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# Stagnation point flow, heat and mass transfer over a nonlinear stretching sheet withSuction/Injection

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*Abstract*— **The present paper is to investigate the combined effects of the heat and mass transfer on the unsteady two-dimensional boundary layer flow over a nonlinear stretching sheet in the presence of Stagnation point and Suction/ Injection.The governing nonlinear partial differential equations have been reduced to the coupled nonlinear ordinary differential equations by the similarity transformations. The resulting equations are solved numerically by using Keller box method. The velocity, temperature and concentration distributions are discussed numerically and presented through graphs.**

**Keywords**— ***Boundary layer flow; nonlinearstretching sheet; Stagnation point; Suction/Injection, Magneto hydrodynamics; Casson fluid.***

## 1. INTRODUCTION

Cassonnanofluid in cylindrical geometry has important in blood flow. Mathematicians as well as medical researchers are widely working on Cassonnanofluid model. Boundary layer flow of Casson fluid over different geometries is considered by many authors in recent years. Nadeem et al. [1] presented the MHD flow of Casson fluid over an exponentially shrinking sheet. The analytical solution arising differential system has been computed by the Adomain Decomposition Method. Fredrickson [2] studied the steady flow of a Casson fluid in a tube. Magyari and Keller [3] provided both analytical and numerical solutions for boundary layer flow over an exponentially stretching surface with an exponential temperature distribution. Hayat et al. [4] investigated the Soret and Dufour effects on the MHD flow of the Casson fluid over a stretchedsurface. The relevant equations are first derived, and the series solutions are constructed by the homotopic procedure. Shehzad et al [5] analysed the effect of mass transfer in the magnetohydrodyanamic flow of a Casson fluid over a porous stretching sheet in the presence of a chemical reaction and suction. It is observed that the Cassion parameter and Hartman number have similar effects on the velocity in a qualitative sense. It is further analysed that the concentration decreases rapidly in comparison to the fluid velocity when the values of the suction parameter are increased.

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Many processes in engineering occur at high temperatures and the full understanding of the effect of radiation on the rate of heat transfer is necessary in the design of equipment. The effect of radiation on the boundary layer flow was studied by Elbashbeshy and Dimian [6], Hossain et al.[7], Bataller [8] and Cortell [9 ]. The radiation effect is considered by Bataller [8] in the study of boundary layer flow over a static flat plate (Blasius flow) andCortell [10]

In the present study, heat and mass transfer in MHD Nano fluid over a nonlinear stretching sheet in presence of suction/injection have been investigated. The governing partial differential equations of the flow are converted into nonlinear coupled ordinary differential equation by the technique of similarity transformation and then solved numerically. The effect of parameters on velocity, temperature and concentration profiles are discussed and presented through graphs.

## 2.MATHEMATICAL FORMULATION

Consider a steady two-dimensional MHD flow of viscous, incompressible, electrically-conducting and radiating fluid over a vertical stretching surface in the presence of heat source/sink and mass transfer. The x-axis is coincident with the vertical surface and the y-axis is perpendicular to the surface. U and v are defined as the velocity components along the x-and y-axes, respectively. The stretching sheet velocity is assumed to be in the form of $u=ax^m$ where $a$ is positive constant. The velocity at short distance from the surface allows a thin boundary layer to develop near the surface. The surface temperature, $T_w$ is assumed to follow the power law $T_w = T_\infty + bx^n$ where $b$ is a constant and $T_\infty$ is the ambient temperature. The surface concentration, $C_w$ is assumed to follow the power law $C_w = C_\infty + cx^n$ where $C_\infty$ is the ambient concentration. It is also assumed that the magnetic Reynolds number is small in such a way that the induced magnetic field is negligible. Both viscous dissipation and Ohmic heating terms are neglected because their values are generally small. Under these assumptions along with Boussinesq and boundary layer approximations, the governing equations of partial differential equations for the conservation of mass, momentum, energy and species are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\left(B+\frac{1}{B}\right)\frac{\partial^2 u}{\partial y^2}+g\beta_T(T-T_\infty)+$$
$$g\beta_C(C-C_\infty)-\frac{\sigma B(x)}{\rho}\left(u_\infty-u\right) \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c}\frac{\partial q_r}{\partial y}+\frac{D_B k_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2}+$$
$$\frac{q_0}{\rho c_p}(T-T_\infty) \quad (3)$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\frac{\partial^2 C}{\partial y^2}+\frac{D_B k_T}{T_B}\frac{\partial^2 T}{\partial y^2} \quad (4)$$

The boundary conditions for the velocity, temperature and concentration fields are

$$u=ax^m, v=-v_w(x), T=T_w(x), C=C_w(x) \ at \ y=0$$
$$u\to 0,\ T\to T_\infty,\ C\to\infty \ as \ y\to\infty \quad (5)$$

where $T$ and $C$ is the fluid temperature and temperature in the boundary layer, $B(x)$ is the variable magnetic field strength, $\nu$ is the kinematic viscosity, $\rho$ is the density of the base fluid , $\sigma$ is the electric conductivity, $k$ is the thermal conductivity, $\alpha=\frac{k}{\rho c_p}$ is the thermal diffusivity, $\beta$ is the coefficient of thermal expansion, $q_0$ is the heat generation or absorption coefficient such that $q_0$ > 0corresponds to heat generation while $q_0$ < 0 corresponds to heat absorption, $g$ is the acceleration due to gravity, $c_p$ is the specific heat at constant pressure, $q_r$ is the radioactive heat flux, $k_T$ is the thermal diffusion ratio, $c_s$ is the concentration susceptibility, $T_B$ is the mean fluid temperature and $D_B$ is the mass diffusivity.

By using the Rosseland approximation is defined as [28], the radiative heat flux $q_r$ is given by

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \quad (6)$$

Where $\sigma^*$ is the Stefan-Boltzmann constant and $k^*$ is the mean absorption coefficient. It is assumed that the temperature differences between the free stream $T_\infty$ and the local temperature $T$ is small enough expanding $T^4$ in a Taylor series about $T_\infty$ and neglecting higher order terms results;

$$T^4\cong 4T_\infty^3 T-3T_\infty^4$$
(7)

After substituting Eqs. (6) and (7) in Eq.(3), it will be reduces to

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\left[\alpha+\frac{16\sigma^* T_\infty^3}{3\kappa^*(\rho c)_f}\right]\frac{\partial^2 T}{\partial y^2}+$$
$$\frac{D_B k_T}{c_s c_p}\frac{\partial^2 C}{\partial y^2}+\frac{q_0}{\rho c_p}(T-T_\infty) \quad (8)$$

The continuity Equ (1) is satisfied by the Cauchys Riemann equation

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x} \quad (9)$$

Where $\psi=\psi(x,y)$ is the stream function. When the variable magnetic field $B(x)=B_0x^{(m-1)/2}$.

The momentum, energy and spices equations along with the boundary conditions can be transformed into a system of coupled ordinary differential equations by the following transformation:

$$\psi=(a\upsilon x^{m-1})^{\frac{1}{2}}f(\eta),\ \eta=\left(\frac{ax^{m-1}}{\upsilon}\right)^{\frac{1}{2}}y,$$
$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\ \phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty} \quad (10)$$

Where $f(\eta)$ is the dimensionless stream function, $\theta(\eta)$ is the dimensionless temperature, $\phi(\eta)$ is the dimensionless concentration, $\eta$ is the similarity variable.

As such Equs.(9 - 10), Eqs. (2), (4) and (8) reduce to the following system of nonlinear ordinary differential equations.

$$\left(B+\frac{1}{B}\right)f'''+\frac{m+1}{2}ff''-mf'^2+m\varepsilon^2+$$
$$\lambda\theta+\delta\phi+M^2(\varepsilon-f')=0 \quad (11)$$

$$\frac{1}{\Pr}(1+R)\theta''+\frac{m+1}{2}f\theta'-nf'\theta+Df\phi''+Q\theta=0 \quad (12)$$

$$\frac{1}{Le}\phi''+\frac{m+1}{2}f\phi'-nf'\phi+Sr\theta''=0 \quad (13)$$

The transformed boundary conditions can be written as

$$f=S,\ f'=1,\ \theta=1,\ \phi=1\ \ at\ \ \eta=0$$
$$f'=\theta=\phi=0\ \ as\ \eta\rightarrow\infty \qquad (14)$$

where

$$M=\frac{\sigma B_0^2}{\rho a},\lambda=\frac{G_{rx}}{\mathrm{Re}_x^2},G_{rx}=\frac{g\beta a(T_w-T_\infty)}{\upsilon^2 x^3},$$

$$\lambda=\frac{G_{cx}}{\mathrm{Re}_x^2},G_{cx}=\frac{g\beta^* a(C_w-C_\infty)}{\upsilon^2 x^3},\mathrm{Pr}=\frac{\nu}{\alpha},\varepsilon=\frac{b}{a},$$

$$Df=\frac{D_B k_T(C_W-C_\infty)}{c_s c_p(T_w-T_\infty)},Sr=\frac{D_B k_T(T_W-T_\infty)}{c_s T_B(C_w-C_\infty)},$$

$$Q=\frac{q_0\nu}{\rho c_p a x^{m-1}},\ Le=\frac{\nu}{D_B} \qquad (15)$$

$M$ - the magnetic parameter, $\lambda$ - the thermal buoyancy parameter, $\delta$ - the solutal buoyancy parameter, $Gr_x$ -the thermal Grashof number, $Gc_x$ - the solutalGrashof number, $\mathrm{Pr}$ - the prandtl number, $Le$ - Lewis number, $R$ - radiation parameter.

The quantities of physical interest for this problem are the skinfrictioncoefficient $C_f$, the local Nusselt number $Nu_x$, and the local Sherwood number $Sh_x$, which are respectively defined as

$$C_f=\frac{\tau_\omega}{\frac{1}{2}pU_\omega^2},\ \ Nu_x=-\frac{x}{(T_w-T_\infty)}\frac{\partial T}{\partial y},$$

$$Sh_x=-\frac{x}{(C_w-C_\infty)}\frac{\partial C}{\partial y}\ .at\ y=0, \qquad (16)$$

$$y=0$$

$$\frac{1}{2}\sqrt{\mathrm{Re}_x}\cdot C_f=f''(0)\ ,\ -\ \theta'(0)=\frac{Nu_x}{\sqrt{\mathrm{Re}_x}}\ ,$$

$$\frac{Sh_x}{\sqrt{\mathrm{Re}_x}}=-\ \phi'(0) \qquad (17)$$

(17)

where $\mathrm{Re}_x=u_\omega(x)\cdot\frac{x}{\nu}$ is the local Reynolds number.

# 3. NUMERICAL PROCEDURE

The set of coupled non-linear governing boundary layer equations (11) – (13) together with the boundary conditions (14) – (15) are solved numerically by using Keller box method. First of all, higher order non-linear differential equations (11) – (13) are converted into simultaneous linear differential equations of first order.Linearize the resulting algebraic equations by using Newton's method and write them in matrix form.Solve the system of Linear equations by the block tridiagonal elimination technique.

In this method the following initial guesses are chosen:

$$f_0(\eta)=(1+s)-e^{-\eta}\ ,\qquad \theta_0(\eta)=e^{-\eta}\ ,$$
$$\phi_0(\eta)=e^{-\eta}$$

(18)

# 4.RESULT AND DISCUSSION

The problem of boundary layer of a Casson fluid over a nonlinear stretching sheet subject to a transverse magnetic field in the presence of Suction/Injection is analysed. The governing non – linear ordinary differential equations are solved using Keller box method.

Table 1 Numerical values of $f''(0)$ and - $-\theta'(0)$ at the sheet for different values of $M$ when $\mathrm{Pr}=\lambda=m=n=1\ and\ \delta=R=Df=Sr=Le=\varepsilon=S=0$ ,Comparison of the present results with that of B. Nagabusanam Reddy et al. [18]

| $M$ | B. Nagabusanam Reddy et al. [18] | | Present results | |
|---|---|---|---|---|
| | $-f''(0)$ | $-\theta'(0)$ | $-f''(0)$ | $-\theta'(0)$ |
| 0.0 | 0.560811 | 1.087250 | 0.560674 | 1.087250 |
| 0.1 | 0.565906 | 1.087250 | 0.565771 | 1.0863327 |
| 0.2 | 0.581081 | 1.086250 | 0.580954 | 1.083322 |
| 0.5 | 0.683073 | 1.083240 | 0.682977 | 0.063052 |
| 1 | 1.000000 | 1.063010 | 1.000000 | 1.000000 |
| 2 | 1.901350 | 1.000060 | 1.896839 | 0.831090 |
| 5 | 4.926900 | 0.758217 | 4.915603 | 0.470235 |

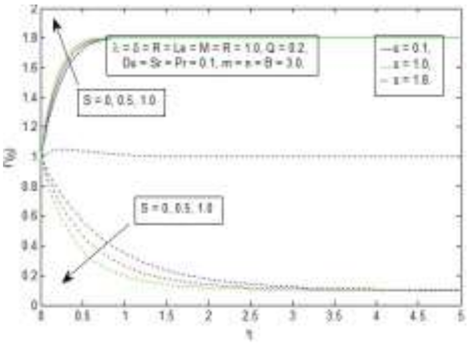


Figure 1: Velocity profile against $\eta$ for different values of $S$

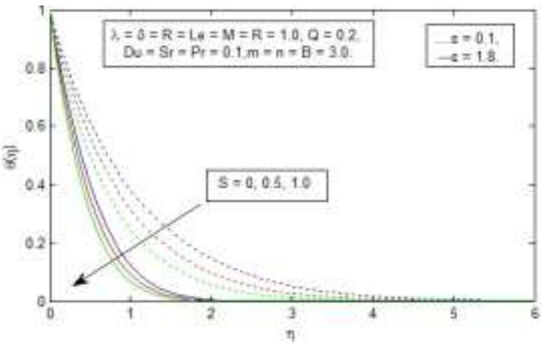


Figure 2: Temperature profile against $\eta$ for different values of $S$

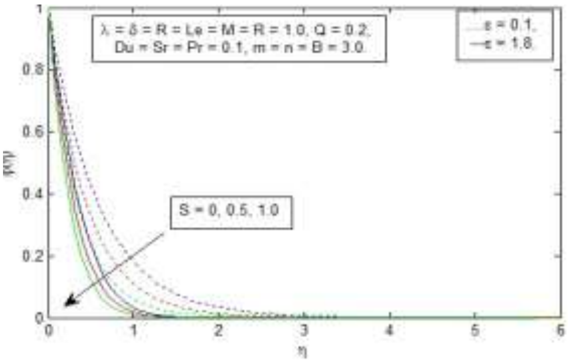


Figure 3: Concentration profile against $\eta$ for different values of $S$

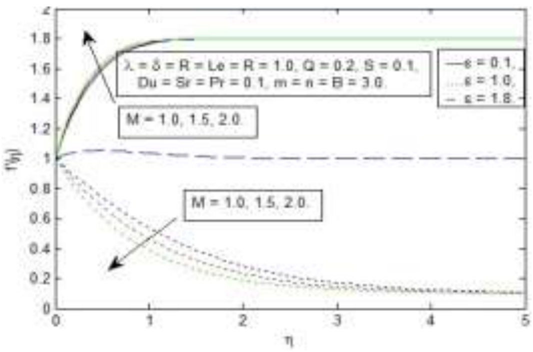


Figure 4: Velocity profile against $\eta$ for different values of $M$

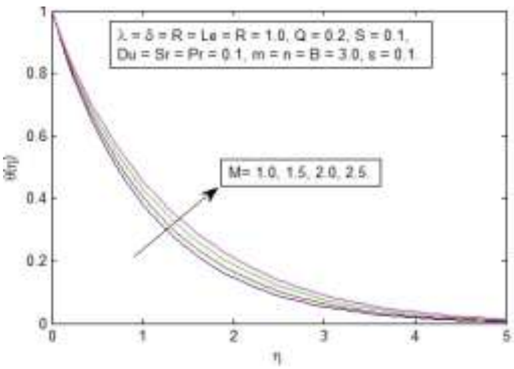


Figure 5: Temperature profile against $\eta$ for different values of $M$

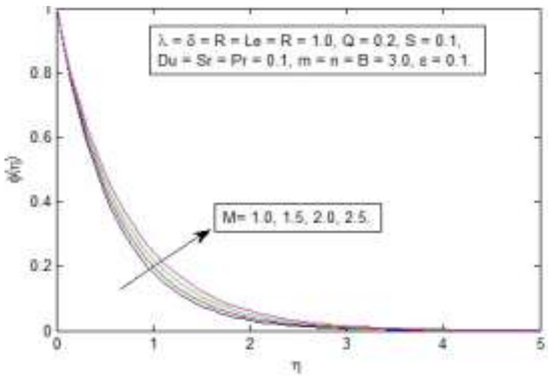


Figure 6: Temperature profile against $\eta$ for different values of $M$

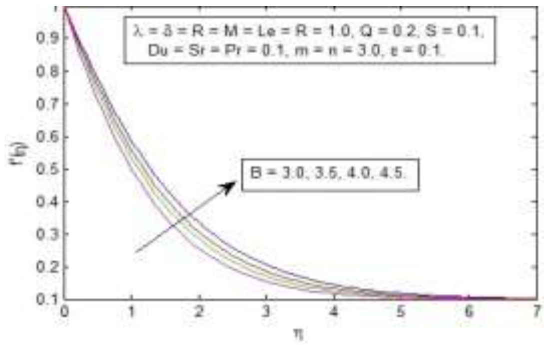


Figure 7: Velocity profile against $\eta$ for different values of $B$

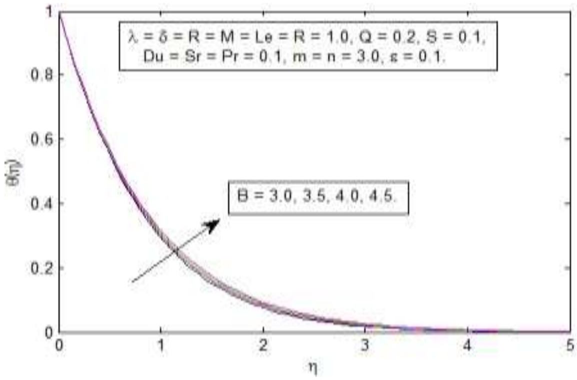


Figure 8: Temperature profile against $\eta$ for different values of $B$

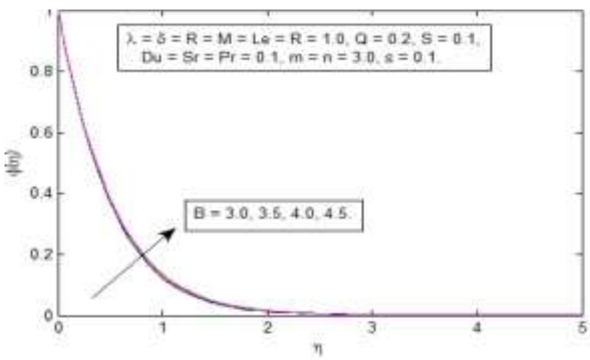


Figure 9: Concentration profile against $\eta$ for different values of $B$

Figures 1 – 3 demonstrate the effects of suction parameter $S$ on velocity, temperature and nanoparticle volume fraction. When $\varepsilon > 1$ & $\varepsilon < 1$ the fluid velocity decreases significantly with increasing values of suction parameter while it is found to enhance with blowing. The presence of suction would result in the reduction of the thickness of the boundary layer. The absence of suction represent the case of non-porous stretching sheet. In the case of blowing $(S<0)$ an opposite effect is observed. The effect of suction parameter on temperature and concentration is similar to that on the velocity.

Figures 4 – 6 present the velocity, temperature and concentration for a variation of the magnetic parameter. It is observed that the presence of a magnetic field reduces the velocity when $\varepsilon>1$ & $\varepsilon<1$. Higher value of Lorentz force further reduces the velocity and consequently the thickness of boundary layer reduces. However the impact of $M$ on temperature and concentration is less when compared to that on velocity.

Figures 7 – 9 indicate the effect of the yield stress parameter/ Casson parameter. It is clear that the velocity decreases with $B$. It may be noted that increased value of $B$ imply a decrease in the yield stress of the Casson fluid and thus facilitates the flow of the fluid. It is observed that increasing values of the Casson parameter enhance the temperature as well as the nanoparticle volume fraction.

# 5. CONCLUSIONS

The present paper is analyses the influence of stagnation point and suction/ injection effects on MHD boundary layer flow, heat and mass transfer flow over a nonlinear stretching sheet. The resulting partial differential equations are nondimensionalised, simplified, and solved by implicit finite difference scheme, known as Keller box method. From the present numerical study the following conclusions can be drawn.

(1) The Casson fluid parameter, magnetic parameter and suction parameter produce a reduction in the velocity and the boundary layer thickness.

(2) The influence of Casson fluid parameter and magnetic parameter on temperature and nanoparticle volume fraction is to enhance temperature and concentration.

(3) Temperature and concentration profiles decreased due to increases inSuction parameter

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