Gyroscope's Internal Kinetic Energy and Properties

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Abstract— **Gyroscope devices are primary units for navigation and control systems that have wide application in aviation, in space, on ships and other industries. The main property of the gyroscope device is maintaining the axis of a spinning rotor. This gyroscope peculiarity is represented in terms of gyroscope effects in which mathematical models have been formulated on the law of kinetic energy conservation and a change in an angular momentum. The known gyroscope theories are represented by numerous publications, which analytical approaches cannot give true results. The nature of gyroscope effects is more complex than represented in known publications. Recent investigations in this area have demonstrated that the external torque acting on a gyroscope generates several inertial torques which are interdependent and simultaneous action. The gyroscope internal torques are generated by the action of several inertial forces as well as a change in an angular momentum. Latter one does not play first role in gyroscope theories. These torques represent internal kinetic energies of the spinning rotor. The internal kinetic energies are combined and manifested the gyroscope resistance and precession torques acting around two axes that perpendicular each other. Combination of the internal torques leads to change in the angular velocities of the gyroscope around axes. This paper represents the mathematical models for the torques of the gyroscope that shows the physics of the gyroscope internal kinetic energies and new properties. The mathematical models practically tested and the results are validated the theoretical approach.**

Keywords— **gyroscope theory, kinetic energy, torque**

I. Introduction

In 1765, L. Euler first lay out the mathematical foundations for the gyroscope theory in his work on the dynamics of rigid bodies. I. Newton, J-L. Lagrange, L. Poinsot, J.L.R. D'Alembert, P-S. Laplace, L. Foucault and other brilliant scientists investigated, developed and added new interpretations for the gyroscope effects, which display in the rotor's persistence of maintaining its plane of rotation. The applied theory of gyroscopes, i.e., the theory of devices and gyroscopic systems, emerged mainly in the twentieth century [1-3]. Numerous valuable publications dedicated to the gyroscope effects and underlined the primary phenomena of a gyroscope is represented as the resistance and precession torques [4-5]. The most fundamental textbooks of classical mechanics have chapters that represent the gyroscope theory [6-9]. There are many publications of the gyroscope theory with severe mathematics, which describe 3D gyroscope motions as well as many other mathematical approaches that quite hard to use for engineering problems. A grate number of valuable publications dedicated to application of the gyroscopes effects and properties in engineering [10]. Some researchers intuitively without mathematical models, pointed on the action of different inertial forces in the gyroscope that also involved in the manifestation of gyroscope effects. However, researchers pointed that analytical equations for the torques and motions of the gyroscopic devices do not match the practical results [11 - 13]. Discussions about gyroscope properties create other problems that need to be solved by producing a clear and understandable presentation of gyroscope motions. Contemporary mathematical models and theories of gyroscope properties contain numerous assumptions and simplifications and explain the gyroscope effect only by the change in the angular momentum of the spinning rotor. Actual reality of gyroscope effects demonstrates that the principle of the change in the angular momentum of the spinning rotor cannot give full picture of the action of gyroscope torques and motions. Most gyroscope properties did not find mathematical explanations. From this, gyroscope effects have spawned terms such as gyroscope resistance, gyroscope couple, gyroscopic effect is strange and weird and gyroscopes are not inertial and non gravitational devices and so forth. This is the reason that gyroscope theory still attracts researchers who seek to discover new properties for these devices and to find true mathematical models.

The nature of gyroscopic effects is more complex than represented by the known theories and mathematical models. The recent investigations of the physical principles of gyroscope motions demonstrate the several inertial forces acting upon a spinning rotor that generate gyroscopic effects [14 -16]. Research shows that the external torque applied to the gyroscope generates the internal torques based on action of several inertial forces that are represented by centrifugal, Coriolis, common inertial forces, and as well as changes in an angular momentum of spinning rotors. All these inertial forces represent one system that is originated by the action of rotating mass-elements of the spinning rotor. Motions of the mass-elements in space generate simultaneously inertial forces acting in different directions at the same time. Action of any internal inertial force cannot be separated from the system of forces. Hence, several internal torques are manifested on the resistance and precession torques acting around two axes. This is physics and fundamental principle of the gyroscope theory. The action of torques generated by centrifugal and Coriolis forces represents the resistance torques that counteract on the action of the external or loads torque applied to the gyroscope. The action of torques generated by common inertial forces and the change in the angular momentum of a spinning rotor represents the precession torques. The precession torques act around axis that perpendicular to axis of action the resistance torques.

The action of the internal torques is combined around two axes due to interdependency. The resistance and precession torques are originated on one axis, but acting around two axes, which are perpendicular each other. This dual interrelation of internal torques cannot be expressed and linked by mathematical equations. The motions of the gyroscope around two axes are formulated by two equations with two variables that cannot solve gyroscope problem. This is specificity of the equations of the gyroscope internal torques that are represented in Table 1.

TABLE I. Internal torques acting on a gyroscope

Type of torque generated by	Equation, (N.m)
Centrifugal forces, T_{ct} Inertial forces, T_{in}	$T_{ct} = T_{in} = 2\left(\frac{\pi}{3}\right)^2 J\omega\omega_i$
Coriolis forces, T_{cr}	$T_{cr} = (8/9)J\omega\omega_i$
Change in angular momentum, T_{am}	$T_{am} = J\omega\omega_i$
Resistance torque, $T_r = T_{ct} + T_{cr}$	$T_r = \left[2\left(\frac{\pi}{3}\right)^2 + \frac{8}{9}\right]J\omega\omega_i$
Precession torque, $T_p = T_{in} + T_{am}$	$T_p = \left[2\left(\frac{\pi}{3}\right)^2 + 1\right]J\omega\omega_i$

The symbols in Table 1 are represented by the following expressions: J is the mass moment of inertia of the spinning rotor, ω is the angular velocity of the spinning rotor, ω_i is the angular velocity of the gyroscope precession around axis i, other components are specified in Table 1.

To solve the problem of gyroscope equations, it is necessary to define the third equation related to gyroscope specificity. This third equation is represented by the equations of equality of the gyroscope's internal kinetic energy around two axes. All internal torques are represented the internal kinetic energy of the spinning rotor generated by the external torque. The internal kinetic energy is the energy associated with the spinning a gyroscope's rotor. The principle of conservation of mechanical energy states that in an isolated system that is only subject to conservative forces the mechanical energy is constant. Hence, the internal kinetic energies that represented by internal gyroscope torques acting around two axes can be used for solving the gyroscope motions.

The simultaneous action of gyroscope's internal torques around two axes manifesting their interrelations and new properties. The external torque activates the resistance torques acting around axis *ox* and the precession torques that acting around axis *oy*. In turn, this precession torques activates the resistance torques acting around axis *oy* and produces the precession torque acting as the resistance torques around axis *ox*. The sequence action and interrelations of the internal torques on two axes are considered on the gyroscope which rotor spins in counter clockwise direction. Graphically the external and internal torques acting in gyroscope and motions are represented in Fig. 1.

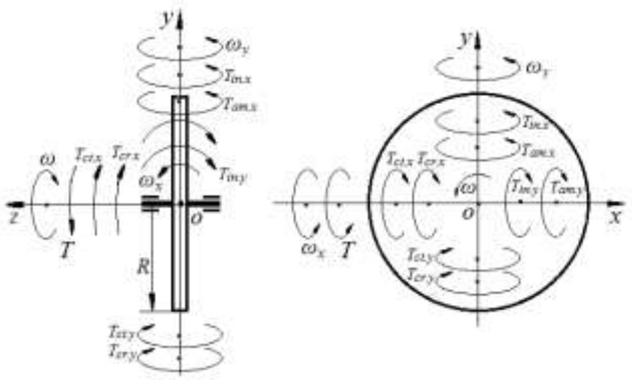

Figure 1. The external and internal torques and motions acting on the gyroscope around axes *ox* and *oy*.

The symbols in Fig. 1 are represented by the following expressions: T is the external torque applied on the gyroscope; R is the radius of the disc type spinning rotor, other components are as specified in Table 1.

The known publications do not give mathematically convincing proofs for several gyroscope properties and effects. Some researchers attract as evidence of these terms the following properties:

- The angular velocity of gyroscope precession is constant, which should be variable under the action of the constant load torque;
- The action of the gyroscope internal torques does not produce the reactive force on the gyroscope supports;
- The action of the gyroscope internal torques is resulting on the different angular velocities of precessions around two axes;
- The action of the internal torques demonstrates antigravity effect;
- Gyroscopes are not inertial systems, etc.

On the first sight, the represented gyroscope properties did not find reasonable explanations and contradicted to the rules of classical mechanics. However, the action of the several internal torques around two axes reveal new gyroscope properties that have not been described. The new mathematical models for gyroscope internal torques and motions enable the physical principles of the gyroscope properties to be described clearly by known laws of classical mechanics.

This paper represents the mathematical models of forces and motions and describes the physics of the gyroscope behavior around two axes. The mathematical models for the gyroscope properties tested and validated experimentally on the Super Precision Gyroscope model "Brightfusion Ltd".

II. Methodology

Recent investigations in the physical principles of the gyroscope motions are represented by the new mathematical models of torques acting on a gyroscope. The action of the external torque on a gyroscope generates two resistance and two precession torques acting simultaneously and interdependently around two axes that strictly perpendicular to each other. These internal torques depends on the speed of a gyroscope's rotor (Table 1) and represent the internal kinetic energy of the spinning rotor that originated and distributed equally on axes *ox* and *oy*. The statement of equality of the kinetic energy of axes is proofed by the following approaches to the gyroscope (Fig. 1).

The external torque applied to the gyroscope produces the resistance torques generated by action of centrifugal and Coriolis forces and precession torques generated by action of common inertial forces and the change in the angular momentum. All these internal torques originated by the external torque acting around axis *ox*. The load torques, which is the precession torques of axis *ox* produces the same internal torques originated on axis *oy*. The precession torque of axis *oy* is acting around axis *ox*. The action of the internal torques around two axes is interrelated. The absolute value of the internal torques originated on axis *ox* is equal the absolute value of the internal torques originated on axis *oy* and expressed by the following equation:

$$T_{ct.x} + T_{cr.x} + T_{in.x} + T_{am.x} = T_{ct.y} + T_{cr.y} + T_{in.y} + T_{am.y} \quad (1)$$

Substituting expressions of the torques represented in Table 1 into Eq. (1), transformation and simplification yields the following equation:

$$\left[2\left(\frac{\pi}{3}\right)^2 + \frac{8}{9}\right]J\omega\omega_x + \left[2\left(\frac{\pi}{3}\right)^2 + 1\right]J\omega\omega_x = \left[2\left(\frac{\pi}{3}\right)^2 + 1\right]J\omega\omega_y + \left[2\left(\frac{\pi}{3}\right)^2 + \frac{8}{9}\right]J\omega\omega_y \quad (2)$$

or $\omega_x = \omega_y$, i.e., angular velocities around axes are equal.

The result of Eq. (2) demonstrates the equality of the angular velocities around two axes, i.e., the equality of kinetic energy of the internal torques of two axes. Each axis contains originated resistance and precession torques only, but these torques act around two different axes interdependently and cannot be separated. This gyroscope peculiarity yields following property. Resistance torques of one axis are combined with precession torques of other axis, which actions can be contradicted. These combinations of internal torques of the gyroscope can change the magnitudes of the angular velocities around axes, but kinetic energies of each axis in absolute values remain constant in time according to the principle of conservation of mechanical energy. This state enables the combination of the internal kinetic energies of the gyroscope around two axes to be described.

Figure 1 demonstrates one example of the action of several internal torques of the gyroscope acting around axes *ox* and *oy*. The equality of the internal kinetic energies of two axes that expresses the internal torques enables the following equation to be represented:

$$-T_{ct.x} - T_{cr.x} - T_{in.y} - T_{am.y} = T_{in.x} + T_{am.x} - T_{ct.y} - T_{cr.y} \quad (3)$$

where all components are as specified above.

Substituting equations from Table 1 into Eq. (3), simplification and transformation yield the following result:

$$-\left[2\left(\frac{\pi}{3}\right)^2 + \frac{8}{9}\right]J\omega\omega_y - \left[2\left(\frac{\pi}{3}\right)^2 + 1\right]J\omega\omega_x = \left[2\left(\frac{\pi}{3}\right)^2 + 1\right]J\omega\omega_y - \left[2\left(\frac{\pi}{3}\right)^2 + \frac{8}{9}\right]J\omega\omega_x \quad (4)$$

Analysis of Eqs. (3) and (4) shows that to the component $T_{ct.y}$ from the left equation and the component $T_{in.y}$ from the right equation has the same expression with the same sign that is $T_{ct.y} = T_{in.y} = \left[2\left(\frac{\pi}{3}\right)^2\right]J\omega\omega_y$. The nature of action of this torques is one. This fact enables removing them in processing of Eq. (4) and others of gyroscope motions. Simplification of Eq. (4) yields the following remarkable equation:

$$\omega_y = -(4\pi^2 + 17)\omega_x$$
$$\frac{\omega_y}{\omega_x} = 56.478416 \quad (5)$$

where the sign (-) means the direction of the resistance torque is negative, all other components are as specified above.

Analysis of Eq. (5) shows the minor turn of the gyroscope around axis *ox* leads to big turn around axis *oy*. This gyroscope property is validated by practice. The ratio of the outer gimbal turn to inner one is the same. This analytical solution gives answer on unexplainable property of the gyroscope's gimbals turn on different angles.

The magnitude of minor turn of the gyroscope around axis *ox* is defined by Eq. (5). Replacing the symbols of the angular velocities by symbols of the angles and substituting the angle $\varphi = 90^0$ of the gyroscope turn around axis *oy* and transformation yields the value of the minor turn on the angle γ of the spin axis turn around axis *ox*:

$$\gamma = \frac{90^0}{4\pi^2 + 17} = 1.593^0 = 1^0 35' 34.8'' \quad (6)$$

Practically, the angle $\gamma = 1^0 35' 34.8''$ is validated by the tests of the Supper Precision Gyroscope model "Brightfusion Ltd".

Analysis of the action of the gyroscope's internal torques clearly demonstrates their interrelation and simultaneous action. The action of the internal torques and the sequence chain of their action and the ratio of the gyroscope angular velocities around two axes are manifested several following properties.

1. The resistance and precession torques ($T_{ct.x}$ and $T_{in.x}$) are acting around axes *ox* and *oy* simultaneously and have one angular velocity of precession ω_x. The action of these torques generated by one external torque and cannot be separated.
2. Blocking of the gyroscope turn around any axis leads to stop the angular velocity of precession. In this case, $\omega_x = 0$ or $\omega_y = 0$, i.e., the resistance and precession torques are deactivated. This property is expressed as $T_{ct.x} = T_{in.x} = 0$.
3. Deactivation of the resistance and precession torques ($T_{ct.x}$ and $T_{in.x}$) leads to free turn of the gyroscope under action of the gravity force.
4. Deactivation of the external torque ($T = 0$) leads to deactivation of all resistance and the precession torques of two axes, in spite of their acting on different axes.
5. All internal torques have limited values that depend on the angular velocity of the spinning rotor and the angular velocity of precession around axes. Increasing and decreasing the angular velocity of the spinning rotor leads to decreasing and increasing respectively the angular velocity of precession. The change in the angular velocity of precession does not lead to change in the angular velocity of

the spinning rotor, but leads to change in the internal kinetic energy of the rotor.

6. The blocking of the gyroscope turn around axis *oy*, and action of the external torque *T* that turns the gyroscope around axis *ox*, leads to the following the action of internal torques:

- The precession torques ($T_{in.x}$ and $T_{am.x}$) are activated with the angular velocity of the gyroscope's free turn around axis *oy* under action of the gravity forced. This free turn is cause of deactivation of the resistance torques ($T_{ct.x}$ and $T_{cr.x}$) around axis *ox*.
- The gyroscope's free turn around axis *ox* generates the action of the resistance torques ($T_{ct.x}$ and $T_{cr.x}$), but at the same time, they are deactivated due to the blocked turn of the gyroscope around axis *oy* and due to duality of action resistance and precession torques ($T_{ct.x}$ and $T_{in.x}$).
- The action of the resistance torque generated by Coriolis forces around axis *ox* is deactivated. The simultaneous action of all torques is generated by one system of rotating mass-elements of the spinning rotor and separating the action of some internal torque is impossible.
- The precession torques ($T_{in.y}$ and $T_{am.y}$) acting around axis *ox* that generated by the common inertial forces and the change in the angular momentum on axis *oy* and resistance torques ($T_{ct.y}$ and $T_{cr.y}$) are deactivated due to blocked motion around axis *oy*.

6. The magnitude of the resistance torques is less than magnitude of load torque. The difference in magnitudes of torques represents minor value of high order that can be neglected. This is the reason that dynamometers and scales do not show the reactive force of acting torques on the gyroscope's supports.

7. The minor differences of high order between the load or external torques and internal resistance torques acting around axes produce minor value of the angular velocity with minor value of acceleration of high order. Latter one can be neglected and the angular velocity of the gyroscope around axes can be accepted as constant. This is the reason that tachometers cannot record the minor value of acceleration.

Represented above gyroscope's properties have defined explanations of their physics. Mathematical models of gyroscope torques and motions based on new analytical approaches are validated by practical tests.

III. Results and discussion

The external torque that applied on the gyroscope generates the several internal torques based on action the centrifugal, common inertial, Coriolis forces and the change in the angular momentum of the spinning rotor. These internal gyroscope torques act simultaneously and interdependently are resulting in the resistance and precession torques, respectively acting around two axes. Combination of the several internal torques produces properties that have physical explanations. All gyroscope's internal torques are generated by action of the inertial forces that are produced by the rotation mass-elements of the spinning rotor. Gyroscope's internal torques acting around two axes represent the internal kinetic energies of the rotor which are equal and constant, but the gyroscope's angular velocities around two axes can be changed. The gyroscope angular velocities are variable with minor acceleration of high order that can be neglected. The minor acceleration of the gyroscope motion is resulting on the minor reactive forces on the supports generated by the action of the gyroscope internal torques. Experimental tests and analytical results for the motions of the gyroscope are well matched.

IV. Conclusion

The gyroscope theory in classical mechanics is one of the most complex and intricate in terms of analytical solutions. The known mathematical models in the gyroscope theory are mainly based on the actions of the load torque and the change in the angular momentum of the spinning rotor. The action of these torques do not do not match practice that produces more questions than answers regarding the gyroscope properties. This is the reason that the gyroscope still attracts researchers that trying to find true theory. The nature of the gyroscope effects is more complex. Actually in the gyroscope are acting the several internal torques generated by inertial forces of rotating mass-elements of the spinning rotor, which represent the fundamental principles of the gyroscope theory. These internal torques are acting simultaneously and interdependently around two axes. The gyroscope's internal torques represent the internal kinetic energy that enables explaining all gyroscope properties. New mathematical models for the gyroscope torques and motions are well matched the results of the practical tests. The known artificial terms of gyroscope theories can be removed and replaced by terms of forces and motions of classical mechanics.

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