*International Journal of Advancements in Communication Technologies – IJACT*
*Volume 1 : Issue 1 [ISSN : 2374-1511]*
*Publication Date : 25 June 2014*

# BER Analysis of OSTBC-MIMO-OFDM Systems using Triple-Polarized Antennas

Geng Wan, Takeshi Higashino, Minoru Okada

***Abstract*—Diversity and data rate over multipath fading channels in wireless communication systems can be significantly enhanced by the combined use of orthogonal space-time block coding (OSTBC) multiple-input multiple-output (MIMO) orthogonal frequency division multiplexing (OFDM) techniques. However, to obtain the maximum benefits of the above schemes, the required antenna spacing at both the transmit and receive sides will result in high-cost antenna deployments. Hence, the use of triple-polarized antennas appears to be a cost- and space-efficient alternative. In this paper, we model the triple-polarized MIMO channel and evaluate the error performance of OSTBC-MIMO-OFDM system using proposed channel model. Simulation result shows error performance improvement when using triple-polarized antennas in OSTBC-MIMO-OFDM systems compared to that of using spatially-separated uni-polarized antennas.***

***Keywords*—OSTBC-MIMO-OFDM, triple-polarized antenna, uni-polarized antenna**

## I. Introduction

In order to achieve higher data rates at high quality of service (QoS) in even bad radio environments, the coupling of multiple-input multiple-output (MIMO) and orthogonal frequency division multiplexing (OFDM) has been considered as the best solution and already been utilized in existing 3/4G wireless systems like Long-Term Evolution (LTE) and LTE-Advanced [1][2]. Orthogonal Space-Time Block Coding (OSTBC) is one of the MIMO schemes, which firstly introduced by Tarokh [3]. Combining it with MIMO and OFDM techniques, full transmit diversity and high link reliability can be obtained in the wireless communication system. At the receive side, OSTBC can be easily decoded by a simpler linear maximum-likelihood (ML) decoder if the channel state information (CSI) is perfectly known at the receiver.

However, to achieve the maximum benefits of schemes mentioned above, an essential fact that the traditional spatially-separated uni-polarized antenna arrays are usually required to be spaced at least ten wavelengths at the base station and at least half a wavelength at the user terminals has been shown by results of many studies. To effectively save the physical antenna spacing needed, the use of co-located orthogonally-polarized antennas appears to be a promising alternative [4]. Recently, a new type of such antennas, namely, triple-polarized antenna has gained much attention. In fact, some researchers have investigated the typical channel characteristics of triple-polarized antenna systems and verified several performance improvements when using triple-polarized antennas through the simulations [5]-[7]. However, these studies only explained some features of triple-polarized MIMO systems and investigated specific technique (e.g., antenna selection) in MIMO systems using triple-polarized antennas. Thus, the main objective of this paper is to propose a relatively simple triple-polarized MIMO channel model. Cross-polar discrimination (XPD) and spatial correlations are taken into consideration when modeling the triple-polarized MIMO channel. Furthermore, we also investigate the BER performance of OSTBC-MIMO-OFDM systems using proposed triple-polarized MIMO channel and compare it with that of using spatially-separated uni-polarized antennas through the numerical simulation.

The rest of this paper is organized as follows. In Section II, we model the triple-polarized MIMO channel. In Section III, we present the overall system model as well as the coding /decoding schemes of OSTBC. We perform numerical simulation for evaluating the BER performance of OSTBC-MIMO-OFDM systems using proposed channel model and compare it with that of using spatially-separated uni-polarized antennas in Section IV. Finally, we make the conclusion in Section V.

In this paper, $(\cdot)^*$ denotes the element-wise conjugate, $E\{\cdot\}$ denotes the expectation, $(\cdot)^H$ denotes the transpose of a matrix/vector. $(\cdot)^T$ stands for conjugate transpose, $\otimes$ is the Kronecker product, $\square$ is the Hadamard product, $\|\cdot\|$ is the Euclidian norm, and $\mathrm{I}_{n\times m}$ denotes a $n\times m$ matrix with unit entries.

## II. Triple-Polarized MIMO Channel

In this section, we model the total channel for Non-line-of-sight (NLOS) mobile scenario. Assuming that the transmitter and receiver have the same even number of triple-polarized antennas and we only model the channel of downlink (from the base station to user terminal) in this work. In fact, the uplink channel matrix (from the user terminal to base station) can be easily obtained by transposing the downlink channel matrix [8]. Fig.1 illustrates the configuration of a $3\times 3$ triple-

**Geng Wan** Graduate School of Information Science
Nara Institute of Science and Technology, Japan

**Takeshi Higashino** Graduate School of Information Science
Nara Institute of Science and Technology, Japan

**Minoru Okada** Graduate School of Information Science
Nara Institute of Science and Technology, Japan

polarized MIMO system. The channel of this system can be described by a $3 \times 3$ matrix, whose elements $h_{ij}, i = j = 3$ represent the channel coefficients from the transmit antenna sub-array $j$ to receive antenna sub-array $i$.

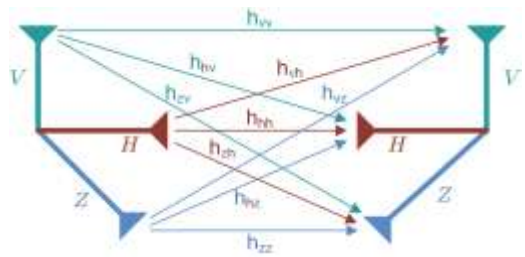


Figure 1. Configuration of $3 \times 3$ triple-polarized MIMO system

## A. Channel XPD

The well-known XPD can be considered as the combined impact of cross-polar isolation (XPI) and cross-polar ratio (XPR) [9]. However, due to the very low value of XPI in real-world condition, we assume that the XPI is negligible in this work (i.e. XPD = XPR). For triple-polarized MIMO channels, the global XPD can be defined as follows,

$$XPR_{ij} = E\left\{|h_{ii}|^2\right\} \Big/ E\left\{|h_{ij}|^2\right\},\ i, j = v, h, z \quad i \neq j\ , \tag{1}$$

Meanwhile, we may also define the co-polar ratio (CPR) as,

$$CPR_{ij} = E\left\{|h_{ii}|^2\right\} \Big/ E\left\{|h_{jj}|^2\right\} i, j = v, h, z \quad i \neq j\ , \tag{2}$$

CPR is the ratio of the signal levels of antennas with the same orientation compared to the ones with different orientations. Based on some experimental results [7][9], we can obtain the matrix $\hat{\mathrm{G}}$ as,

$$\hat{\mathrm{G}} = \frac{1}{\sqrt{1+\sigma}} \begin{pmatrix} 1 & \sqrt{\mu\sigma}e^{j\phi} & \sqrt{\mu\sigma}e^{j\phi} \\ -\sqrt{\mu\sigma}e^{j\phi} & \sqrt{\mu} & \sqrt{\mu\sigma}e^{j\phi} \\ -\sqrt{\mu\sigma}e^{j\phi} & -\sqrt{\mu\sigma}e^{j\phi} & \sqrt{\mu} \end{pmatrix}, \tag{3}$$

where $\sigma$ and $\mu$ are both lognormal variables and represent respectively the random CPR and XPR, $\phi$ is a time-varying angle randomly distributed over $[0, 2\pi)$. Note that $\hat{\mathrm{G}}$ will only yield desired polarized-related correlations. Then, we first let matrix $\Psi_\theta$ has the following structure,

$$\psi_\theta = \begin{pmatrix} 1 & \cos\theta & -\sin\theta \\ \sin\theta & 1 & -\cos\theta \\ \sin\theta & -\cos\theta & 1 \end{pmatrix}, \tag{4}$$

If we assume that one of the three sub-arrays is vertically polarized, for alternative polarization schemes of another two sub-arrays, the Rayleigh channel matrix can be represented by,

$$\mathrm{G} = \psi_\alpha \hat{\mathrm{G}} \psi_\gamma^T\ , \tag{5}$$

where $\alpha$ and $\gamma$ respectively denote the polarization angle of one sub-array relative to the vertical direction at the transmit and receive sides.

## B. Spatial Correlations

For multiple triple-polarized antennas used at both the transmit and receive sides, the spatial correlation existed if there is not too much scattering and/or the antenna spacing is too small. Then, the transmitter and receiver spatial correlation matrices can be respectively modeled as [10],

$$\Gamma_t = \begin{pmatrix} 1 & \delta_t^* & \delta_t^{2^*} & \cdots & \delta_t^{N_t-1^*} \\ \delta_t & 1 & \delta_t^* & \cdots & \delta_t^{N_t-2^*} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \delta_t^{N_t-1} & \delta_t^{N_t-2} & \delta_t^{N_t-3} & \cdots & 1 \end{pmatrix}, \tag{6}$$

$$\Gamma_r = \begin{pmatrix} 1 & \delta_r^* & \delta_r^{2^*} & \cdots & \delta_r^{N_r-1^*} \\ \delta_r & 1 & \delta_r^* & \cdots & \delta_r^{N_r-2^*} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \delta_r^{N_r-1} & \delta_r^{N_r-2} & \delta_r^{N_r-3} & \cdots & 1 \end{pmatrix}, \tag{7}$$

where $N_t$ and $N_r$ are the numbers of triple-polarized antenna sub-arrays deployed at the transmitter and receiver, respectively. Note that $|\delta_t| < 1$ and $|\delta_r| < 1$.

Then, we can obtain the correlated Rayleigh channel matrix $\mathrm{D}$ as,

$$\mathrm{D} = \Gamma_r^{1/2} \mathrm{H} \Gamma_t^{1/2} \tag{8}$$

where H is the $N_r \times N_t$ classical independent identically distributed (i.i.d.) complex Gaussian matrix.

## C. Total Channel

As a conclusion to the above analysis, the total triple-polarized MIMO channel matrix can be represented as follows,

$$\mathrm{H}_{TP} = \mathrm{I}_{N_r/3 \times N_t/3} \otimes \mathrm{G} \square\ \mathrm{D}\ , \tag{9}$$

# III. Triple-Polarized OSTBC-MIMO-OFDM system

## A. System Model

Fig.2 and Fig.3 respectively depict the block diagrams of overall system transmitter and receiver. At the transmitter, the input data stream is first modulated and passed through the OSTBC encoder. Then, the encoded data stream is split into six data streams and all of them are multiplexed into parallel data streams and passed through the inverse fast Fourier transform (IFFT). Next, every data stream is added with guard

interval (GI) which serves to eliminate interference between OFDM symbols and turns linear convolution into circular

1) **Linearity**: all elements $a_{ij}$ are linear combinations of the transmitted symbols and their conjugates;

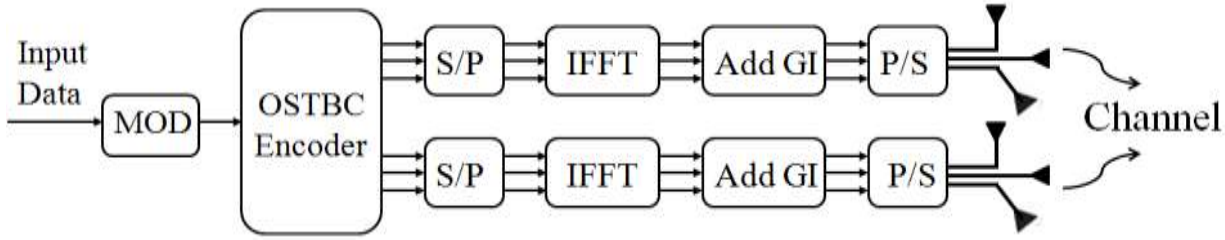


Figure 2. Block Diagram of OSTBC-MIMO-OFDM System Transmitter ( $N_t$ = 6)

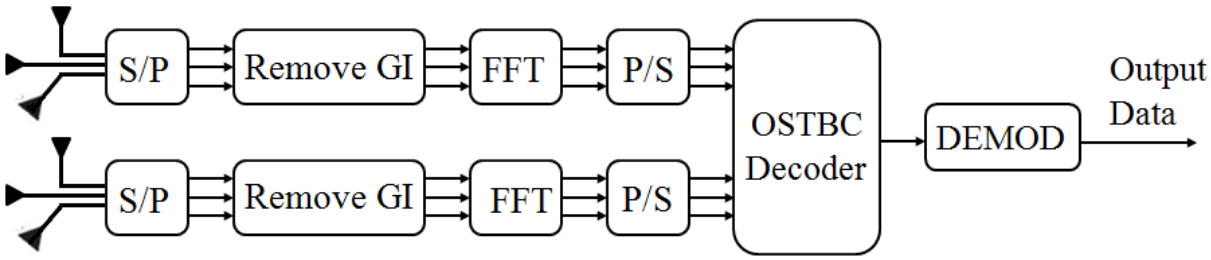


Figure 3. Block Diagram of OSTBC-MIMO-OFDM System Receiver ( $N_r$ = 6)

convolution such that the channel is diagonalized by the fast Fourier transform (FFT). Finally, all the data streams are transformed into serial ones and transmitted by every triple-polarized antenna sub-array. At the receive side, the received signals are passed through OFDM demodulators which first remove the GI and then perform the FFT. The outputs of the OFDM demodulators are finally separated and decoded. Thus, the received signal can be represented as.

$$\mathrm{r} = \mathrm{H}_{TP}\mathrm{x}+\mathrm{n}, \tag{10}$$

The triple-polarized MIMO channel $\mathrm{H}_{TP}$ is a $6\times 6$ matrix whose entries have been represented in Section II, $\mathrm{r}$ denotes the received signal, and $\mathrm{x}$ represents the transmitted signal whose energy is $E_s[|s|^2]=1$. The 6-dimensional noise vector $\mathrm{n}$ has i.i.d. entries with normal distribution $\mathcal{CN}$ (0, 1).

## B. *OSTBC Encoding and Decoding*

***Encoding*** At the input of the STBC encoder, we consider a $k$-dimensional vector $\mathrm{s}=[s_1, s_2, s_3, \cdots, s_k]^T$ with entries from a certain complex signal constellation. At the output, we obtain the codeword matrix $\mathrm{C}_6=[a_{ij}]$ of size $T\times 6$, whose entries are combinations of the input symbols and their conjugates. During each $i$-th time slot, the symbols $a_{ij}$ are sent simultaneously from the transmit antenna sub-arrays $j=[1,2,3,4,5,6]$. For each $j$ th transmit antenna, symbols $a_{ij}$ are transmitted successively at $i=1,2,\cdots,T$ time slots.

In practice, we are interested in OSTBC mentioned in the introduction, for which the codeword matrix $\mathrm{C}_6$ should satisfy two conditions as shown below [11]:

2) **Orthogonality**:

$$\mathrm{C}_6^H\mathrm{C}_6 = \|\mathrm{x}\|^2\,\mathrm{I}_6 = \left(|s_1|^2+|s_2|^2+\cdots+|s_k|^2\right)\mathrm{I}_6, \tag{11}$$

According to (11), the columns of the matrix $\mathrm{C}_6$ are orthogonal to each other, which means that in each block, the symbols transmitted by any two transmit antenna sub-arrays are orthogonal. The orthogonality enables the system to achieve the full transmit diversity. Actually, $\mathrm{C}_6$ can be presented as follows [12],

$$\mathrm{C}_6=\sum_{k=1}^{K}\left(s_k\mathrm{A}_k+s_k^*\mathrm{B}_k\right), \tag{12}$$

where matrices $\mathrm{A}_k$ and $\mathrm{B}_k$ both have a size of $T\times 6$ with constant complex entries, and $K$ is the number of symbols transmitted in one block.

Assuming $M$-ary phase-shift keying (MPSK) modulation format and a constant transmitted energy per information bit $E_b$. Therefore, the total energy assigned to one block is $E_bK\log_2 M$. From the orthogonality condition (11), it can be seen that the total energy for one block is given by $\sum_m^6\sum_k^K|s_k|^2$. Hence, the transmitted energy per MPSK symbol can be represented as,

$$E_s=\frac{E_bK\log_2 M}{\sum_m^6\sum_k^K|s_k|^2}, \tag{13}$$

Next, we first let matrix $\mathrm{C}$ has the following structure as shown below,

$$\mathrm{C}=\begin{pmatrix} s_1 & s_2 & s_3 & s_4 & s_5 & s_6 \\ -s_2 & s_1 & s_4 & -s_3 & s_6 & -s_5 \\ -s_3 & -s_4 & s_1 & s_2 & s_7 & s_8 \\ -s_4 & s_3 & -s_2 & s_1 & s_8 & -s_7 \\ -s_5 & -s_6 & -s_7 & -s_8 & s_1 & s_2 \\ -s_6 & s_5 & -s_8 & -s_7 & -s_2 & s_1 \\ -s_7 & s_8 & s_5 & -s_6 & -s_3 & s_4 \\ -s_8 & -s_7 & s_6 & s_5 & -s_4 & -s_3 \end{pmatrix}, \quad (14)$$

Now, the codeword matrix $\mathrm{C}_6$ can be obtained, with a coding rate of 1/2, given by [13],

$$\mathrm{C}_6=\begin{pmatrix} \mathrm{C} \\ \tilde{\mathrm{C}} \end{pmatrix}, \quad (15)$$

where all the entries of $\tilde{\mathrm{C}}$ are the conjugates of that in $\mathrm{C}$. Obviously, $\mathrm{C}_6$ has a size of $16\times 6$, the value of $K$ in equation (12) is 8 and $T$ equals 16.

***Decoding*** At the output of OFDM demodulator, the received signal can be reasonably represented as follows,

$$\mathrm{R}=\mathrm{C}_6\mathrm{H}_{TP}+\mathrm{N}, \quad (16)$$

where N denotes the noise matrix of a size $16\times 6$, whose entries are i.i.d. complex Gaussian random variables with zero mean and variance $N_o/2$ per dimension. $\mathrm{H}_{TP}$ stands for the $6\times 6$ triple-polarized MIMO channel matrix we discussed in section II. Assuming the knowledge of channel matrix $\mathrm{H}_{TP}$ is perfectly known at the receiver, the ML decoding rule is given by,

$$\hat{\mathrm{C}}_6=\arg\max_{\mathrm{C}_6} p\left(\mathrm{R}\left|\mathrm{C}_6,\mathrm{H}_{TP}\right.\right), \quad (17)$$

Therefore, the conditional probability density function (PDF) of the received signal R is given by,

$$\begin{aligned} &p\left(\mathrm{R}\left|\mathrm{C}_6,\mathrm{H}_{TP}\right.\right)=\det{}^6\left(\pi N_o\mathrm{I}_{16}\right) \\ &\exp\left(-Tr\left[\left(\mathrm{R}\text{-}\mathrm{C}_6\mathrm{H}_{TP}\right)^H\left(N_o\mathrm{I}_{16}\right)^{-1}\left(\mathrm{R}-\mathrm{C}_6\mathrm{H}_{TP}\right)\right]\right) \end{aligned}, \quad (18)$$

Where $\det(\cdot)$ denotes the determinant of a matrix and $Tr(\cdot)$ is the trace of a matrix. Thus, the ML decoding rule (17) can be simplified to

$$\hat{\mathrm{C}}_6=\arg\min_{\mathrm{C}_6}\left\|\mathrm{R}-\mathrm{C}_6\mathrm{H}_{TP}\right\|^2, \quad (19)$$

Substituting (12) into (19), (17) can be further simplified to a symbol-by-symbol detector as follows,

$$\hat{s}_k=\arg\max_{s\in MPSK}\Re\left[z_{k'}s^*\right], \forall k'=1,2,\cdots,8, \quad (20)$$

where $\Re(\cdot)$ denotes the real part of the argument and $z_k$ has the following structure,

$$z_k=Tr\left[\mathrm{R}^H\mathrm{B}_k\mathrm{H}_{TP}+\mathrm{H}_{TP}^H\mathrm{A}_k^H\mathrm{R}\right]. \quad (21)$$

# IV. Performance Evaluation

In this section, we use numerical simulation to evaluate the BER performance of OSTBC-MIMO-OFDM system using proposed channel model and compare it with that of using spatially-separated uni-polarized antennas.

For triple-polarized antenna system, the values of $\mu$ and $\sigma$ appeared in (3) are chosen to be 1 and 15dB, respectively. Except the vertically polarized sub-array, we assume that the polarization angles of the other two sub-arrays of transmit and receive triple-polarized antennas are the same and set them to $\alpha=\gamma=\pi/4$. For the elements in spatial correlation matrices, as shown in (6) and (7) are both chosen to be $\delta_t=\delta_r=0.4$.

For spatially-separated uni-polarized antenna system, we also pay respect to the spatial correlations that exist at both the transmitter and receiver. Correlation coefficients are chosen to be as the same as that in triple-polarized antenna system.

For the common MIMO-OFDM schemes, the main simulation parameters are given in TABLE I.

TABLE I. MIMO-OFDM PARAMETERS

| Channel Type | Frequency-Selective Fading |
|---|---|
| Multipath Number | 4 |
| Modulation Format | QPSK |
| Nosie Type | AWGN |
| FFT Size | 64 |
| Subcarrier Number | 48 |
| Guard Intervel | 16 |

Fig.4 illustrates the result of system performance comparison. For certain channel conditions, we can find that the performance of OSTBC-MIMO-OFDM system using triple-polarized antennas outperforms that of using spatially-separated uni-polarized antennas.

# V. Conclusion

In this paper, we proposed a general model for triple-polarized MIMO channels. Furthermore, we also investigated the BER performance of OSTBC-MIMO-OFDM system using proposed triple-polarized MIMO channel model and compared it with that of using spatially-separated uni-polarized antennas. According to the simulation result, the performance of triple-polarized OSTBC-MIMO-OFDM system outperforms that of uni-polarized system.

One direction of the future work is to investigate the antenna selection scheme for triple-polarized OSTBC-MIMO-OFDM systems in order to further improve the system error performance.

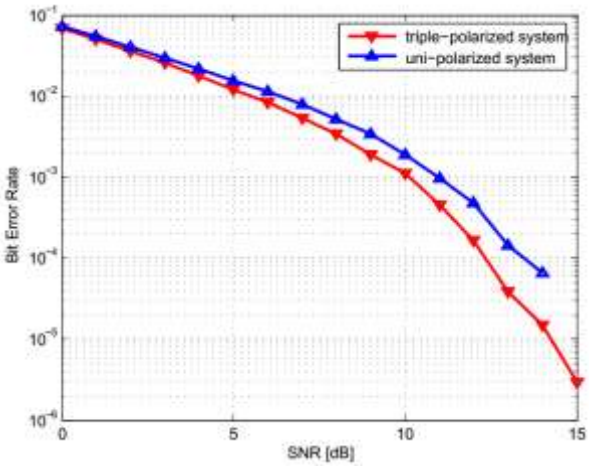


Figure 4. BER performance comparison of triple- and spatially-separated uni-polarized OSTBC-MIMO-OFDM systems

## *References*

[1] 3GPP TS 36.201, "Long Term Evolution (LTE) physical layer; General description," Release 8, V8.3.0, Mar. 2009.

[2] 3G Americas White Papers, "3GPP Mobile Broadband Innovation Path to 4G: Release 9, Release 10 and Beyond: HSPA+, SAE/LTE and LTEAdvanced," Feb. 2010.

[3] V. Tarokh, N. Seshadri, A. R. Calderbank, "Space-Time Codes for High Data Rate Wireless Com-munication: Performance Criterion and CodeConstruction," *IEEE Trans. Inform. Theory*., vol. 44, no. 2, pp. 744-765, Mar. 1998.

[4] C. Oestges, M. Guillaud, and M. Debbah, "Multi-polarized MIMO communications: Channel model, mutual information and array optimization," in *Proc. IEEE Wireless Commun. Netw. Conf*., pp. 1057-1061, 2007.

[5] G. Gupta, B. Hughes, and G. Lazzi, "On the degrees of freedom in linear array systems with tri-polarized antennas," *IEEE Transactions on Wireless Communications*, vol. 7, no. 7, pp. 2458-2462, 2008.

[6] A. Habib, "Mulitple Polarized MIMO with Antenna Selection", in *18th IEEE Symposium on Communications and Vehicular Technology* (*SCVT'11*), Belgium. Nov. 2011.

[7] J. Gutierrez, A. Habib, and M. Rupp, "Indoor measurements by dual tripole antennas," in *2012 Loughborough Antennas and Propagation Conference*, (*LAPC'12*), UK. Nov. 2012

[8] C. Oestges, B. Clerckx, M. Guillaud and M. Debbah, "Dual-polarized wireless communications: from propagation models to system performance evaluation," *IEEE Trans. Wireless Commun*., vol. 7, no. 10, pp.4019-4031, Oct. 2008.

[9] B. Clerckx, C. Oestges, *MIMO Wireless Networks, Second Edition: Channels, Techniques and Standards for Multi-Antenna, Multi-User and Multi-Cell Systems*. UK: Academic Press, 2013

[10] C. Martin and B. Ottersten, "Asymptotic eigenvalue distributions and capacity for MIMO channels under correlated fading," *IEEE Trans. Wireless Commun*., vol. 3, no. 4, pp. 1350-1359, Jul. 2004.

[11] S. M. Alamouti, "A simple transmit diversity technique for wireless communications," *IEEE Journal Selected Areas Commun*., vol. 16, no.8, pp. 1451-1458, Oct. 1998.

[12] G. Ganesan and P. Stioca, "Space-time block codes: A maximum SNR approach, *IEEE Trans. Inform. Theory*, pp. 1650-1656, Apr. 2001.

[13] V. Tarokh, H. Jafarkhani and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inform. Theory*, vol. 45, no. 5, pp. 1456-1467, Jul. 1999.