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$M^x/G/1$ Queueing System With Breakdowns and Repairs

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***Abstract*— We consider an M^X/G/1 queuing system with breakdown and repairs, where batches of customers are assumed to arrive in the system according to a compound poisson process. While the server is being repaired, the customer in service either remains the service position or enters a service orbit and keeps returning, after repair the server must wait for the customer to return. The server is not allowed to accepte new customers until the customer in service leaves the system. We find a stability condition for this system. In the steady state the joint distribution of the server state and queue length is obtained, and some performance mesures of the system, such as the mean number of customers in the retrial queue and waiting time, and some numerical results are presented to illustrate the effect of the system parameters on the developed performance measures.**

Keywords—*batch arrival*, break down, *repair*.

I. Introduction

Retrial queuing systems have been widely used to model many practical problems arising in telephone switching systems, telecommunication networks, and computer systems. The main characteristic of these queues is that a customer who find the sever busy upon arrival joins the retrial group called orbit to repeat his request for service after some random time. For a systematic account of the fundamental methods and results on this topic the reader can refer to the survey papers of (Artalejo and Gomez, 1998, 2008) (Falin and Templeton , 1997)and (Farahman, 1990).

Many of the queueing systems with repeated attempts operate under the classical retrial policy, where the intervals between successive repeated attempts are exponentially distributed, In contrast, there are other types of queueing situations in which the intervals separating successive repeated attempts are independent of the number of customers in orbit size.

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This second kind of policy is called constant retrial policy. which is introduced by (Fayolle, 1986) who investigated a telephone exchange model as an M/M/1 retrial queue where retrial groupe form a queue and only the customer at the head of the orbit queue can access the service after a retrial time following an exponentially .(Farahmand, 1996)calls the discipline of the orbit queue, First Come First Served (FCFS).

(Atencia and Moreno, 2008) generalized this retrial policy by considering an M/G/1 retrial queue with general retrial times.The linear policy is introduced by (Arltalejo and Atencia, 2004).

In recent years, several studies on retrial queueing models enhanced with various concepts such as vacation (Maraghi, 2009), (Choudhury, 2012)and(Baruah and all, 2013), discouragement (Chan and all, 1993) bulk (Jain and all, 2014), breakdown (Aissani, 1993) have been carried out.

The repair of broken down server is also an important factor.Practically, the reliable server may breakdown or stops working during any phase of service and will need to be repaired.

Among some earlier paper on broken down server(service interruption), we cite the paper of (MAraghi,Madan and Dowman, 2009). These authors investigated a bath arrival queueing system with random breakdowns and Bernoulli server vacations. (Roszik and Sztrik, 2004) studied the effet

II. Description Of The Model

We consider an $M^X/G/l$ queueing system, with breakdowns and repairs. We assume that the number of primary customers arrive to the system according to a compound Poisson process with rate λ. The size of successive arriving batches is $X_1,X_2,...$, where $X_1,X_2,...$ are identically and independently distributed (i.i.d) random variables with probability mass function (p.m.f) $g_n = P\{X = n\}, n > 1$, probability generating function (PGF)

$g(z) = E(z^X)$ and we denote $g_{[n]}$ as *nth* factorial moments. There is no waiting room in the front of the server. If the customers find the service busy or broken, then all the arriving customers join the orbit (retrial group) with probability p or leave the system with. Probability $q = 1 - p$. But if the server is free then he starts serving the customers from the batch which is on the head of the queue, whereas others leave the service area and join the orbit in accordance with an FCFS discipline, in order to seek service again and again until it find the server free. the time of successive repeated attempts of any customers in orbit follow an arbitrary law with probability distribution function *A(x)*, density function *a(x)* and Laplace-Stieldjer transform $L_A(s)$. The retrial customer is required to cancel its attempt for service if a primary customers arrives first. In this case, the

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retrial customers either returns to its position in the retrial queue with probability q or leaves the system with probability 1 - q. The service time of the customers are independently and identically distributed with a probability distribution function $B(x)$, a density function $b(x)$, Laplace-Stieldjes transform $L_B(s)$ and nth moments β_n.

We assume that the server may fail in a time exponentially distributed with mean $\frac{1}{\mu}$, but failure can occur only when a customer is being served.

When the server fails, repair begins immediately.The repair time has distribution function $C(x)$, density function c(x), Laplace transform LC(s) and first two moments γ_1 and γ_2.

Upon breakdown, the customer in service either remains in the service position with probability r until the server is up or enters a retrial orbit with probability 1 - r and keeps repeating its request for service continuation at times exponentially distributed with mean $\frac{1}{\theta}$, until the server is repaired. The server is not allowed to accept new customers until the customer in service leave the system. the server is said to be blocked if the server is busy, under repair or reserved.Service for a customer resumes after the repair time and reserved time. At any service completion, the server becomes idle. The length of the idle period of the server is determined by the competition between an exponential law of rate λ (a primary customer)and the general retrial time distribution (the retrial customer at the head of the orbit) which determines the next customer who accesses the server. Inter-arrival times,retrial times, service times, repair times and reserved times are assumed to be mutually independent.The time until failure is independent of the other times. The function $\alpha(x)$, $\beta(x)$ and $\gamma(x)$ are the conditional completion rates for repeated attempts, for service, for repair, respectively;

$$\alpha(x)=\frac{a(x)}{1-A(x)},\beta(x)=\frac{b(x)}{1-B(x)},\gamma(x)=\frac{c(x)}{1-C(x)}.$$

our work here is generalized the article of Wu, Brill, Hlynka and Wang "An M/G/1 Retrial Queue with Retrials of the Customer and Balking" (August 2004), at batch arrival of customers. And also calculates the stability condition with the method of normalization, we found the performance of this model

Remark 1:

Special cases of our model can be deduced by setting appropriate parameters as follows:

If $g_1=1,\theta\to\infty$,then this retrial queue become a classical M/G/1 retrial queue With reparable sever and balking;

If $g_1=1,q=p=1,\mu\neq0,\theta\to\infty$ This retrial queue reduce to the M/G/1 retrial queue with persistent customer and repairable server where the customer whose service is interrupted remains in service;

If $g_1=1,\mu=0$, this retrial reduce to the M/G/1 retrial queue With impatient customer.

Let N(t) be the orbit size (i.e., number of customers in the retrial group) at time t $\xi_1(t),\xi_2(t)$ and $\xi_3(t)$ be the elapsed retrial time, elapsed service time of customers, elapsed repair time and elapsed reserved time. Further, we introduced the following random variable:

III. STEADY-STATE DISTRIBUTION

The size of successive arriving batches is X1,X2,· · · where X1,X2,· · · are identically and independently distribution (i.i.d) random variables with probability mass function (p.m.f) gj = P{X = j}; j ≥ 1 Let N(t) be the orbit size (i.e., number of customers in the retrial group) at time t, $\xi_0(t)$, $\xi_1(t)$ and $\xi_2(t)$,$\xi_3(t)$ be the elapsed retrial time, elapsed service time and elapsed repair time,elapsed reserved time, Further, we introduced the following random variable:

$$J(t)=\begin{cases}0, \text{ If the server is idle at time t;}\\ 1, \text{ If the server is busy at time t ;}\\ 2, \text{ If the server is under repair at time t;}\\ 3, \text{ If the server is reserved at time t.}\end{cases}$$

$$J^*(t)=\begin{cases}0 \text{ customer in service remains in service position after server failure;}\\ 1 \text{ customer in service enters a retrial queue orbit after server failure;}\end{cases}$$

Q(t): number of customers in the retrial queue at time t.

$\{X(t), t\geq0\} = \{J(t); J^*(t); Q(t); \xi_0(t), \xi_1(t), \xi_2(t), \xi_3(t)\}$

we first definite the state probabilistes,state densities and joint state probability densities for the Markov process X(t), t ≥ 0 By considering transitions of the process between time t and t + Δt and letting Δt → 0, we derive the following system of equations that govern the dynamics of the system behavior (equations of chapman Kolmongorov): Relating the states of the system at time ***t*** and t +dt and taking t → ∞, we obtain the following differential equations in steady-state as follows :

$$(\frac{\partial}{\partial t}+\lambda)P_{0,0}(t)=\int_0^\infty P_{1,0}(t,x)\beta(x)d(x)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial w}+\lambda+\alpha(w)\right)p_{(0,n)}(t,w)=0$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+p\lambda+\mu+\beta(x)\right)p_{1,n}t(x)=\lambda p\sum_{j=1}^{n}g_j p_{1,n-j}(t,x)$$

$$+\int_0^\infty p_{2,0,n}(t,x,y)\gamma(y)d(y)+0\int_0^\infty p_{3,n}(t,x,\tau)d\tau$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+p\lambda+\gamma(y)\right)2,0,nP(t,x,y)=p\lambda\sum_{J=1}^{n}g_j p_{2,0,n-j}(t,x,y)$$

$$p_{2,1,n}(t,x,y)=p\lambda\sum_{j=1}^{n} g_j p_{2,1,n-j}(t,x,y)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial \tau}+p\lambda+\theta\right)p_{3,n}(t,x,y)=p\lambda\sum_{j=1}^{n}g_j p_{3,n-j}(t,x,\tau)$$

The boundary conditions are given as follows:

$$p_{0,n}(t,0)=\int_0^\infty p_{1,n}(t,x)\beta(x)dx$$

$$p_{1,n}(t,0)=\int_0^\infty p_{0,n+1}(t,w)\beta(w)dw$$

$$+(1-q)\lambda\sum_{j=1}^{n} g_j\int_0^\infty p_{0,n-j+2}(t,w)dw$$

$$+q\lambda\sum_{j=1}^{n} g\int_0^n p_{0,n-j+1}(t,w)dw+\lambda g_{n+1}p_{00}(t)$$

$$p_{2,0,n}(t,x,0)=r\mu P_{1,n}(t,x)$$

$$p_{2,1,n}(t,x,0)=(1-r)\mu\, p_{1,n}(t,x)$$

$$p_{3,n}(t,x,0)=\int_0^\infty p_{2,1,n}(t,x,y)\gamma(y)dy$$

The normalizing condition is :

$$P_{0,0}(t)+\sum_{i=1}^{\infty}\int_0^\infty p_{0,n}(t,w)dw+\sum_{i=0}^{\infty}\int_0^\infty p_{1,n}(t,x)dx$$

$$+\int_0^\infty\int_0^\infty p_{2,0,n}(t,x,y)dxdy$$

$$+\int_0^\infty\int_0^\infty p_{2,1,n}(t,x,y)dxdy$$

$$+\int_0^\infty\int_0^\infty p_{3,n}(t,x,\tau)dxd\tau)=1$$

The Following Theorem Gives The Joint Distribution Of The Server State And Queue Length In Terms Of Probability Generating Function.

$Theoreme1: If\ p\lambda\beta_1 g'(1)\beta_1(1+\mu(\frac{1-r}{\theta}+\gamma_1))\prec$ $1-(1-L_A(\lambda))(g'(1)+q-1)$ then there exists the following steady state solution of the model

$$P_{00}=\frac{(1-L_A(\lambda))(g'(1)+q-1)+(1-K'(1))}{(1-q+(q-pg'(1))L_A(\lambda))(1+\dfrac{K'(1)}{pg'(1)})-L_A(\lambda)(pg'(1)-K'(1)(\dfrac{1}{pg'(1)}-\dfrac{1}{p}))}$$

$$P_0(z,w)=\frac{\lambda z(z-K(z)g(z))\exp(-\lambda w)(1-A(w))P_{00}}{D(z)}$$

$$P_1(z,x)=\frac{\lambda(1-q+qL_A(\lambda))(1-z)g(z)+L_A(\lambda)(z-g(z))P_{00}}{D(z)}$$
$$\times\exp(-G(p\lambda(1-g(z))x))(1-B(x))$$

$$P_{20}(z,x,y)=\frac{r\mu\lambda(1-q+qL_A(\lambda))(1-z)g(z)+L_A(\lambda)(z-g(z))P_{00}}{D(z)}$$
$$\times\exp(-G(p\lambda(1-g(z))x)-p\lambda(1-g(z))y)(1-B(x))(1-C(y))$$

$$P_{21}(z,x,y)=\frac{(1-r)\mu\lambda(1-q+qL_A(\lambda))(1-z)g(z)+L_A(\lambda)(z-g(z))P_{00}}{D(z)}$$
$$\times\exp(-G(p\lambda(1-g(z))x)-p\lambda(1-g(z))y)(1-B(x))(1-C(y))$$

$$P_3(z,x,\tau)=\frac{(1-r)\mu\lambda(1-q+qL_A(\lambda))(1-z)g(z)+L_A(\lambda)(z-g(z))L_C(p\lambda(1-g(z)))P_{00}}{D(z)}$$

$$P_3(z,x,\tau)=\frac{(1-r)\mu\lambda(1-q+qL_A(\lambda))(1-z)g(z)+L_A(\lambda)(z-g(z))L_C(p\lambda(1-g(z)))P_{00}}{D(z)}$$

$$\times\exp(-(p\lambda(1-g(z))y)-(p\lambda(1-g(z)+\theta\tau))$$

$$were\quad G(x)\equiv x+\mu\frac{rx+\theta}{x+\theta}L_C(x)$$

$$K(x)\equiv L_B(G(p\lambda(1-x)$$

$$D(z)=(1-q+qL_A(\lambda))(1-z)K(z)g(z)-z(z-K(z)g(z))+K(z)L_A(\lambda)(z-g(z))$$

IV.PERFORMANCE MEASURES

In this section; we derive some performance measures of the system under the steady state condition. *Theorem 2:* The generating function of the number of customers in the system and in the orbit are given by respectively:

$$P_q(z)=\frac{P_{00}}{D(z)\lambda p(1-g(z))}\times\left\{\begin{array}{l}\left[(1-q+qL_A(\lambda))(1-z)g(z)+L_A(\lambda)(z-g(z))\right]\\ \times\left[\lambda p(1-g(z))K(z)+\lambda(1-K(z))\right]\\ -\lambda p(1-g(z))zL_A(\lambda)(z-K(z)g(z))\end{array}\right\}$$

$$P(z)=\frac{P_{00}}{D(z)\lambda p(1-g(z))}\times\left\{\begin{array}{l}\left[(1-q+qL_A(\lambda))(1-z)g(z)+L_A(\lambda)(z-g(z))\right]\\ \times\left[\lambda p(1-g(z))K(z)+\lambda z(1-K(z))\right]\\ -\lambda p(1-g(z))zL_A(\lambda)(z-K(z)g(z))\end{array}\right\}$$

Proof: Let denotes Nq ,N the number of: customers in the orbit and customers in the system respectively.

Then the generating function of each them are respectively given by :

$$P_q(z)=E(z^{\mathrm{Nq}})=P_{00}+P_0(z)+P_1(z)+P_{20}(z)+P_{21}(z)+P_3(z)$$

$$P(z)=E(z^{\mathrm{N}})=P_{00}+P_0(z)+z\left[P_1(z)+P_{20}(z)+P_{21}(z)+P_3(z)\right]$$

Then the results follow directly

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