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Level-Crossing Rate and Average Duration of Fades for TWDP Fading Channel

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***Abstract*—This paper mainly proposes the Level-Crossing Rate (LCR) and Average Duration of Fades (ADF) of the Two Wave with Diffuse Power (TWDP) fading process. Both reference model and deterministic model are presented. The reference model is derived based on the description of TWDP process while the deterministic model is derived based on the Rice's sum-of-sinusoids (SOS). The derived deterministic model is proved to consist with the reference model based on the central limit theorem. The simulation results show an excellent agreement between these two models and the simulations.**

Keywords*—*Fading channels; TWDP; Level-Crossing Rate; Deterministic simulation; Doppler frequency

I. Introduction

The Two Wave with Diffuse Power (TWDP) fading channel model, first proposed by Durgin [1], can be used to describe received signals consisted of two specular multipath waves with the diffuse components. This may occur for the typical narrow-band receiver, directional antennas and wide-band signals which may increase the ratio of the specular to the diffuse power [1]. The envelope probability density functions (PDF) and its approximated form of TWDP fading model was also presented in [1]. Based on that, the BPSK system performance was analyzed by Soon H. Oh [2][3]. It points out the system performance was even worse under the TWDP channel condition than the Rayleigh channel condition when the amplitudes of two specular waves are equal and much larger than the diffuse power's [2].

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The performance of QAM system for this fading channel was also analyzed by Suraweera [4], Dixit [5] and Yao Lu [6], respectively, in which the results indicate that the system exhibits a poorer performance under the TWDP fading channel than the Rayleigh channel [6]. The statistics of TWDP fading model have also been studied in [7], where new expressions of the PDF and the cumulative distribution functions (CDF) are presented and these expressions can provide applicable results when the amplitudes of the two specular waves are much larger than the variance of the scattering components. Rao pointed out in [8] that the TWDP fading model can be regard as an extension of Rice fading model from the derivation of its PDF, in which the closed-form expression for the Moment Generating Function of TWDP fading model is given. The expression of the Level-Crossing Rate (LCR) and Average Duration of Fades (ADF) for a special case that the incident angles of the two specular waves are perpendicular to the moving direction of receiver was also given in [8]. However, it needs to be pointed out that all the simulation parameters were obtained by using a stochastic method, the Monte Carlo Method. Besides, the higher order statistics of TWDP fading model still haven't been well investigated.

In this paper, the expressions of LCR and ADF for the TWDP fading model are given and verified with situations which show that the LCR and ADF are correlated to the Doppler frequencies of the two specular paths. If both of the two specular waves' Doppler frequencies approach to zero, the expression of its LCR can be simplified to its PDF multiplied by a constant, which was given in [8]. The simulation results of the LCR and ADF for the deterministic TWDP model is shown by using the Jakes power density. The LCR and ADF of the TWDP deterministic model is verified by the simulation results, and is also proved to be a good-fit for the reference model.

The rest of this paper is organized as follows. Section II derives the LCR and ADF of the TWDP reference model for both the general and special case. In Section III, the deterministic TWDP process is implemented by using the SOS method and parameterized by the method of exact Doppler spread (MEDS) under Jakes power spectral density [9]. The LCR and ADF for the deterministic TWDP model which is proved to match the reference model are derived from this deterministic process in Section IV. The comparisons between these two models and the simulation results are also given in this Section. Finally, the conclusions of this paper are drawn in Section V.

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II. LCR and ADF Expression of The Reference Model for The TWDP Fading Channel

A. *The derivation of LCR*

The description of TWDP channels can be denoted by the following equation:

$$V = V_1 \exp\left(j\phi_1\right) + V_2 \exp\left(j\phi_2\right) + V_{dif} \tag{1}$$

where

$$V_{dif} = \mu_1 + j\mu_2 \tag{2}$$

The phases of the two special waves ϕ_1, ϕ_2 are assumed to be independent from each other and uniformly distributed (ϕ_1, $\phi_2 \sim U(0,2\pi)$) . The envelopes of the two special waves V_1, V_2 are constant. The μ_1, μ_2 are independent zero-mean Gaussian random variables (μ_1, $\mu_2 \sim N(0,\sigma^2)$) whose derivatives with respect to time are also independent Gaussian random variables with zero-mean, and independent from μ_i [9]. The variance can be expressed as:

$$\mathrm{Var}(\dot{\mu}_i) = r_{\mu_i\mu_i}(0) = -r_{\ddot{\mu}_i}(0) = \beta_i \tag{3}$$

By using the Jakes power spectral density [10], this formula is given by:

$$\beta_1 = \beta_2 = \beta = 2\left(\pi f_{max}\sigma\right)^2 \tag{4}$$

The f_{max} represents the max Doppler frequency and the Doppler frequency of each specular component is f_i . The joint pdf of μ_i and its derivation can be expressed as follows:

$$\begin{aligned} & p_{\mu_{\rho 1}\mu_{\rho 2}\dot{\mu}_{\rho 1}\dot{\mu}_{\rho 2}}(x_1, x_2, \dot{x}_1, \dot{x}_2) \\ & = \frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1}{2\pi}\int_{-\pi}^{\pi} p_{\mu_1}\left(x_1 - h_1(\phi_1,\phi_2)\right)\cdot \\ & p_{\mu_2}\left(x_2 - h_2(\phi_1,\phi_2)\right)\cdot p_{\dot{\mu}_1}\left(\dot{x}_1 + 2\pi s_1(\phi_1,\phi_2)\right)\cdot \\ & p_{\dot{\mu}_2}(\dot{x}_2 - 2\pi s_2(\phi_1,\phi_2))\, d\phi_2\, d\phi_1 \end{aligned} \tag{5}$$

where

$$p_{\mu_i}\left(x_i\right) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left\{\frac{-x_i^2}{2\sigma^2}\right\}, i = 1,2 \tag{6}$$

$$p_{\dot{\mu}_i}(\dot{x}_i) = \frac{1}{\sqrt{2\pi\beta}}\exp\left\{\frac{-\dot{x}_i^2}{2\beta}\right\}, i = 1,2 \tag{7}$$

$$h_1(\phi_1,\phi_2) = V_1\cos\phi_1 + V_2\cos\phi_2 \tag{8}$$

$$h_2(\phi_1,\phi_2) = V_1\sin\phi_1 + V_2\sin\phi_2 \tag{9}$$

$$s_1(\phi_1,\phi_2) = f_1V_1\sin\phi_1 + f_2V_2\sin\phi_2 \tag{10}$$

$$s_2(\phi_1,\phi_2) = f_1V_1\cos\phi_1 + f_2V_2\cos\phi_2 \tag{11}$$

By transforming the Cartesian coordinates to the polar coordinates, the equation (5) can be expressed by [10]:

$$\begin{aligned} p_{\xi\theta\dot{\xi}\dot{\theta}}(z,\theta,\dot{z},\dot{\theta}) = \frac{1}{z^2} p_{\mu_{\rho 1}\mu_{\rho 2}\dot{\mu}_{\rho 1}\dot{\mu}_{\rho 2}}(z\cos\theta, z\sin\theta, \dot{z}\cdot \\ \cos\theta - \dot{\theta}z\sin\theta, \dot{z}\sin\theta + \dot{\theta}z\cos\theta) \end{aligned} \tag{12}$$

With the integrations as follows:

$$p_{\xi\dot{\xi}}\left(z,\dot{z}\right) = \int_{-\pi}^{\pi}\int_{-\infty}^{+\infty} p_{\xi\vartheta\dot{\xi}\dot{\vartheta}}\left(z,\theta,\dot{z},\dot{\theta}\right) d\dot{\theta}\, d\theta \tag{13}$$

LCR [9] can be obtained by using the following formula:

$$N_{\xi}\left(r\right) = \int_{0}^{+\infty} \dot{z}\, p_{\xi\dot{\xi}}\left(r,\dot{z}\right) d\dot{z} \tag{14}$$

Further, the LCR of the reference model can be expressed as follows:

$$\begin{aligned} N_{\xi}\left(r\right) = \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} \frac{r}{4\pi^2\sigma^2}\cdot e^{-\frac{r^2+V_1^2+V_2^2}{2\sigma^2}}\cdot e^{-\frac{V_1V_2\cos(\phi_1-\phi_2)+rh_1(\phi_1,\phi_2)}{\sigma^2}}\cdot \\ \left(\begin{aligned} & \sqrt{\frac{\beta}{2\pi}}e^{-\frac{2\pi^2 s_1^2(\phi_1,\phi_2)}{\beta}} + \\ & \pi s_1(\phi_1,\phi_2)\cdot \mathrm{erf}\left(\sqrt{\frac{2}{\beta}}\pi s_1(\phi_1,\phi_2)\right) \end{aligned}\right) d\phi_2\, d\phi_1 \end{aligned} \tag{15}$$

The results with the different Doppler frequencies of the two waves are shown in Figure.1, in which the LCR $N_{\xi}(r)$ is normalized by f_{max} in order to remove the influence of the max Doppler frequency.

It shows that the LCR of TWDP fading channel model has two peaks, and the LCR will increase with the increase of the two specular waves' Doppler frequencies f_1, f_2.

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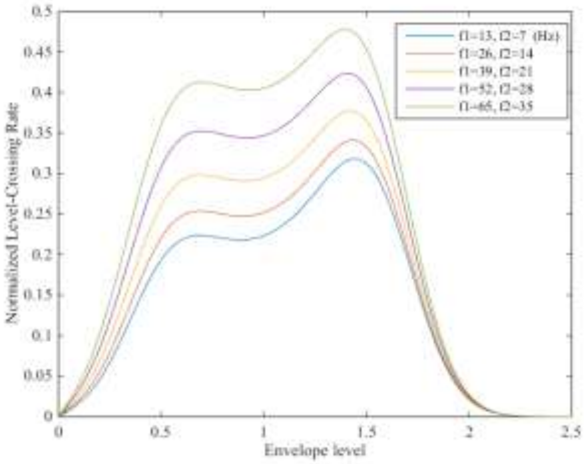


Figure.1. the normalized LCR $N_\xi(r)/f_{max}$ ($V_1 = 1$, $V_2 = 0.6$, $\sigma = 0.2$, $f_{max} = 91$)

B. *The LCR for The Special Case*

When the incident directions of the two specular components are perpendicular to the receiver moving direction, f_1, f_2 are equal to zero according to the Doppler frequency shift equation [9]. Thus, (15) can be simplified as:

$$N_\xi(r)=\sqrt{\frac{\beta}{2\pi}}\cdot\frac{r}{2\pi\sigma^2}\cdot e^{-\frac{r^2+V_1^2+V_2^2}{2\sigma^2}}\int_{-\pi}^{\pi} e^{\frac{V_1V_2\cos(\theta)}{\sigma^2}}\cdot I_0\left(\frac{r\sqrt{V_1^2+V_2^2-2V_1V_2\cos(\theta)}}{\sigma^2}\right)d\theta \tag{16}$$

where $I_0(x)$ is zero order modified Bessel's function. The PDF of the TWDP model [1] is given in equation (17), which can be regarded as a part of the equation (16):

$$f_{TWDP}(r)=\frac{r}{2\pi\sigma^2}\cdot e^{-\frac{r^2+V_1^2+V_2^2}{2\sigma^2}}\int_{-\pi}^{\pi} e^{\frac{V_1V_2\cos(\theta)}{\sigma^2}}\cdot I_0\left(\frac{r\sqrt{V_1^2+V_2^2-2V_1V_2\cos(\theta)}}{\sigma^2}\right)d\theta \tag{17}$$

After that, the N_ξ (r) can be obtained as follows:

$$N_\xi(r)=\sqrt{\frac{\beta}{2\pi}}\cdot f_{TWDP}(r) \tag{18}$$

C. *The ADF*

The ADF [11] is defined as follows [9]:

$$T_{\xi\_}(r)=\frac{F_{\xi\_}(r)}{N_\xi(r)} \tag{19}$$

where, $F_{\xi\_}(r)$ is the CDF of envelope r, which is the integration of the PDF. Thus, the CDF of TWDP model can be expressed as follows:

$$F_{\xi\_}(r)=\int_0^r f_{TWDP}(r)\,dr \tag{20}$$

It should be pointed out that the ADF can be only influenced by the LCR, since the CDF doesn't depend on the Doppler frequencies f_1, f_2. The ADFs of the reference model with different Doppler frequencies are shown in Figure.2.

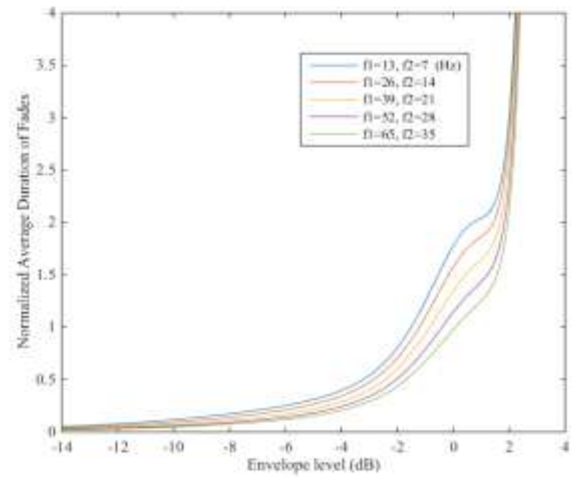


Figure.2 the normalized ADF $T_{\xi\_}(r)\cdot f_{max}$ ($V_1 = 1$, $V_2 = 0.6$, $\sigma = 0.2$, $f_{max} = 91$)

III. The Deterministic Process of TWDP Channel

From the TWDP channel model's description denoted by formula (1), the real part and imaginary part of the diffusion part are expressed by two independent zero-mean Gaussian random processes which can be modeled by using a superposition of a finite of sinusoids with given correlation properties based on the Rice's sum-of-sinusoids (SOS) [9]. According to the SOS theory, a stochastic Gaussian process $\mu_i(t)$ can be expressed by using an infinite SOS:

$$\mu_i(t)=\lim_{N_i\to\infty}\sum_{n=1}^{N_i} c_{i,n}\cos(2\pi f_{i,n}t+\theta_{i,n}), i=1,2 \tag{21}$$

If $N_i < \infty$ and all the model parameters $c_{i,n}$, $f_{i,n}$ and $\theta_{i,n}$ are realizations of random variables, the above stochastic model becomes a deterministic process denoted by:

$$\tilde{\mu}_i(t)=\sum_{n=1}^{N_i}c_{i,n}\cos(2\pi f_{i,n}t+\theta_{i,n}), i=1,2 \quad (22)$$

where these parameters $c_{i,n}$, $f_{i,n}$ and $\theta_{i,n}$ can be computed by Method of Exact Doppler Spread (MEDS) [9]. This method is used because it can product an autocorrelation function which is quasi-optimal to given correlation properties like the Jakes power spectral density.

$$c_{i,n}=\sigma\sqrt{\frac{2}{N_i}}, i=1,2 \quad (23)$$

$$f_{i,n}=f_{\max}\sin[\frac{\pi}{2N_i}(n-\frac{1}{2})], i=1,2 \quad (24)$$

$$\begin{aligned}\vec{\theta}_i&=(\theta_{i,1},\theta_{i,2},\ldots,\theta_{i,N_i})\\&=(2\pi\frac{1}{N_i+1},2\pi\frac{2}{N_i+1},\cdots,2\pi\frac{N_i}{N_i+1}), i=1,2\end{aligned} \quad (25)$$

Where, $N_2 = N_1 + 1$ is implemented to make the real part and the imaginary part uncorrelated. The specular components can be parameterized as follows:

$$V_i\cos(2\pi f_it+\theta_{vi})+jV_i\sin(2\pi f_it+\theta_{vi}), i=1,2 \quad (26)$$

The Doppler frequencies f_1, f_2 are equal to zero for the special case since the incident directions of the two specular components are perpendicular to the receiver motion direction. However, if $f_1 = f_2 = 0$, the V_1, V_2 can be regarded as one path, and the deterministic process turns out to be a Rice process. In order to avoid the above situation, f_1 can be set to zero, while f_2 can be equal to a very small value approaching to zero.

IV. The Exact Solution of The LCR and ADF for The Deterministic TWDP Process

The time derivative of the deterministic Gaussian process (22) is given as follows:

$$\dot{\tilde{\mu}}_i(\mathrm{t})=-2\pi\sum_{n=1}^{N_i}c_{i,n}f_{i,n}\sin(2\pi f_{i,n}t+\theta_{i,n}) \quad (27)$$

The PDF of (22) and (27) can be expressed by using (28) and (29), respectively [12].

$$\tilde{p}_{\mu_i}(x)=2\int_0^\infty j_{hi}(v)\cos(2\pi vx)dv \quad (28)$$

$$\tilde{p}_{\dot{\mu}_i}(\dot{x})=2\int_0^\infty j_{si}(v)\cos(2\pi v\dot{x})dv \quad (29)$$

$$j_{hi}(v)=\prod_{n=1}^{N_i}J_0(2\pi c_{i,n}v) \quad (30)$$

$$j_{si}(v)=\prod_{n=1}^{N_1}J_0[(2\pi)^2c_{i,n}f_{i,n}v] \quad (31)$$

$J_0(x)$ is zero order Bessel's function here. μ_1, μ_2 and their corresponding derivatives are considered to be independent from each other [12]. Thus, their joint probability density function with the two specular components can be expressed by using a similar form as the reference model:

$$\begin{aligned}&\tilde{p}_{\mu_1\mu_2\dot{\mu}_1\dot{\mu}_2}(x_1,x_2,\dot{x}_1,\dot{x}_2)\\&=\frac{1}{4\pi^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\tilde{p}_{\mu_1}(x_1-h_1(\phi_1,\phi_2))\cdot\\&\tilde{p}_{\mu_2}(x_2-h_2(\phi_1,\phi_2))\cdot\\&\tilde{p}_{\dot{\mu}_1}(\dot{x}_1+2\pi s_1(\phi_1,\phi_2))\cdot\\&\tilde{p}_{\dot{\mu}_2}(\dot{x}_2-2\pi s_2(\phi_1,\phi_2))d\phi_1d\phi_2\end{aligned} \quad (32)$$

Similarly, the Cartesian coordinates can be transformed to polar coordinates as the derivation of reference model (12). After that, this joint PDF can be reformulated as follows:

$$\begin{aligned}&\tilde{p}_{\xi\dot{\xi}}(z,\dot{z})\\&=\frac{2z}{\pi^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\int_0^\infty j_{h1}(v)\cos[2\pi v(z\cos\theta-\\&h_1(\phi_1,\phi_2)]dv\cdot\int_0^\infty j_{h2}(v)\cos[2\pi v(z\sin\theta-\\&h_2(\phi_1,\phi_2))]dv\cdot\int_0^\infty j_{s1}(v\cos\theta)j_{s2}(v\sin\theta)\cdot\\&\cos[2\pi v(\dot{z}+s_1(\phi_1,\phi_2)\cos\theta-s_2(\phi_1,\phi_2)\sin\theta)]\cdot\\&dvd\phi_1d\phi_2d\theta\end{aligned} \quad (33)$$

According to (14), the LCR of the deterministic model can be derived as follows by using the joint PDF in (33):

$$\begin{aligned}&\tilde{N}_\xi(r)\\&=\frac{2r}{\pi^2}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\int_0^\infty j_{h1}(v)\cos[2\pi v(r\cos\theta-\\&h_1(\phi_1,\phi_2))]\cdot dv\int_0^\infty j_{h2}(v)\cos[2\pi v(z\sin\theta-\\&h_2(\phi_1,\phi_2))]dv\cdot\int_0^\infty j_{s1}(v\cos\theta)j_{s2}(v\sin\theta)\cdot\\&\dot{z}\cos[2\pi v(\dot{z}+s_1(\phi_1,\phi_2)\cos\theta-s_2(\phi_1,\phi_2)\sin\theta)]\cdot\\&dvd\dot{z}d\phi_1d\phi_2d\theta\end{aligned} \quad (34)$$

In order to prove the correctness of above expression, a theoretical verification is given in the following context of this section. Assuming N_i tends to infinity, the following equations can be given based on the central limit theorem [9].

$$j_{h1}(v) = j_{h2}(v) = e^{-2(\pi\tilde{\sigma}v)^2} \tag{35}$$

$$j_{s1}(v\cos\theta)\cdot j_{s2}(v\sin\theta) = e^{-2(\tilde{\beta}_1\cos^2\theta+\tilde{\beta}_2\sin^2\theta)(\pi v)^2} \tag{36}$$

$$\tilde{\sigma}^2 = \lim_{N_i\to\infty}\sum_{n=1}^{N_i}\frac{{c_{i,n}}^2}{2} = \sigma^2 \tag{37}$$

$$\tilde{\beta}_1 = \tilde{\beta}_2 = \lim_{N_i\to\infty} 2\pi^2\sum_{n=1}^{N_i}(c_{i,n}f_{i,n})^2 = 2(\pi f_{\max}\sigma)^2 = \beta \tag{38}$$

Combining (35), (36), (37) and (38) with (34), the following expression can be given:

$$\lim_{N_i\to\infty}\tilde{N}_{\xi}(r) = \int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\frac{r}{4\pi^2\sigma^2}\cdot e^{-\frac{r^2+V_1^2+V_2^2}{2\sigma^2}}\cdot e^{-\frac{V_1V_2\cos(\phi_1-\phi_2)+rh_1(\phi_1,\phi_2)}{\sigma^2}}\cdot \left(\sqrt{\frac{\beta}{2\pi}}e^{-\frac{2\pi^2 {s_1}^2(\phi_1,\phi_2)}{\beta}} + \pi s_1(\phi_1,\phi_2)\cdot \mathrm{erf}\left(\sqrt{\frac{2}{\beta}}\pi s_1(\phi_1,\phi_2)\right)\right)d\phi_1 d\phi_2 \tag{39}$$

it proves that the LCR expression of the deterministic model agrees with LCR expression of the reference model (15).

Since the CDF is the integral of its PDF, the CDF of the TWDP deterministic model (shown in Fig. 3) can be written as:

$$\tilde{F}_{\xi\_}(r) = \frac{r}{2\pi^2}\int_0^{\infty}\int_0^{\pi} j_{h1}(v\cos\theta)j_{h2}(v\sin\theta)d\theta\cdot \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} J_1(2\pi rv)\cos(2\pi v h_1(\phi_1,\phi_2))d\phi_1 d\phi_2 dv \tag{40}$$

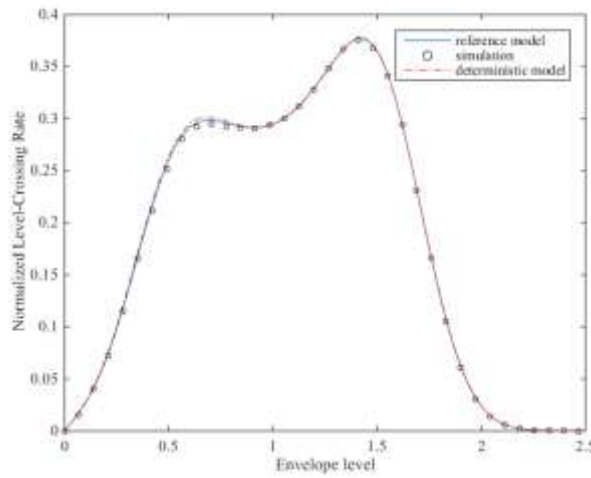
According to the definition of ADF given in (19), the ADF can be calculated from the ratio of the LCR (34) and CDF (40). The LCR and ADF of the deterministic model, simulation and reference model are simulated by using MATLAB and shown in Figure.3, from which it can be found that the deterministic model, reference model and simulation agrees with each other perfectly.

For a special case that the f_1 is zero and the ϕ_1 is set to be constant, the CDF and LCR of the deterministic model can be simplified as:

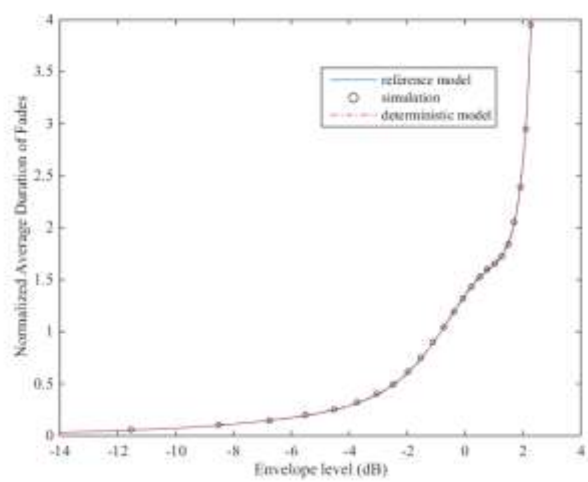
$$\tilde{F}_{\xi\_}(r) = \frac{r}{\pi}\int_0^{\infty}\int_0^{\pi} j_{h1}(v\cos\theta)j_{h2}(v\sin\theta)\int_{-\pi}^{\pi} J_1(2\pi rv)\cdot \cos[2\pi v(V_1\cos\theta+V_2\cos\phi_2)]d\phi_2 dv d\theta \tag{41}$$

$$\tilde{N}_{\xi}(r) = \frac{4r}{\pi}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}\int_0^{\infty} j_{h1}(v)\cos[2\pi v(r\cos\theta - h_1(\theta_{v1},\phi_2))]dv \int_0^{\infty} j_{h2}(v)\cos[2\pi v(r\sin\theta - h_2(\theta_{v1},\phi_2))]dv d\phi_2\cdot \int_0^{\infty}\int_0^{\infty} j_{s1}(v\cos\theta)j_{s2}(v\sin\theta)\cdot\dot{z}\cos(2\pi v\dot{z})dv d\dot{z} d\theta \tag{42}$$

Similarly, the simulation results for this special case are shown in Figure. 4. It needs to be mentioned that the LCR and ADF expression of the reference model can be found in (16).

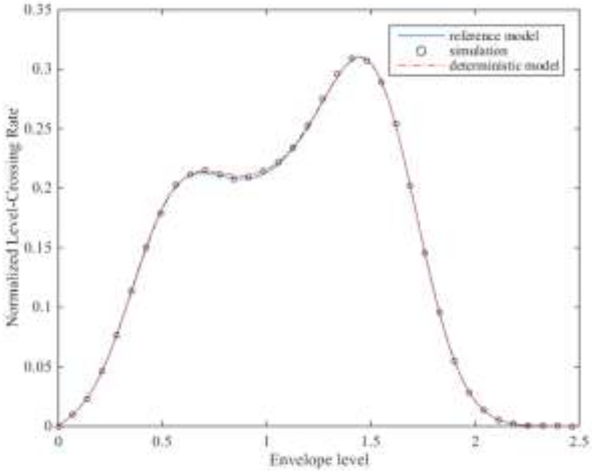


(a)

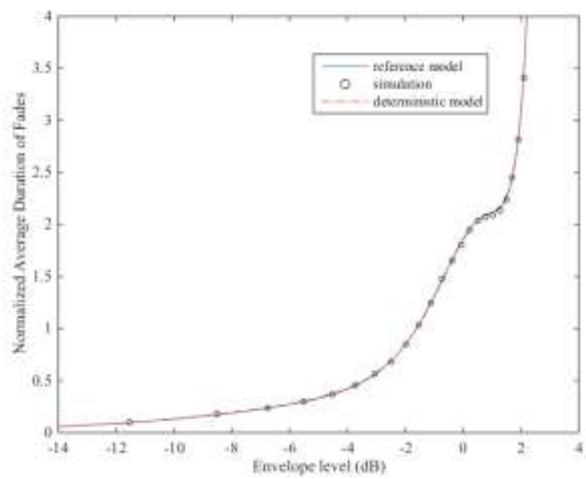


(b)

Figure.3. (a) the simulation results of normalized LCR (N_{ξ}(r)/$f_{\max}$) and (b) the simulations of normalized ADF ($T_{\xi\_}$ (r) $\cdot f_{\max}$) (V_1 = 1, V_2 = 0.6, σ = 0.2, $f_{\max}$ = 91, f_1 = 39, f_2 = 21, N_1 = 25, N_2 = 26, θ_{v1} = π/3, θ_{v2} = π/6.)



(a)



(b)

Figure.4. (a) the simulation results of normalized LCR ($N_{\xi}(r)/f_{max}$) for the special case and (b) the simulation results of normalized ADF ($T_{\xi\_}(r) \cdot f_{max}$) for the special case ($V_1 = 1$, $V_2 = 0.6$, $\sigma = 0.2$, $f_{max} = 91$, $N_1 = 25$, $N_2 = 26$, $\theta_{v1} = \pi/3$, $\theta_{v2} = \pi/6$.)

The simulation results of LCR and ADF expression of the reference model, deterministic model and simulation show perfect agreement in above figures for both general and special cases.

V. Conclusion

In this paper, the LCR and the ADF expressions of the reference model and the deterministic model for the TWDP fading process are derived for both general and special cases. The simulation results of the reference model shows the LCR will increase together with an increase of the frequencies, while the ADF will decrease. It indicates the values of the two specular waves' Doppler frequencies will result in influences on the higher order statistics of TWDP fading model. For the special case when the incident direction of the two specular paths are perpendicular to the receiver moving direction, the specular waves' Doppler frequencies turn out to be zero and the LCR expression becomes proportion to its corresponding PDF. Finally, the simulation results show that our LCR and ADF expression of the deterministic model matches with the reference model and simulation perfectly for both general and special case.

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