that the initial state of the medium is quiescent at a temperature T_0. An impulsive mechanical source is assumed to act at the origin of the cylindrical co-ordinate system (r,φ,z) having isothermal boundary and the chemical potential is a known function of time.

The problem is thus two-dimensional with all considered functions depending on the spatial variables r and z as well as on the time variable t.

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The displacement vector, thus, has the form $\vec{u} = (u, 0, w)$.

For the two dimensional problem, the components of strain tensors can be written in the form,

$$e_{rr} = \frac{\partial u}{\partial r}, \; e_{\varphi\varphi} = \frac{u}{r}, \; e_{zz} = \frac{\partial w}{\partial z}, \; e_{rz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) \qquad (1)$$

and e is the cubical dilatation given by,

$$e = \frac{u}{r} + \frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z}$$

where $\nabla^2 \equiv \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$

The governing equations are of the form,

$$\mu \nabla^2 u - \frac{\mu}{r^2} u + (\lambda + \mu)\frac{\partial e}{\partial r} - \beta_1 \frac{\partial \theta}{\partial r} - \beta_2 \frac{\partial C}{\partial r} = \rho \frac{\partial^2 u}{\partial t^2} \qquad (2)$$

$$\mu \nabla^2 w + (\lambda + \mu)\frac{\partial e}{\partial z} - \beta_1 \frac{\partial \theta}{\partial z} - \beta_2 \frac{\partial C}{\partial z} = \rho \frac{\partial^2 w}{\partial t^2} \qquad (3)$$

$$k \nabla^2 \theta = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)(\rho C_E \theta + \beta_1 \theta_0 \, div \, \underline{u} + a\theta_0 C) \qquad (4)$$

$$D\beta_2 \nabla^2 (div \, \vec{u}) + Da\nabla^2\theta + \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right) C - Db\nabla^2 C = 0 \qquad (5)$$

$$\sigma_{\varphi\varphi} = 2\mu e_{\phi\phi} + \lambda e - \beta_1(\theta - \theta_0) - \beta_2 C \qquad (6a)$$

$$\sigma_{rr} = 2\mu e_{rr} + \lambda e - \beta_1(\theta - \theta_0) - \beta_2 C \qquad (6b)$$

$$\sigma_{zz} = 2\mu e_{zz} + \lambda e - \beta_1(\theta - \theta_0) - \beta_2 C \qquad (6c)$$

$$\sigma_{rz} = \mu e_{rz} \qquad (6d)$$

$$\sigma_{r\varphi} = \sigma_{z\varphi} = 0 \qquad (6e)$$

$$P = -\beta_2 e + bC - a(\theta - \theta_0) \qquad (7)$$

To facilitate the solution, the following dimensionless variables are introduced

$$r' = c_1\eta\, r, z' = c_1\eta\, z, u' = c_1\eta\, u, w' = c_1\eta\, w, t' = c_1^2\eta\, t,$$
$$\tau_1' = c_1^2\,\eta\,\tau, \; P' = P/\beta_2, \; \sigma_{ij}' = \sigma_{ij}/(\lambda + 2\mu), \; \tau_0' = c_1^2\,\eta\,\tau_0$$
$$\theta' = \beta_1(\theta - \theta_0)/(\lambda + 2\mu), \; C' = \beta_2 C/\lambda + 2\mu \qquad (8)$$

where $\eta = \rho C_E / k$ is the dimensionless characteristic length, $c_1 = \sqrt{\lambda + 2\mu/\rho}$, is the speed of propagation of isothermal elastic waves.

The boundary conditions of the problem in dimensionless form at $z = 0$ are taken as

$$\theta(r, 0, t) = 0 \quad , \quad 0 < r < \infty \qquad (9)$$

$$\sigma_{zz}(r, 0, t) = -\frac{Q_0 \delta(r)\delta(t)}{2\pi r} \quad , \quad 0 < r < \infty \qquad (10)$$

$$\sigma_{rz}(r, 0, t) = 0 \quad , \quad 0 < r < \infty \qquad (11)$$

$$P(r, 0, t) = H(t)\, f(r) \quad , \quad 0 < r < \infty \qquad (12)$$

where $\delta(.)$ denotes the Dirac delta function, Q_0 is the magnitude of the force, $f(r)$ is a known function and $H(t)$ is a Heaviside unit step function.

II. Analytic Solution

Applying the Laplace and Hankel transform to a function $f(r, z, t)$ defined by

$$\bar{f}(r, z, s) = L[f(r, z, t)] = \int_0^\infty e^{-st} f(r, z, t)\,dt$$

$$\bar{f}^*(\alpha, z, s) = H[\bar{f}(r, z, s)] = \int_0^\infty \bar{f}(r, z, s)\, r\, J_0(\alpha r)\,dr$$

On taking Laplace and Hankel transform of both sides of equations (2)-(5), after using equation (8) (supressing the primes for convenience) , we get,

$$(D^2 - \alpha^2)\bar{\theta}^* = (s + \tau_0 s^2)(\bar{\theta}^* + \varepsilon \bar{e}^* + \varepsilon \alpha_1 \bar{C}^*) \qquad (13)$$

$$(D^2 - \alpha^2)\,\bar{e}^* + \alpha_1(D^2 - \alpha^2)\bar{\theta}^* + \alpha_2(s + \tau_0 s^2)\bar{C}^* - \alpha_3(D^2 - \alpha^2)\bar{C}^* = 0 \qquad (14)$$

$$(D^2 - \alpha^2 - s^2)\bar{e}^* = (D^2 - \alpha^2)\bar{\theta}^* + (D^2 - \alpha^2)\bar{C}^* \qquad (15)$$

Eliminating the transformed $\bar{e}^*$, $\bar{C}^*$ and $\bar{\theta}^*$ from (13), (14) and (15), we obtain the following six order differential equation

$$(D^6 - a_1 D^4 + a_2 D^2 - a_3)(\bar{\theta}^*, \bar{e}^*, \bar{C}^*) = 0 \qquad (15)$$

where the coefficients a_1, a_2, a_3 are giving by

$$a_1 = -\frac{s}{(\alpha_3 - 1)}\left\{\begin{array}{l}(s\tau_0 + 1)(\alpha_1\varepsilon(\alpha_1 + 2) + \alpha_3(\varepsilon + 1) - 1) \\ + \alpha_2(s\tau + 1) + \alpha_3 s\end{array}\right\}$$

$$a_2 = \frac{s^2}{(\alpha_3 - 1)}\left\{\begin{array}{l}(s\tau_0 + 1)(\alpha_1^2 \varepsilon s + \alpha_3 s + \alpha_2(s\tau + 1)(\varepsilon + 1)) \\ + \alpha_2 s(s\tau + 1)\end{array}\right\}$$

$$a_3 = \frac{s^4 \alpha_2}{(\alpha_3 - 1)}(s\tau_0 + 1)(s\tau + 1)$$

Equation (15) can also be written as,

$$(D^2 - k_1^2)(D^2 - k_2^2)(D^2 - k_3^2)(\bar{\theta}^*, \bar{e}^*, \bar{C}^*) = 0 \qquad (16)$$

where $\pm k_1, \pm k_2$ and $\pm k_3$ are the roots of the characteristic equation given by,

$$k^6 - a_1 k^4 + a_2 k^2 - a_3 = 0 \qquad (17)$$

The roots k_1, k_2 and k_3 are given by,

$$k_1 = \sqrt{\frac{1}{3}(2p_1 \sin(p_2) + a_1)}$$

$$k_2 = \sqrt{\frac{1}{3}\left[a_1 - p_1\left(\sqrt{3}\cos(p_2) + \sin(p_2)\right)\right]}$$

$$k_3 = \sqrt{\frac{1}{3}\left[a_1 + p_1\left(\sqrt{3}\cos(p_2) - \sin(p_2)\right)\right]}$$

where

$p_1 = \sqrt{a_1^{\,2} - 3a_2}$, $p_2 = \dfrac{\sin^{-1}(\gamma)}{3}$ and $\gamma = -\left(\dfrac{2a_1^{\,3} - 9a_1a_2 + 27a_3}{2p^3}\right)$

The solutions of equation (16) are of the form,

$$\bar{\theta}^* = \sum_{i=1}^{3} A_i(\alpha, s)e^{-q_i z} \tag{18}$$

$$\bar{e}^* = \sum_{i=1}^{3} A_i'(\alpha, s)e^{-q_i z} \tag{19}$$

$$\bar{C}^* = \sum_{i=1}^{3} A_i''(\alpha, s)e^{-q_i z} \tag{20}$$

where $A_i; A_i'$ and A_i'', $i = 1, 2, 3$ are parameters depending on α and s. Substituting from (18), (19), (20) into (14) and (15), the parameters $B_i(\alpha, s)$ and $C_i(\alpha, s)$, $i = 1, 2, 3$ can be expressed in terms of $A_i(\alpha, s)$ as

$$A_i'(\alpha, s) = f_i\, A_i(\alpha, s) \ , \quad A_i''(\alpha, s) = d_i\, A_i(\alpha, s) \tag{21}$$

where $f_i = \dfrac{\left\{k_i^{\,4} - \left[\left(s + \tau_0 s^2\right)(1 - \alpha_1\varepsilon)\right]k_i^{\,2}\right\}}{\varepsilon\left(s + \tau_0 s^2\right)\left\{k_i^{\,2}(1 + \alpha_1) - \alpha_1 s^2\right\}}$,

$d_i = \dfrac{\left\{k_i^{\,4} - \left[\left(s + \tau_0 s^2\right)(1 + \varepsilon) + s^2\right]k_i^{\,2} + \left(s + \tau_0 s^2\right)s^2\right\}}{\varepsilon\left(s + \tau_0 s^2\right)\left\{k_i^{\,2}(1 + \alpha_1) - \alpha_1 s^2\right\}}$

Applying the inversion of Hankel transform to equations (18), (19) and (20), we get,

$$\bar{\theta} = \int_0^\infty \left\{\sum_{i=1}^{3} A_i(\alpha, s)e^{-q_i z}\right\}\alpha\, J_0(\alpha r)\, d\alpha \tag{22}$$

$$\bar{e} = \int_0^\infty \left\{\sum_{i=1}^{3} A_i'(\alpha, s)e^{-q_i z}\right\}\alpha\, J_0(\alpha r)\, d\alpha \tag{23}$$

$$\bar{C} = \int_0^\infty \left\{\sum_{i=1}^{3} A_i''(\alpha, s)e^{-q_i z}\right\}\alpha\, J_0(\alpha r)\, d\alpha \tag{24}$$

Applying Laplace transform to eqns. (2)–(3) and making use of eqns. (22)–(24), the solutions for the displacement components in the Laplace transform domain as

$$\bar{u}(r, z, s) = \int_0^\infty -\alpha^2 J_1(\alpha r)\begin{bmatrix} B(\alpha, s)e^{-qz} \\ +\sum_{i=1}^{3} \dfrac{\lambda_i}{(q_i^{\,2} - q^2)} e^{-q_i z}\end{bmatrix} d\alpha \tag{25}$$

$$\bar{w}(r, z, s) = \int_0^\infty \alpha J_0(\alpha r)\begin{bmatrix} C(\alpha, s)\, e^{-qz} \\ +\sum_{i=1}^{3} \dfrac{\lambda_i\, q_i}{(q_i^{\,2} - q^2)} e^{-q_i z}\end{bmatrix} d\alpha \tag{26}$$

where the parameters $B(\alpha, s)$ and $C(\alpha, s)$ depend on α and s only.

Also, $q^2 = \alpha^2 + \beta^2 s^2$, $C(\alpha, s) = \dfrac{\alpha^2 B(\alpha, s)}{q}$,

$\lambda_i = \left\{\left(1 - \beta^2\right)f_i + \beta^2(1 + d_i)\right\}A_i$

Applying Laplace transform to eqns. (6a)-(7) and making use of the solutions given in eqns. (22)-(26), we obtain the stress components and the chemical potential in the Laplace transform domain,

$$\bar{\sigma}_{\varphi\varphi} = -\frac{2}{\beta^2 r}\int_0^\infty \alpha^2 J_1(\alpha r)\begin{bmatrix} B(\alpha, s)e^{-qz} \\ +\sum_{i=1}^{3} \dfrac{\lambda_i}{(q_i^{\,2} - q^2)} e^{-q_i z}\end{bmatrix} d\alpha + \bar{G} \tag{27}$$

$$\bar{\sigma}_{rr} = -\frac{2}{\beta^2}\int_0^\infty \alpha^3 \begin{bmatrix} \dfrac{1}{\alpha r} J_1(\alpha r) \\ - J_0(\alpha r)\end{bmatrix}\begin{bmatrix} B(\alpha, s)\, e^{-qz} \\ +\sum_{i=1}^{3} \dfrac{\lambda_i}{(q_i^{\,2} - q^2)} e^{-q_i z}\end{bmatrix} d\alpha + \bar{G} \tag{28}$$

$$\bar{\sigma}_{zz} = \frac{2}{\beta^2}\int_0^\infty \begin{bmatrix} -q\, C(\alpha, s)\, e^{-qz} \\ -\sum_{i=1}^{3} \dfrac{\lambda_i q_i^{\,2}}{(q_i^{\,2} - q^2)} e^{-q_i z}\end{bmatrix} \alpha J_0(\alpha r) d\alpha + \bar{G} \tag{29}$$

$$\bar{\sigma}_{rz} = \frac{1}{\beta^2}\int_0^\infty \begin{bmatrix} \left(\dfrac{\alpha^2 + q^2}{q}\right) B(\alpha, s)\ e^{-qz} \\ +\sum_{i=1}^{3} \dfrac{\lambda_i q_i (1 + q_i)}{(q_i^{\,2} - q^2)} e^{-q_i z}\end{bmatrix} \alpha^2 J_1(\alpha r) d\alpha \tag{30}$$

$$\bar{P}(r, z, s) = \frac{2}{\beta^2}\int_0^\infty \left[\sum_{i=1}^{3} \mu_i A_i\, e^{-q_i z}\right] \alpha\, J_0(\alpha r) d\alpha \tag{31}$$

$$\bar{G} = \int_0^\infty \alpha\, J_0(\alpha r)\left(\sum_{i=1}^{3} \xi_i e^{-q_i z}\right) d\alpha$$

$\mu_i = (-f_i + \alpha_3 d_i - \alpha_1)$, $\xi_i = \left(\left(\beta^2 - 2\right)f_i / \beta^2 - d_i - 1\right)A_i$

Applying Laplace transform and Hankel transform on both sides of boundary conditions (9)-(12) and using equations (22)-(31), we get,

$$\sum_{i=1}^{3} A_i(\alpha, s) = 0 \tag{32}$$

$$\frac{2}{\beta^2}\left[\alpha^2 B(\alpha, s) + \sum_{i=1}^{3} \frac{\lambda_i q_i^{\,2}}{(q_i^{\,2} - q^2)}\right] = -Q_0 \tag{33}$$

$$\left(\frac{\alpha^2 + q^2}{q}\right) B(\alpha, s) + 2\sum_{i=1}^{3} \frac{\lambda_i q_i (1 + q_i)}{(q_i^{\,2} - q^2)} = 0 \tag{34}$$

$$(2/\beta^2)\left[\sum_{i=1}^{3} \mu_i A_i\right] = f^*(\alpha) \tag{35}$$

Equations (32)-(35) is a system of linear equations with A_1, A_2, A_3 and B as unknown parameters. Solving the above system of linear equations, the complete solution of the problem is obtained in the Laplace transform domain.

III. Inversion of Double transforms

The formula for the inverse of the Laplace transform as obtained by Gaver [17] and Stehfast [18, 19] is given below.

By this method the inverse $f(t)$ of the Laplace transform $\bar{f}(s)$ is approximated by,

$$f(t)=(\ln 2/t)\sum_{j=1}^{K} D(j,K)\,F\,(j\ln 2/t) \qquad (36)$$

With

$$D(j,K)=(-1)^{j+M}\sum_{n=m}^{\min(j,M)}\frac{n^M(2n)!}{(M-n)!n!(n-1)!(j-n)!(2n-j)!} \qquad (37)$$

where K is an even integer, whose value depends on the word length of the computer used. $M=K/2$ and m is the integer part of the $(j+1)/2$. The optimal value of K was chosen as described in Gaver-Stehfast algorithm, for the fast convergence of results with the desired accuracy. This method is easy to implement and very accurate for functions of the type $e^{-\alpha t}$. The Romberg numerical integration technique [20] with variable step size was used to evaluate the integrals involved. All the programs were made in mathematical software Matlab.

IV. Numerical results and discussion

The chemical potential is taken as

$$f_2(r)=P_0H(r_1-r)$$

where P_0 is constant.

On taking Hankel transform, we get,

$$f_2^*(\alpha)=(r_1P_0/\alpha)\,J_1(\alpha).$$

Mathematical model is prepared with Copper material for purposes of numerical computations. The material constants of the problem are thus given in S.I. units [9]:

$T_0=293\,K$, $\rho=8954\,kg\,m^{-3}$, $\tau_0=0.02$, $\tau=0.2$, $k=386\,JK^{-1}m^{-1}s^{-1}$,
$\alpha_t=1.78\times10^{-5}K^{-1}$, $\alpha_C=1.78\times10^{-5}K^{-1}$,
$\mu=3.86\times10^{10}Nm^{-2}$, $\lambda=7.76\times10^{10}Nm^{-2}$, $c_1=4.158\times10^{3}ms^{-1}$,
$a=1.2\times10^4\,m^2/s^2k$, $b=0.9\times10^6\,m^5/kg\,s^2$,
$D=0.88\times10^{-8}\,kg\,s/m^3$, $c_E=383.1\,J.Kg^{-1}K^{-1}$,

Using these values it was found that, $\eta=8886.73\,s.m^{-2}$
$\varepsilon=0.0168\,Nm.J^{-1}$, $\beta^2=4$, $\alpha_1=5.43$, $\alpha_2=0.533$, $\alpha_3=36.24$.
It should be noted that a unit of non-dimensional time corresponds to $6.5\times10^{-12}s$, while a unit of non-dimensional length corresponds to $2.7\times10^{-8}m$. The computations were carried out for non-dimensional time $t=0.05$.

Figures 1-3 exhibit the variations of θ, C and σ_{zz} with distance r. The variations of the various components with distance r are shown a) Solid line for TEDCT theory b) Dotted line for TEDLS theory. The numerical simulations are done at the bounding plane i.e. $z=0$.

Fig. 1 exhibits the variation of θ as a function of radius. It is observed that θ follows a non-uniform pattern as distance r increases. TEDLS and TEDCT theories show large variations throughout the medium. It is seen that for TEDLS theory, temperature has a positive value at $r=0$ and then follows an oscillatory pattern whereas for TEDCT theory, the values of temperature at $r=0$ is negative and then it gradually increases and follows an oscillatory pattern thereafter. As the disturbance travels through the medium, it encounters sudden changes, resulting in a non uniform pattern of the curves which shows the effect of coupling of the fields of temperature, diffusion and strain.

In figure 2, the concentration C shows an oscillatory behavior throughout the medium. The values of concentration fall sharply till $r\le3$ and then gradually decrease to zero with the increase in radial distance. The magnitudes of values of concentration for TEDLS theory are more than TEDCT theory throughout the medium. Particularly, if we observe the region $3\le r\le6$, the non-uniformity in the graphs is clearly visible. This can be attributed to the effect of coupling between the fields of temperature, diffusion and strain.

Figure 3 exhibits the variation of σ_{zz} along the radial direction. One can observe that the variation in values of σ_{zz} for TEDLS and TEDCT theory is seen throughout the medium. The axial stress values are tensile in the medium. A sharp fall in σ_{zz} values is seen till $r\le3$ and then is gradually decreases. It is also observed that the values of axial stress are more for TEDLS theory than TEDCT theory up to $r\le4$ and then TEDCT theory predicts higher axial stress as compared to TEDLS theory. It is also observed that the axial stresses for TEDLS theory become compressive after $r\ge8$.

V. Conclusion

In this work, the effect of an impulsive mechanical source on a two dimensional thermoelastic half-space in contact with a permeating substance was investigated. The method used in this study provides quite a successful approach in dealing with thermoelastic diffusion problems without any assumed restriction on the field variables. Coupling of the diffusion field, temperature and strain plays an important role in the deformation of an elastic body. As the disturbance travels through the medium, it encounters sudden changes, resulting in a non uniform pattern of the curves. It was observed that the chemical potential of the diffusive material attains a steady state. The results of this problem are very useful in the two dimensional problems in axisymmetric half-space which have various geophysical and industrial applications.

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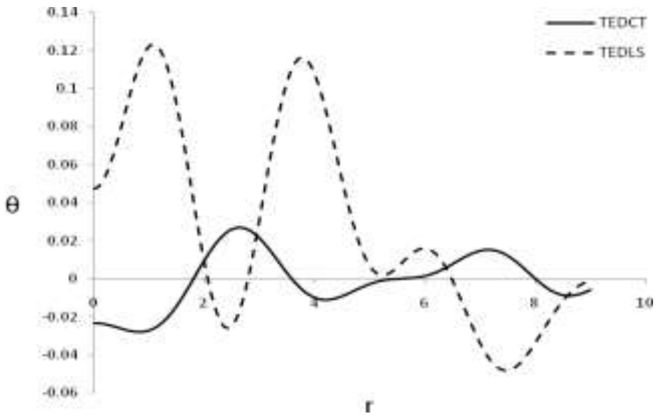

Fig.1.Temperature θ distribution along the radial direction

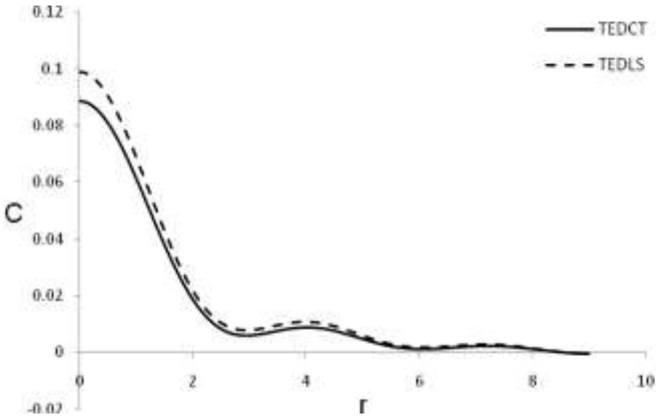

Fig.2. Concentration C distribution along the radial direction

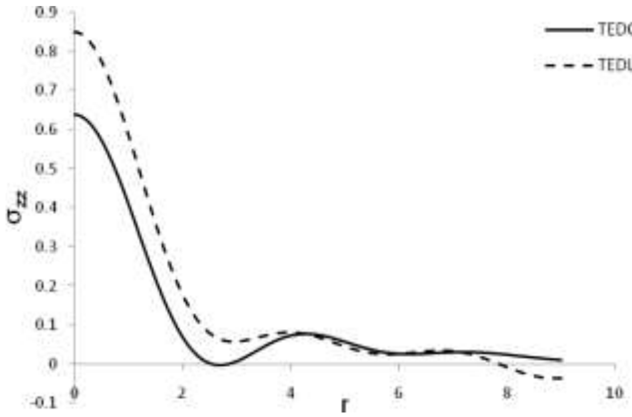

Fig.3. Axial stress σ_{zz} distribution along the radial direction

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