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Uncertainty quantification of settlements in granular soils using Pseudo-response surface method

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Abstract:

Soil, owing to its complex process of deposition, exhibits spatial uncertainty. Deterministic methods of settlement predictions are far from accurate and generally rely on site specific test data. In order to carry out stochastic analysis and get an estimate of the probability of failure, one needs a limit state function expressing the relationship between all input and output variables. Often to preserve the simplicity of a limit state function, there will be many input variables that need to be read from empirical charts and make the limit state function very complex if incorporated in one equation. This paper presents a method where in till a pseudo response surface is constructed, a limit state function that refers to empirical charts is used, and a stochastic analysis based on Monte Carlo simulations in conjunction with the developed response surface is carried out to arrive at the probability of failure. This way, the engineer assessing the settlement need not build a finite element model and construct a response surface out of it. In this paper, this technique is applied on a Level III reliability based design example solved in the literature using Finite Element Method.

INTRODUCTION

Since soil is not a homogeneous material, its parameters are most of the times associated with some uncertainty. Keeping this in mind if a geotechnical engineer has to make an engineering prediction about the reliability of a design, he has to either resort to simple and approximate probabilistic uncertainty propagation methods like FORM/SORM [1], response surface (RSM)[2] methods or more complicated and robust methods like stochastic response surface method (SRSM) [3] or its variants like collocation based stochastic response surface method (CSRSM) [4], or more conventional simulation based methods like Monte Carlo simulations [6] Subset simulation [5]etc. Among the methods mentioned above, Monte Carlo simulation is straight forward when it comes to implementation but is computationally expensive. In order to gain some numerical efficiency, methods like RSM replace the numerical model with an approximated less expensive surrogate model which can be used to study the systems' response and analyze uncertainty propagation. In order to build a response surface, normally a finite element model should be built and run it certain number of times with different input variables and use the output data for response surface generation. Honjo et al [7] solved 3 out of six problems set by ETC10- Evaluation of Euro code 7 – of ESSMGE. In this paper we leveraged his work and applied the proposed way of constructing a response surface on the same examples.

PROBLEM DEFINITION

A square pad foundation shown in Figure 1 is made of concrete with a weight density of 25 kN/m3 and has an embedment depth of 0.8 m. The ground surface shown can reliably be assumed to be below any topsoil and disturbed ground.

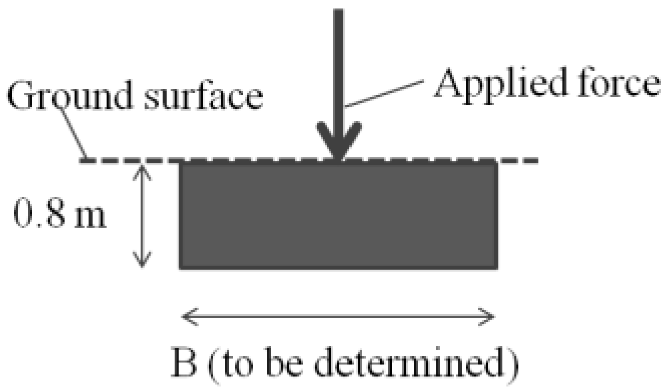

Fig 1: Square Pad foundation on sand

The foundation is required to support a permanent vertical load of 1000 kN, excluding weight of foundation and a variable vertical load of 750 kN. The pad foundation is built at an embedded depth of 0.8 m. There are 4 CPT (q_c) tests within 15 m radius from the point the pad foundation is to be constructed. Digitized version of the data is available at http://www.eurocode7.com/etc10.The data presented in Figure 2 is for the sake of completeness.

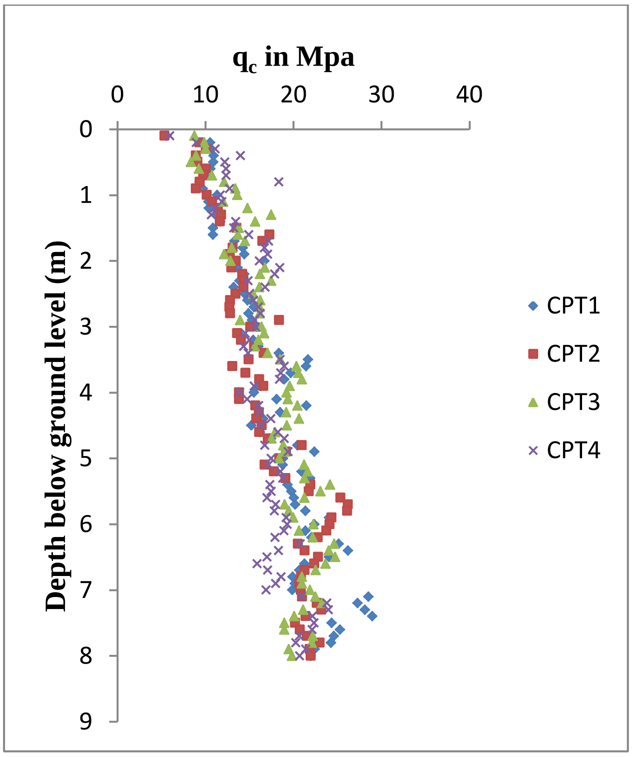

Fig 2: CPT q_c profile

A trend model with constant variance is fitted to the data. Thus, the trend component of q_c

$$q_c = 10.54 + 1.66z \quad (1)$$

With the residual standard error being 2.28, R^2=0.74. To find the distance over which CPT values exhibit spatial correlation, the autocorrelation function is estimated in the vertical direction for each CPT data and it is found that the correlation distance lies between 0.4 to 0.5 m and that there is no horizontal correlation.

The following are the list of basic variables for the problem defined

Basic variables	Notation	mean	SD	Distribution type
Estimation of error of spatial average of E for 2(m) depth	E	E=47.43+ 7.38z Mpa	7.2 Mpa	Normal
Transformation error on E	δ_E	1.14	0.94	lognormal
Permanent load	δ_{Gk}	1	0.1	Normal
Variable load	δ_{Qk}	0.6	0.21	Gumbel

Table 1: List of basic variables, assumed distributions and their parameters

Honjo et al [7] also has taken into account the statistical estimation error caused by limited number of soil samples (Listed in Table 1.0), statistical estimation error caused by the relative positioning of the samples, the variance reduction by taking spatial average of soil parameters for a certain volume, and has arrived at the characteristic value of CPT q_c value at the site and reported them as

Mean Value : $q_c = 10.54 + 1.66z$ Mpa

Standard Deviation : 1.60 Mpa

In order to predict settlement using the theory of elasticity, Young's Modulus of soil as a function of depth based on the CPT values should be found. The following relation is arrived at

$$E = 47.43 + 7.38z \text{ Mpa} \quad (2)$$

Also due to the transformation of q_c to E, we also have to take into account the transformation error of the spatial average of E. The error is listed in Table 1. Calculation of these errors is out of the scope of discussion in this paper and interested readers can refer Honjo et al [7] for details.

RESPONSE SURFACE GENERATION

In order to generate a response surface, all input variables should be passed through a model to arrive at the output variables. Honjo et al [7] built a finite element model of the problem under definition and generated the response surface. Braja.M.Das [8] in his critical review of elastic settlement of shallow foundations on granular soils has proposed the use of Steinbrenner and Fox's equation as the best method. In this work, we hence use this method. Theoretically, if the foundation is perfectly flexible (Figure 3), the settlement can be expressed as

$$S_e = q(\alpha' \beta') \frac{1-\mu_s^2}{E_s} I_s I_f \tag{3}$$

Where

q = net applied pressure on the foundation

μ_s= Poisson's ratio of soil

E_s = average modulus of elasticity of the soil under the foundation, measured from $z = 0$ to about $z = 4B$

B' = B/2 for centre of foundation (= B for corner of foundation)

I_s = shape factor (Steinbrenner, 1934)

I_f = depth factor (Fox, 1948)

I_s depends on the shape of the footing and the depth of rigid boundary below the footing. I_f is a function of $\frac{D_f}{B}, \mu_s, \frac{L}{B}$ and these have to be read from tables (Table 1) or from some charts. Hence calculation of settlement invariably needs a geotechnical engineer to refer to the charts and tables. In the proposed method in this paper, this process should be done until sufficient points required to develop a response surface are available. Using that response surface, a geotechnical engineer can conduct uncertainty analysis via Monte Carlo Simulations. The above mentioned procedure is illustrated in the following section.

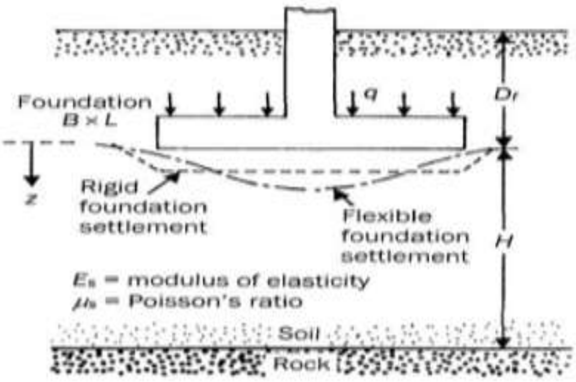

Fig 3: Settlement profile for shallow and flexible foundation

	L/B						
D_f/B	1.0	1.2	1.4	1.6	1.8	2.0	5.0
Poisson's ratio μ=0.30							
0.05	0.979	0.980	0.982	0.982	0.983	0.985	0.990
0.1	0.954	0.958	0.958	0.962	0.964	0.968	0.977
0.2	0.902	0.911	0.911	0.917	0.923	0.930	0.951
0.4	0.808	0.823	0.823	0.834	0.843	0.857	0.899
0.6	0.738	0.754	0.754	0.767	0.779	0.796	0.852
0.8	0.687	0.703	0.703	0.716	0.728	0.747	0.813
1	0.650	0.655	0.665	0.678	0.689	0.709	0.780
2	0.562	0.571	0.571	0.580	0.588	0.603	0.675
Poisson's ratio μ=0.40							
0.05	0.989	0.990	0.991	0.992	0.992	0.993	0.995
0.1	0.973	0.976	0.978	0.980	0.981	0.982	0.988
0.2	0.932	0.940	0.945	0.949	0.952	0.955	0.970
0.4	0.848	0.862	0.872	0.881	0.887	0.893	0.927
0.6	0.779	0.795	0.808	0.819	0.828	0.836	0.886
0.8	0.727	0.743	0.757	0.769	0.779	0.788	0.849
1	0.689	0.704	0.718	0.730	0.740	0.749	0.818
2	0.596	0.606	0.615	0.624	0.632	0.640	0.714

Table 2: Variation of I_f (Fox, 1948)

Using equation (3), settlements of pad foundation for widths of 4,3,2,1 and 0.5 m on the average values of Young's modulus given by equation (2) and applied loads are calculated and tabulated in Table 2

Steinbrenner's (1934) AND Fox's (1948)					
B in m	4	3	2	1	0.5
S_e in mm	4.36	5.87	9.16	18.77	37.95

Table 2: Settlement of pad foundation calculated using Equation (3)

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A regression analysis is carried out to obtain the relationship between B and S_e, which has resulted as below

$$s = 23.106 - 15.86\log(B) \quad (4)$$

With $R^2 = 0.9314$. Thus, one obtain,

$$s = 23.106 - 15.86\log(B)\ / IE \quad (5)$$

Note that I_E is a normalized Young's modulus which is equal to one when it is at the mean. By introducing the Young's modulus transformation error, δ_E, defined in Table 1 and also describing the fluctuations of permanent and variable loads from their characteristic values by δG_k and δQ_k, the final response curve employed in this analysis is obtained as follows:

$$s = \frac{23.106 - 15.86\log(B)}{I_E.\delta_E}\left(\frac{(\gamma.D_f.B^2 + G_k.\delta_{Gk} + Q_k.\delta_{qk})}{\gamma.D_f.B^2 + G_k + Q_k}\right) \quad (6)$$

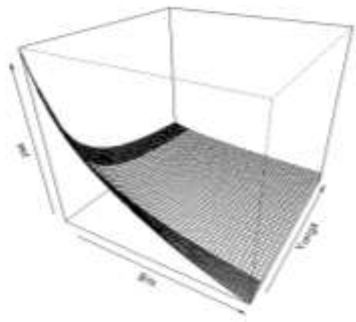

Fig 4: View of response surface generated

UNCERTAINITY QUANTIFICATION AND RESULTS

The results of Monte Carlo simulation are presented in Figure 5

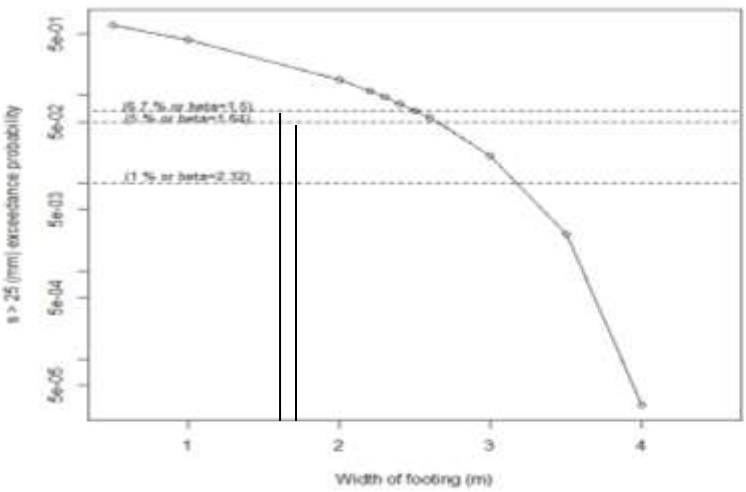

Fig 5: Results of Monte Carlo simulation

Since the settlement exceedance of 25 (mm) is a serviceability limit state, if the reliability index, β, of 1.5 (i.e. 6.7% exceedance in 50 years) should be satisfied (EN 1990, Annex C), the foundation width of more than 2.5 (m) is required. If the exceedance probability of more than 5 % (i.e. β > 1.64) is to be satisfied, the width more than 2.7 (m) is required.

CONCLUSION

Results obtained by the proposed pseudo response surface method are compared with the ones reported by Honjo et al. They are in reasonably good agreement and are presented in Figure 6

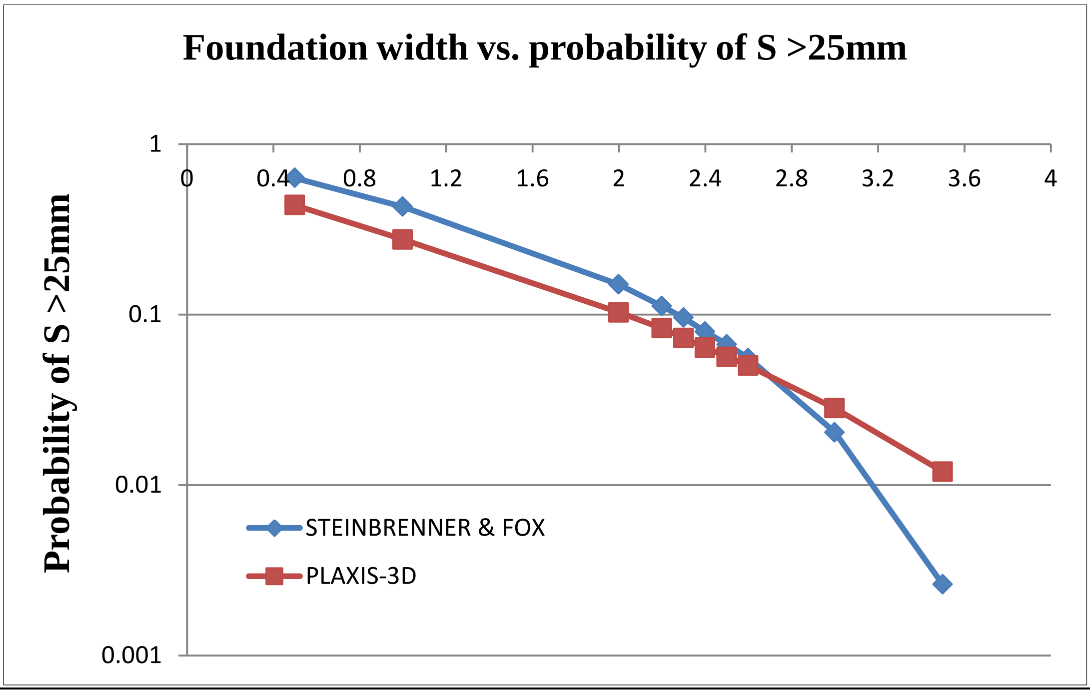

Fig 6: Comparison of results from the proposed method with that reported in HONJO et al using Finite element method

With the proposed method, a geotechnical engineer need not build a Finite element model to generate a response surface and hence carry out an uncertainty quantification study.

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