

# The effect of single and double curvature in optimum design of slender reinforced concrete columns

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**Abstract**— Slenderness is an important factor in the design of long reinforced concrete (RC) columns. It is also a major problem for RC columns which are restricted to lateral displacement. In that case, buckling is effective on the second the second order effects. The elastic curve of the columns may have single or double curvature. The type of curvature is effective on slenderness of the column. In ACI318-Building code requirements for structural concrete-, an effect of slenderness is considered by using a moment magnification factor. In this factor, the type of curvature is also considered by a correction factor ( $C_m$ ) which is defined according to end moments. For that reason, different flexural moment cases are presented in the present study. By employing teaching learning based optimization, the cost optimization of the columns was done. According to the results, the type of curvature is effective on the optimum design and cost.

**Keywords**— Reinforced concrete, slenderness, columns, optimization, metaheuristic methods, teaching learning based optimization

## I. Introduction

The optimum design of reinforced concrete (RC) member is an important area for engineers because the economy and safety are the main aims of engineers. Optimization is challenging problem for reinforced concrete members considering of concrete and steel. These two materials have different strength behaviors and costs.

The optimization of RC columns has been investigated in several studies [1-6]. Also, metaheuristic algorithms are very effective on optimum design of RC member. The nature inspired Genetic algorithm (GA) [7-8] was employed in the optimum design of RC columns [9], RC frames [10-13], RC continuous beams [14], RC T-shaped beams [15] and various RC members [16]. GA is also combined with methods such as sequential quadratic programming [17], Hook and Jeeves method [18] and Simulated Annealing (SA) [19] in the optimum design of RC members.

SA is also a metaheuristic algorithm inspired by the annealing process of a material [20] and it is employed in the optimum design of RC frames [21] and RC bridges [22]. Most of the optimization studies about RC design consider the minimization of the total cost. In addition to that, the minimum embedded CO<sub>2</sub> emission was also considered in the methodologies employing SA [23] and big bang-big crunch (BB-BC) algorithm [24]. BB-BC inspired from the evolution of the universe [25] was also employed by Kaveh and Sabzi for RC frames [26]. Additionally, the RC retaining wall is an important member in structural engineering. In order to consider the geotechnical and structural constraint in design of RC retaining walls, SA [27-28], BB-BC [29], harmony search (HS) [30], charged system search (CSS) [31] and teaching learning based optimization [32] were employed. The music inspired HS [33] was employed in the optimum design of RC continuous beams [34], RC frames [35], T-shaped RC beams [36-37], RC slender columns [38]. TLBO is an education inspired method [39] and it was also employed in the optimum design of slender RC columns [40].

In this study, the optimum design of RC slender columns was investigated for different end moment cases. In that case, it is possible to consider the type of the curvature in the optimum design. In the optimization, TLBO based method [40] was used by considering ACI 318-Building code requirements for structural concrete [41].

## II. Methodology

The effect of slenderness can be taken into consideration by using the approximate procedure defined in ACI 318 [41]. According to the procedure, the maximum flexural moment of the column is multiplied by the moment magnification factor ( $\delta_s$ ). This factor is calculated according to the buckling behavior of the column. The effective length in buckling ( $k$ ) is calculated by considering  $\Psi_A$  and  $\Psi_B$  (Eq. 1) for upper and lower ends of the columns, respectively.

$$\Psi_{A,B} = \frac{\sum (EI/l)_{column}}{\sum (EI/l)_{beam}} \quad (1)$$

$$\Psi_m = 0.5(\Psi_A + \Psi_B) \quad (2)$$

$$k = \frac{20 - \Psi_m}{20} \sqrt{I + \Psi_m} \quad \text{if } \Psi_m < 2 \quad (3)$$

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$$k = 0.9\sqrt{I + \Psi_m} \quad \text{if } \Psi_m \geq 2 \quad (4)$$

In Eq. (1), E, I, and l are elasticity modulus, moment of inertia and length of the RC members, respectively. The rigidity (EI) of columns and beams are multiplied by 0.65 and 0.3 in order to consider the cracking of the concrete sections. Then k is calculated according to end conditions of the column. For example, k is obtained according to following equations if the column is free to make lateral displacement

The moment magnification factor ( $\delta_s$ ) is found according to Eq. (5) in which the design axial force, the buckling force and correction factor considering actual moment diagram to end equivalent moment diagram are shown with  $P_u$ ,  $P_c$  and  $C_m$ , respectively.

$$\delta_s = \frac{C_m}{1 - \frac{P_u}{0.75P_c}} \quad (5)$$

The correction factor defined in Eq. (6) is calculated according to end moment defined as  $M_1$  and  $M_2$ .  $M_2$  is the biggest one in absolute value In that case,  $C_m$  is taken between 0.4 and 1.0.

$$C_m = 0.6 + 0.4 \frac{M_1}{M_2} \quad (6)$$

The buckling load can be calculated by the equation of Euler as seen in Eq. (7).

$$P_c = \frac{\pi^2 EI}{(kl)^2} \quad (7)$$

During the optimization, the maximum design flexural moment is factored with  $\delta_c$ . The column is subjected to flexural moment, shear force (V) and axial force ( $P_u$ ). These

forces are defined with the other design constant such as clear cover of concrete; cc, maximum aggregate diameter;  $D_{max}$ , length of column; l, elasticity modulus of steel; Es, cost of the concrete per m<sup>3</sup>; Cc, cost of the steel per ton; Cs, compressive strength of concrete; yield strength of steel;  $f_y$ , specific gravity of steel;  $\gamma_s$  and specific gravity of concrete;  $\gamma_c$ . The ranges of design variables are also defined and the design variables are breadth, height and reinforcements (number, diameter and space) of column. The optimization is an iterative process and TLBO algorithm is employed as done by Bekdas and Nigdeli [40]. The two phases such as teacher and learner phases are consequently used in generation of new possible solutions. For all generations, the analyses are done considering ACI318 constraints and the total material cost is calculated. The aim of the optimization is to minimize the total cost.

### III. Numerical Example

The investigation is done for seven cases of the external forces as seen in Table 1. In these cases, the minimum flexural moment is different. According to the different values,  $C_m$ , values and shear forces (V) change. In the first two cases, the elastic curve of the column has double curvature. In the other cases, the single curvature is observed and the second order effects are increasing. The design constants; length of column (l), clear cover (cc), maximum aggregate diameters ( $D_{max}$ ), yield strength of steel ( $f_y$ ), compressive strength of concrete (fc), elasticity modulus of steel (Es), specific gravity of steel ( $\gamma_s$ ), cost of concrete per m<sup>3</sup> and cost of steel per ton were taken as 10.0 m, 30 mm, 16 mm, 420 MPa, 25 MPa, 200000 MPa, 7.86 t/m<sup>3</sup>, 40 \$ and 400 \$, respectively. The design variable ranges of breadth (bw), height (h), longitudinal reinforcement diameter and shear reinforcement diameter were taken as 250 mm- 400 mm, 300 mm-600 mm, 16 mm-30 mm and 8 mm- 14 mm, respectively. The dimension values were assigned with the multiples of 10 mm for practical design. Also, the sizes of reinforcement were assigned with even values. The optimum results are presented in Table 2.

TABLE I. THE DESIGN CASES OF OPTIMIZATION PROCESS

Cases	1	2	3	4	5	6	7
$M_1$ (kNm)	-250	-125	0	125	250	375	500
$M_2$ (kNm)	500	500	500	500	500	500	500
$P_u$ (kN)	3500	3500	3500	3500	3500	3500	3500
V (kN)	75.0	62.5	50.0	37.5	25.0	12.5	0
$C_m$	0.4	0.5	0.6	0.7	0.8	0.9	1.0

TABLE II. THE OPTIMUM RESULTS AND COSTS

Cases	1	2	3	4	5	6	7
Breadth of column ( $b_w$ ) (mm)	470	500	530	570	600	600	600
Height of column (h) (mm)	600	600	600	600	600	600	600
Bars in each face	2 $\Phi$ 24	1 $\Phi$ 26+1 $\Phi$ 20+ 1 $\Phi$ 18	1 $\Phi$ 16+1 $\Phi$ 20+ 0+1 $\Phi$ 22	1 $\Phi$ 18+1 $\Phi$ 20+1 $\Phi$	1 $\Phi$ 20+1 16	2 $\Phi$ 20+ 1 $\Phi$ 28+1 $\Phi$ 1	4 $\Phi$ 20+ $\Phi$ 16
Web reinforcements	2 $\Phi$ 18	2 $\Phi$ 16	2 $\Phi$ 16	2 $\Phi$ 18	2 $\Phi$ 16	2 $\Phi$ 16	2 $\Phi$ 16
Shear reinforcement diameter (mm)	$\Phi$ 8	$\Phi$ 8	$\Phi$ 8	$\Phi$ 8	$\Phi$ 8	$\Phi$ 8	$\Phi$ 8
Shear reinforcement distance (mm)	250	240	220	210	200	200	200
Optimum cost (\$)	214.09	224.48	242.25	258.62	272.06	303.26	335.64

## IV. Conclusions

According to the optimum results of numerical example, the optimum cost has an increasing manner by the increase of the minimum end moment. In that case, the effect of the type of the curvature and the slenderness can be seen. This situation shows the importance of the slenderness although the value of the shear force is decreasing by the reduction of the difference of the end moments.

The aim of the optimization process is to reduce the slenderness because the height of the column is at the upper bound of the solution range for all cases. By the increase of the slenderness, the breadth of the columns are also increasing and it is also at the upper bound for the last three cases.

The employed metaheuristic algorithms called TLBO is also effective to find the optimum design supporting the effect of the slenderness.

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