Influence of vertical movement in bathymetries and its influence on the measurement conditions.

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Abstract—**Bathymetries constitute a fundamental element of building objects settled on land submerged land: dikes, docks, underwater pipelines; in addition to dredges, volume of reservoir, etc. In civil engineering the bathymetry affect areas of small extent in which measures should be the most accurate as possible, influencing many factors in its precision. Among these factors can be distinguished: speed of sound in water, positioning system, vessel movements (pitch and roll) and vertical movement of the boat.**

Vertical movement of the vessel is perhaps one of the least studied factors, but their influence on the accuracy of bathymetries could be very important depending on swell conditions and depth that exists at the time of measurement.

This paper defines a mathematical model based on measurements of bathymetry observed with GPS that lets you define wave conditions that found the boat according to its trajectory and speed. From this point, the article quantifies the effect of the vertical movement of the boat to correct its effect on the measured depths with an echo-sounder.

Finally, from the results it is possible to obtain a set of recommendations about operations to measure a bathymetry depending on the accuracy that you should obtain.

Keywords— **Bathymetry, echo-sounder, Doppler effect, GPS.**

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I. Introducion.

Echo sounding bathymetries constitute a common technique for obtaining mapping underwater. Their applications in civil engineering are many (construction of dams, decks or submarine emissaries, dredging of harbours, determination of the volume of operating water reservoirs, definition of profiles for flood calculation in rivers, etc.) and they usually require detailed bathymetries which are constrained to small areas and they must be as accurate as possible.

In this sense, the accuracy is affected by a number of errors that occur during the different phases of modeling; [Maleika 2013]:

- Depth Reading error caused by measurement devices.

- Errors resulting from adopted certain operating parameters.

- Errors in determining the position.

- Errors that occur during the interpolation processes.

- Errors due to surface smoothing process.

- Errors that occur during the editing and publishing of cartography maps.

Among these, the errors due to navigation have been studied in depth and this is the case of pithc and roll; but less another ones like vertical movement due to the waves.

This article establishes a mathematical model to define this error solely on the data obtained by the element that allows to calculate the position (GPS) and, from it, set the corrections that could be applied and the conclusions that could give place.

II. Instrumental y condicionantes.

Vertical movement of the boat does not produce any deviation in the direction of the sounder but can equally influence the depth measurement.

It consists of the vertical movement of the boat due mainly to swell; and its consequence is a delay defined as the time that the wave of the sounder takes to travel the vertical movement that the vessel has made since it was issued until it is received its echo, that is, during depth measurement . Long period of vertical movements of the boat, such as tide, can be

considered second order compared to small period because depth readings require very short times.
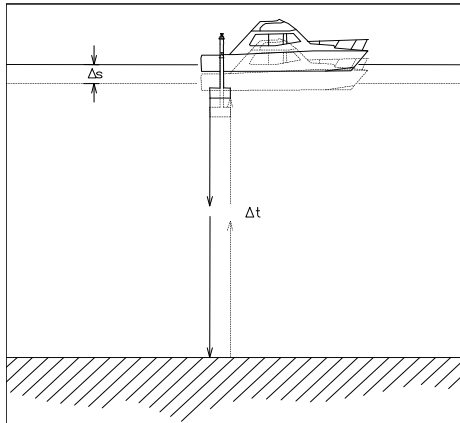

Figure 1.   Vertical movement of the boat.

Thus, the vertical movement of the vessel, corresponding to the instant of measurement echo sounder with a duration Δt, has a value Δs, and can be quantified as:

$$\Delta t = \frac{p_m + \Delta s}{v_c} \approx \frac{p_m}{v_c} \qquad (1)$$

Being v_c the velocity of the sound in the water (calibrated) and p_m the measured depth.

The vertical offset of the vessel will be defined, therefore, from this movement that has been suffered in this interval of time.

Since the depth is calculated from the time that a pulse takes to reach the bottom and go back, this is going to be affected by an error caused by the displacement of the ship, and therefore, the sounder. To quantify the value of this correction to be applied to each depth readings must model the vertical movement of the boat. To this end, the proposed model only uses data obtained by the GPS which is placed on the sounder, allowing both the definition of the path and the movement to be studied.

III.   Mathematical model.

A.   *Mathematical model for vertical movement.*

Assuming that the main component of the vertical motion is due to swell, it is regarded as a periodic motion of a certain amplitude and period during the time that the depth reading is performed. Linear interpolation constitute an excessively rough approximation especially if the recording cadence is high, on the order of a second or higher, and generally comparable to the wave period.

The movements of pitch and roll can cause an increase or decrease of height sounder, especially if it is located on one side of the vessel, being included in the proposed model. However its amplitude is generally much smaller than the wave so, in principle, the proposed approach will be close to the conditions of movement of the water surface at each instant and in the direction of advance of the boat.
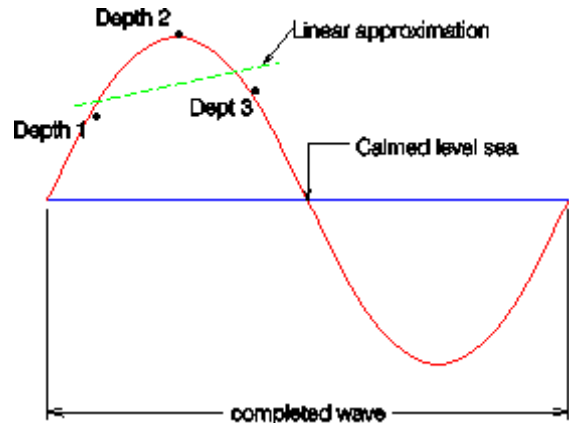

Figure 2.   Mathematical model.

$$z = F(t) = z_i + A \cdot \cos Bt \qquad (2)$$

However, none of these factors is constant: the amplitude because swell conditions in the different work areas can vary significantly; and period depend on the angle that forms the vessel track with wave direction and the relative velocity between the ship and the swell, that is, because of Doppler effect.

Thus, you must determine the conditions of movement at all times, for which an approach is proposed by least squares discrete choosing the measures, the GPS receiver located on sounder, before and after which it is to be corrected. This way, these positions define the wave motion to be studied that we want to study.

In this way, if for three specific sound readings are known, the height of the GPS antenna receiver located on the echo sounder can be modeled, being the second the measure to be studied:

$$(t_1, z_1); (t_2, z_2); (t_3, z_3)$$

Applying the method of least squares to determine the parameters A, B should minimize the expression:

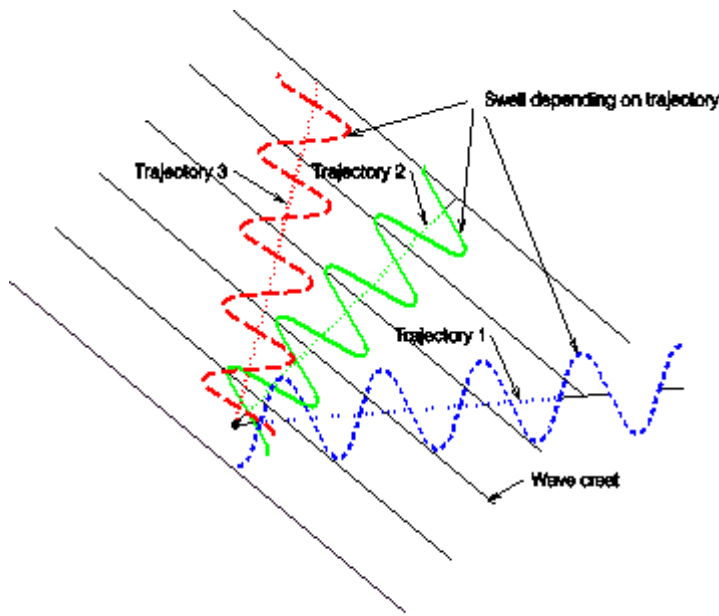$$M = \sum_{i=1}^{3} \left[z_i - (z_1 + A \cdot \cos Bt) \right] \qquad (3)$$

The values of the parameters A, B will be defined by solving the above equations system. Since it is a nonlinear system for resolution may apply some of the techniques of numerical analysis: Method of Newton, Quasi-Newton method, or another techniques. In this case we have chosen the technique known as Newton Method [R.L.BURDEN], which provides a solution by iteration with rapid convergence towards the solution. This requires determining the Jacobian matrix from the functions defined by the above expression:

$$2 \cdot k_2 \cdot \cos Bx_2 + 2 \cdot k_3 \cdot \cos Bx_3 -$$
$$2 \cdot A \cdot (1 + \cos^2 Bx_2 + \cos^2 Bx_3) = f_1$$
$$2 \cdot A \cdot [k_2 \cdot x_2 \cdot senBx_2 + k_3 \cdot x_3 \cdot senBx_3] -$$
$$A^2 \cdot (x_2 \cdot sen2Bx_2 + x_3 \cdot sen2Bx_3) = f_2 \tag{8}$$

In this way:

$$\frac{\partial f_1}{\partial A} = -2 \cdot (1 + \cos^2 Bx_2 + \cos^2 Bx_3) \tag{9}$$

Figure 3.   Swell depending on trajectory.

Derivating and equating to zero:

$$\frac{\partial M}{\partial A} = 2 \cdot (z_i - z_1 - A \cdot \cos Bt) \cdot \cos Bt = 0$$
$$\frac{\partial M}{\partial B} = 2 \cdot (z_i - z_1 - A \cdot \cos Bt) \cdot A \cdot t \cdot senBt = 0 \tag{4}$$

Applying both expressions for the data referred in the approximation you can obtain these equations:

$$2 \cdot (z_2 - z_1) \cdot \cos Bt_2 + 2 \cdot (z_3 - z_1) \cdot \cos Bt_3 -$$
$$- 2 \cdot A \cdot (\cos^2 Bt_1 + \cos^2 Bt_2 + \cos^2 Bt_3) = 0$$
$$2 \cdot A \cdot [(z_2 - z_1) \cdot t_2 \cdot senBt_2 + (z_3 - z_1) \cdot t_3 \cdot senBt_3] -$$
$$- A^2 \cdot (t_1 \cdot sen2Bt_1 + t_2 \cdot sen2Bt_2 + t_3 \cdot sen2Bt_3) = 0 \tag{5}$$

Making the following change of variable:

$$k_i = z_i - z_1$$
$$x_i = t_i - t_1 \tag{6}$$

The expressions are simplified:

$$2 \cdot k_2 \cdot \cos Bx_2 + 2 \cdot k_3 \cdot \cos Bx_3 -$$
$$- 2 \cdot A \cdot (1 + \cos^2 Bx_2 + \cos^2 Bx_3) = 0$$
$$2 \cdot A \cdot [k_2 \cdot x_2 \cdot senBx_2 + k_3 \cdot x_3 \cdot senBx_3] -$$
$$- A^2 \cdot (x_2 \cdot sen2Bx_2 + x_3 \cdot sen2Bx_3) = 0 \tag{7}$$

$$\frac{\partial f_1}{\partial B} = -2 \cdot (k_2 \cdot x_2 \cdot senBx_2 + k_3 \cdot x_3 \cdot senBx_3) +$$
$$+ 2 \cdot A \cdot (x_2 \cdot sen2Bx_2 + x_3 \cdot sen2Bx_3) \tag{10}$$

$$\frac{\partial f_2}{\partial A} = 2 \cdot (k_2 \cdot x_2 \cdot senBx_2 + k_3 \cdot x_3 \cdot senBx_3) -$$
$$- 2 \cdot A \cdot (x_2 \cdot sen2Bx_2 + x_3 \cdot sen2Bx_3) \tag{11}$$

$$\frac{\partial f_2}{\partial B} = 2 \cdot A \cdot (k_2 \cdot x_2^2 \cdot \cos Bx_2 + k_3 \cdot x_3^2 \cdot \cos Bx_3) -$$
$$- 2 \cdot A^2 \cdot (x_2^2 \cdot \cos 2Bx_2 + x_3^2 \cdot \cos 2Bx_3) \tag{12}$$

Therefore the Jacobian matrix is:

$$J = \begin{bmatrix} \dfrac{\partial f_1}{\partial A} & \dfrac{\partial f_1}{\partial B} \\ \dfrac{\partial f_2}{\partial A} & \dfrac{\partial f_2}{\partial B} \end{bmatrix} \tag{13}$$

Calling:

$$F = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \tag{14}$$

Newton's method proposes the iterative resolution of the linear system of equations:

$$\begin{bmatrix} A^k \\ B^k \end{bmatrix} = \begin{bmatrix} A^{k-1} \\ B^{k-1} \end{bmatrix} - \left(J^{k-1} \cdot F^{k-1} \right) \qquad (15)$$

That is to say:

$$J^{k-1} \cdot \begin{bmatrix} A^k - A^{k-1} \\ B^k - B^{k-1} \end{bmatrix} = -F^{k-1} \qquad (16)$$

Calling:

$$a = 1 + \cos^2 B^{k-1} x_2 + \cos^2 B^{k-1} x_3 \qquad (17)$$

$$b = x_2 \cdot sen2B^{k-1}x_2 + x_3 \cdot sen2B^{k-1}x_3 \qquad (18)$$

$$c = 2 \cdot (k_2 \cdot x_2 \cdot senB^{k-1}x_2 + z_3 \cdot k_3 \cdot senB^{k-1}x_3) \qquad (19)$$

$$d = 2 \cdot (k_2 \cdot \cos B^{k-1}x_2 + k_3 \cdot \cos B^{k-1}x_3) \qquad (20)$$

$$e = \left(\frac{\partial f_2}{\partial B} \right)^{k-1} = 2A^{k-1}(k_2 \cdot x_2^2 \cdot \cos B^{k-1}x_2 +$$

$$+ k_3 \cdot x_3^2 \cdot \cos B^{k-1}x_3) - 2A^{k-1^2}(x_2^2 \cdot \cos 2B^{k-1}x_2 + x_3^2 \cdot \cos 2B^{k-1}x_3)$$

The system of equiations is:

$$\begin{bmatrix} -2 \cdot a & -c + 2 \cdot A^{k-1} \cdot b \\ c - 2 \cdot A^{k-1} \cdot b & e \end{bmatrix} \begin{bmatrix} A^k - A^{k-1} \\ B^k - B^{k-1} \end{bmatrix} = \begin{bmatrix} d - 2 \cdot A^{k-1} \cdot a \\ A^{k-1} \cdot c - A^{k-1^2} \cdot b \end{bmatrix} \quad (21)$$

Calculating:

$$\begin{bmatrix} A^k \\ B^k \end{bmatrix} = \frac{1}{\left(c - 2 \cdot A^{k-1} \cdot b\right)^2 - 2 \cdot a \cdot e} \cdot \begin{bmatrix} e & -c + 2 \cdot A^{k-1} \cdot b \\ c - 2 \cdot A^{k-1} \cdot b & -2 \cdot a \end{bmatrix} \cdot (22)$$

$$\cdot \begin{bmatrix} 2 \cdot A^{k-1} \cdot B^{k-1} \cdot b - d - c \cdot B^{k-1} \\ e \cdot B^{k-1} - A^{k-1^2} \cdot b \end{bmatrix}$$

The first value for variables A and B are recommended to be: A = 1 m, , B=1/2π corresponding to a wave amplitude of one meter and a period of one second because with these initial values the solution converges quickly for common situations.

B. *Mathematical model for correction of depth measurements.*

Once modeled vertical movement of the boat according to equation (2): $z = F(t) = z_i + A \cdot \cos Bt$; for the time of depth reading t_2, the depth read by the echo sounder p_2 should be corrected by the travel time of the signal, effect of the vertical movement of the vessel.

Thus the instant output pulse of the sounder corresponds to the instant:

$$t_i = t_2 - \delta t \qquad (23)$$

The value δt can be calculated with sufficient accuracy as the time it takes the pulse emitted by the transducer to return to him:

$$\delta t = \frac{p_2}{v_c} \qquad (24)$$

The height of the boat for measurement moments t_2 and $t_i = t_2 - \delta t$ have these values:

$$F_2 = z_1 + A \cdot \cos Bt_2 \qquad (25)$$

$$F_i = z_1 + A \cdot \cos Bt_i \qquad (26)$$

Since the boat has undergone a vertical movement during measurement, the correction will be:

$$\Delta p = F_2 - F_i = A \cdot (\cos Bt_2 - \cos Bt_i) \qquad (27)$$

Therefore, the depth corrected by the effecto of the vertical movement has a value of:

$$p_{ca} = p_m + \Delta p \qquad (28)$$

IV. **Results and conclusions.**

From the proposed model, it is possible to affirm that a sine-wave model to reflect the vertical movement is the most appropriate given their relative simplicity of calculation and correspondence with the physical reality of the movement, variable depending on both the swell and the relationship of this one with the boat.

Depth reading correction depends largely on the depth obtained. That is, since the speed of sound in water is

relatively high (about 1500m/s) and the depths obtained by the echo sounder are usually of low magnitude in applications of civil engineering, the duration of a measure is very small and therefore, the vertical movement of the boat as well.

Therefore, one can ensure that the incidence of movement studied in small depths (up to 30m) can be neglected. Not so for great depths, where their value can approach even the full amplitude of motion.

One possible approach to the value of this correction is presented in the table and graph below, calculated from a vertical variable speed boat, a maximum depth of 30m and a speed of sound in water of 1500 m/s.

TABLE I.   CORRECTION DUE TO VERTICAL MOVEMENT. (CM)

Depth(m)	Vertical speed (m/s)				
	0.4	0.6	0.8	1	1.5
5	0.3	0.4	0.5	0.7	1.0
10	0.5	0.8	1.1	1.3	2.0
15	0.8	1.2	1.6	2.0	3.0
20	1.1	1.6	2.1	2.7	4.0
25	1.3	2.0	2.7	3.3	5.0
30	1.6	2.4	3.2	4.0	6.0

In view of the results obtained it can be concluded that for depths up to 30 meters can reach neglected the effect of vertical movement, especially in inland areas where the swell is very small.
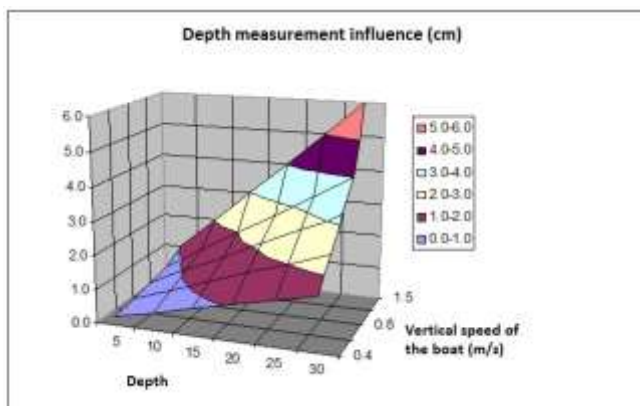

Figure 4.   Depth measurement influence of vertical speed (cm).

In any case, it is recommended not to run bathymetric surveying campaigns when swell conditions are unfavorable, with speeds of ascent or descent of the upper vessel at 1m / s, provided the required accuracy so dictates.

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