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# The Using of ANN at Design of Window Function Having Useful Spectral Parameters

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***Abstract*—The window functions are proposed structures at many application areas. The windows with helping two or more independent parameters show useful spectral characteristic. Kaiser window has been used at many areas because of the fact that it has good characteristic specification such as mainlobe width and the ripple ratio for various applications but Kaiser window having two parameters cannot control the sidelobe roll-off ratio. The ultraspherical window having three independent parameters has better sidelobe roll-off ratio than other windows in literature. At work, the coefficients of a new window function that has side-lobe roll-off ratio characteristic of ultraspherical window and Kaiser window's main-lobe width and ripple ratio were obtained by helping of artificial neural networks (ANN). 3000 data are used for training and testing of ANN. Sixty-five percentage of these data was used for learning of ANN and thirty-five percentage of that was used for testing process. It was observed that between the desired values and the obtained values using ANN were good agreements. Hence, a new window having better mainlobe width ripple ratio and side lobe roll-off ratio than other windows in spectral parameters at literature was modeled with ANN.**

***Keywords*—Window functions, Kaiser window, Ultraspherical window, Neural networks.**

## I. Introduction

Window functions (or windows for short) are time-domain functions that are used to reduce Gibbs' oscillations resulting from the truncation of a Fourier series at nonrecursive digital filter design (FIR, Finite Impulse Response). Windows are widely preferred at many application areas such as digital filter design, classification of cosmic data [1,2], improving the reliability of weather prediction models [3], facilitating the detection of disease at biomedical areas [4] and speech processing [5].

The window functions are two types for fixed and adjustable according to their parameters [6]. In literature, many windows has been proposed [7-16]. Fixed windows control the mainlobe width since these windows have one independent parameter, namely, the window length.

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On the other hand, adjustable windows provide more useful window amplitude spectrum with helping of two or more independent parameters. Dolph-Chebyshev [3], Kaiser [9] and Saramaki [10] are most used windows in literature and these windows are window functions having two adjustable parameters. Other improved windows based on Kaiser window equation were given at [11, 12]. Dolph-Chebyshev window having two parameters provides the minimum mainlobe width for a specified maximum side-lobe level. Saramaki window minimizes the sidelobe energy better than Kaiser window. Kaiser window which is preferred in many areas due to its parameters has maximum energy concentration in the mainlobe and provides better mainlobe width characteristic and ripple ratio than other windows. A well-known two parameter windows don't control side-lobe roll-off ratio in the window spectral parameters. Alternatively, Ultraspherical window having three parameters has been proposed to solve this problem [13-15]. This window has window spectral characteristic such as mainlobe width, ripple ratio, null-to-null width, and a user-defined side-lobe pattern.

In literature, there are many intelligent methods such as GA [7,16], fuzzy [17,18] and ANN. The ANNs are used to solve complex problems and to obtain solution of constrained optimization problems [19]. The canonical nonlinear programming circuit was developed by Chua and Lin [20]. In literature, ANNs are used in signal processing areas such as one [21] and two dimensional [22] filter design and image processing [23].

In this study, the calculation of coefficient values for a new window function that provides together side-lobe roll-off ratio characteristic of Ultraspherical window and Kaiser window's main-lobe width and ripple ratio was proposed by using of ANN. Thus, a new structure used at more large application areas with a new window having useful spectral parameters has been presented.

## II. Window Function Spectrum

Windows are frequently classified according to their spectrum characteristics and they are compared with other windows in terms of these characteristics.

$$\begin{aligned} W(e^{j\omega T}) &= |A(\omega)|\, e^{j\theta(\omega)} \\ &= e^{-j\omega(N-1)T/2} W_0(e^{j\omega T}), \\ &= \omega(0) + 2\sum_{n=1}^{(N-1)/2} \omega(nT)\cos\omega nT \end{aligned} \qquad (1)$$

where $W_0(e^{j\omega T})$ is the amplitude function, N is the window length, and T is called sample period. $A(\omega) = |W_0 e^{jwT}|$ and $\theta(w) = -w(N-1)T/2$ are amplitude and phase spectrum of window at equation (1), respectively. A typical window's frequency spectrum in dB range can be shown as Figure 1. Normalized amplitude spectrum of window [12],

$$W_N\ (e^{jwT})|=20\log_{10}(|A(w)|/|A(w)|_{max}) \quad (2)$$

Spectral parameters that determine the performance of the windows are mainlobe width ($\omega_M$), null-to-null width ($\omega_N$), ripple ratio (R) and sidelobe roll-off ratio (S).

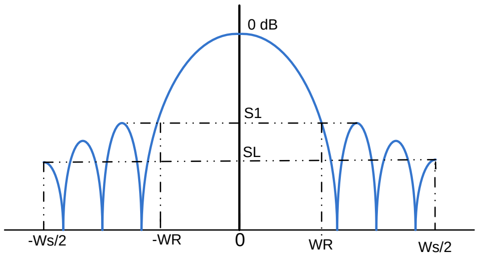


Figure 1. Window's spectrum and spectral characteristic

These parameters can be defined as below from window amplitude spectrum [12].
$\omega_M$ = two times half mainlobe width $= 2\omega_R$
R = maximum sidelobe amplitude in dB – mainlobe amplitude in dB = *S1*
S = maximum sidelobe amplitude in dB – minimum sidelobe amplitude in dB = *S1 – SL*
The desired properties at window amplitude spectrum are narrow mainlobe width, small ripple ratio and large sidelobe roll-off ratio.
Mainlobe width determines the transition width between the passband and stopband, the ripple ratio affects the ripples in the passband and stopband and the side-lobe roll-off ratio determines the distribution of stopband energy.

## III. Window Functions

In literature, the different windows have been proposed both digital filter design and other different applications. The improved windows in literature are two types as fixed and adjustable according to their parameters.

### A. *Kaiser Window*

Kaiser window equation in discrete time is defined by

$$w[n]=\begin{cases}\dfrac{I_0\left(\alpha_k\sqrt{1-\left(\frac{2n}{N-1}\right)^2}\right)}{I_0(\alpha_k)} & |n|\le\dfrac{N-1}{2}\\ 0 & otherwise\end{cases} \quad (3)$$

where $\alpha_k$ is the adjustable parameter, and $I_0(x)$ is the modified Bessel function of the first kind of order zero which can be described by power series expansion as [12].

$$I_0(x)=1+\sum_{k=1}^{\infty}\left[\frac{1}{k!}\left(\frac{x}{2}\right)^k\right]^2 \quad (4)$$

An increase in $\alpha_k$ at Kaiser window causes wider mainlobe width and smaller ripple ratio.

### B. *Ultraspherical Window*

The coefficients of Ultraspherical window of length N can be calculated with [14].

$$w_u(nT)=\frac{A}{p-n}\binom{\mu+p-n-1}{p-n-1}\sum_{m=0}^{n}\binom{\mu+n-1}{n-m}\binom{p-n}{m}B^m \quad \text{for } n=0,1,\ldots,N-1 \quad (5)$$

where

$$A=\begin{cases}\mu x_\mu^p & \mu\neq 0\\ x_\mu^p & \mu=0\end{cases},$$
$$B=1-x_\mu^{-2}, \quad (6)$$
$$p=N-1$$

Ultraspherical window has three independent parameters, $\mu$, $x_\mu$ and $N$. With appropriate selection of these variables, Ultraspherical window control side-lobe roll-off ratio, ripple ratio and mainlobe width which are window spectral parameters. Parameter $\mu$ alters the side-lobe roll-off ratio, $x_\mu$ provides a trade-off between the ripple ratio and a width characteristic, and N allows different ripple ratios to be obtained for a fixed width characteristic and vice versa [14].
Normalized Ultraspherical window can be obtained as below.

$$\hat{w}(nT)=w_u(nT)/w(CT) \quad (7)$$

$$C=\begin{cases}\dfrac{(N-1)}{2} & for\ odd\ N\\ (N/2)-1 & for\ even\ N\end{cases} \quad (8)$$

The binomial coefficients at Equation 5 can be defined as

$$\binom{\alpha}{0}=1,\ \binom{\alpha}{p}=\frac{\alpha(\alpha-1)\ldots(\alpha-p+1)}{p!} \quad for\ \ p\ge 1 \quad (9)$$

The amplitude function of window is given by

$$W_0(e^{jwT})=C_{N-1}^{\mu}\left[x_\mu\cos\left(\frac{wT}{2}\right)\right] \quad (10)$$

where $C_n^\mu(x)$ is Ultraspherical polynomial and it can be found by using recurrence relationship given in (11).

$$C_r^\mu(x)=\frac{1}{r}\left[2x(r+\mu-1)C_{r-1}^\mu(x)-(r+2\mu-2)C_{r-2}^\mu(x)\right] \quad for\ r=2,3,\ldots,n \quad (11)$$

where $C_0^\mu(x)=1$ and $C_1^\mu(x)=2\mu x$

## IV. Neural Networks

ANNs are used at many application areas due to specifications such as recognition, learning and decision making. ANNs are different types according to learning methods, connection structure and architecture. Due to connection structures, ANNs are two types as kind of feed-forward ANN and back-forward ANN. At back-forward ANNs, because of the fact that the connections have loops, the data get result by processing. Thus, learning time is so long at this network.

ANNs are implemented with different layer and different neuron values in layers. While the neuron values in layers used in ANN structure raise successful, that decrease the time reaching the result.

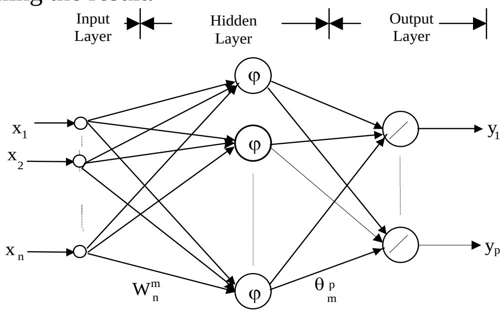


Figure 2. A three layer ANN structure

## V. Analyses of ANN and Comparison Examples

In this study, the feed forward back-propagation network model was used. This method is the most popular network structure in literature [24]. The layer values and the neuron values in layers used for neural network are determined with trial-error method. The best structure is network with three-layer.

At work, Ultraspherical's parameters $\mu$, $x_\mu$, Kaiser's parameter $\alpha_k$ and filter length N were used as inputs to the ANN model. The output layer is the coefficients of new window containing together amplitude response of Kaiser and Ultraspherical windows. At work, 3000 data are used for training and testing of ANN. Sixty-five percentage of these data was used for learning of ANN and thirty-five percentage of that was used for testing process. These data are the most suitable values for training and testing of ANN due to trial-error method.

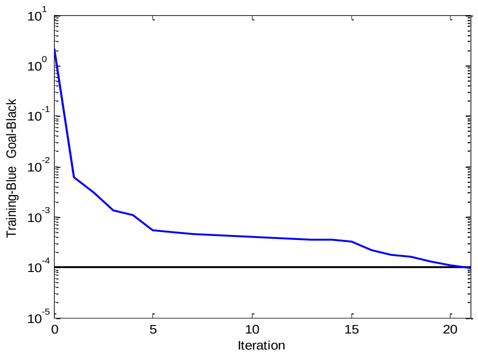


Figure 3. The training error graph for the ANN model

At work, a new window having together good characteristic of Kaiser and Ultraspherical windows was obtained with an alternative method which is elimination the difficulty of combining the equations of those windows. The coefficients of the improved this window were calculated by using of ANN. The obtained window has better spectral parameters than windows common used in literature.

***Example 1.*** Kaiser, Ultraspherical and the obtained results from the improved method in MATLAB (license number: 585775) by using of ANN for large mainlobe width N=21 are shown in Figure 4, and the corresponding data are given at Table I.

Kaiser window has best ripple ratio as seen at Figure 4 and Table I. The improved method by using of ANN has best characteristic in term of side-lobe roll-off ratio.

TABLE I. THE OBTAINED DATA FOR IMPROVED METHOD, KAISER AND ULTRASPHERICAL WINDOWS

| Parameter | Window type | | |
|---|---|---|---|
| | ANN | Ultraspherical | Kaiser |
| N | 21 | 21 | 21 |
| $\alpha_k$ | - | - | 3.95 |
| $w_R$ | 0.509 | 0.478 | 0.478 |
| R (dB) | 31.39 | 29.44 | 31.58 |
| S (dB) | -29.81 | -27.15 | -15.17 |

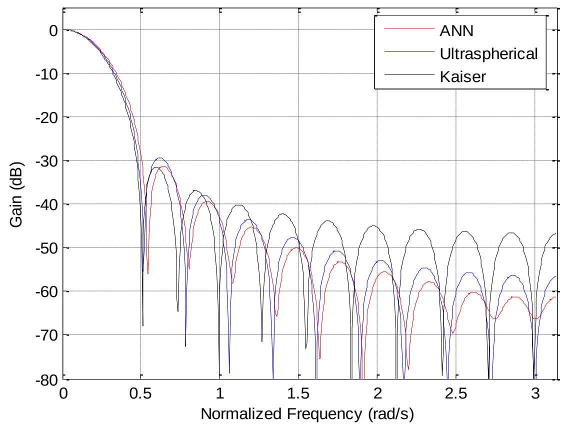


Figure 4. The amplitude comparison of the ANN, Kaiser and Ultraspherical window for N=21

But, mainlobe width characteristic of the improved method is bad. Kaiser window has better ripple ratio than improved method, but the improved method has best side-lobe roll-off ratio and it provides as well as better ripple ratio and much better side-lobe roll-off ratio than the Ultraspherical window. The Ultraspherical window parameters for this example $\mu$= 2.503 and $x_\mu$= 1.00752

***Example 2.*** In this example, the spectrum comparison for exponential window has been done. The success of improved method for narrow mainlobe width and different window length was compared with windows much preferred in literature. The simulation results for same mainlobe width and window length (N=27 and $w_R$=0.227) were illustrated at Figure 5 and corresponding data was given at Table II.

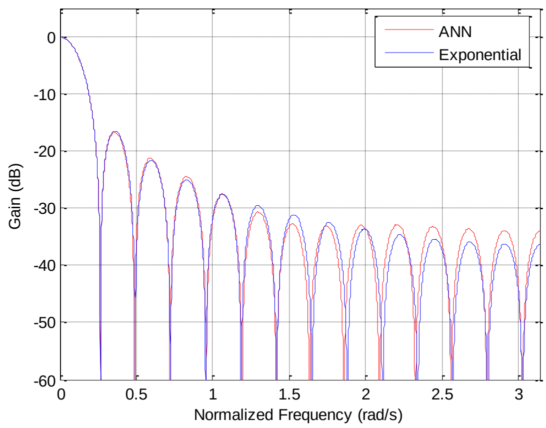


Figure 5. The amplitude comparison of ANN and exponential window for N=27 and $w_R$= 0.227

TABLE II. THE OBTAINED DATA FOR IMPROVED METHOD AND EXPONENTIAL WINDOW

| Parameter | Window type | |
|---|---|---|
| | ANN | Exponential |
| N | 27 | 27 |
| $\alpha_e$ | - | 0.77 |
| $w_R$ | 0.227 | 0.227 |
| R (dB) | -16.7 | -16.45 |
| S (dB) | 17.32 | 19.9 |

Figure 5 and Table II show that ANN has much better ripple ratio characteristic than exponential window for a larger window length and narrower mainlobe width. But, exponential window provides better side-lobe roll-off ratio than ANN model.

***Example*** 3. In this example, the comparison of spectrum for same window length and mainlobe width ($w_R$=0.327) has been performed with cosh window [12] that based on Kaiser window and which has two independent parameters. The results of the simulation are shown at Figure 6 and the corresponding data are given in Table III.

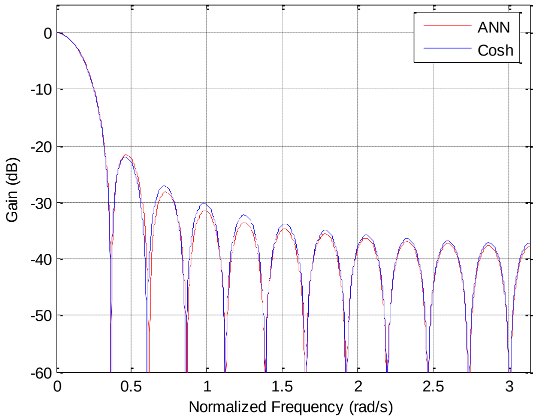


Figure 6. The amplitude comparison of ANN and Cosh window for N=23, $w_R$= 0.327

TABLE III. THE OBTAINED DATA FOR IMPROVED METHOD AND COSH WINDOW

| Parameter | Window type | |
|---|---|---|
| | ANN | Cosh |
| N | 23 | 23 |
| $\alpha_e$ | - | 1.97 |
| $w_R$ | 0.327 | 0.327 |
| R (dB) | -21.61 | -21.97 |
| S (dB) | 16.26 | 15.22 |

Figure 6 and Table III show that the obtained results from ANN have better side-lobe roll-off ratio characteristic than cosh window. But, the ripple ratio of cosh window is a little better than results from ANN model.

# VI. Discussion and Conclusion

At work, the using of ANN at design of a new window which has together characteristic of two windows preferred due to good properties in literature was proposed. The obtaining the design equation of window which provides together spectral characteristics of the Kaiser and Ultraspherical windows having good properties is very difficult. Thus, the using of ANN which is an alternative method to eliminations this difficulty was proposed.

Hence, a new method to design of window having useful spectral parameters has been improved. ANN used in method has eliminated difficulty at design of a new window function containing together side-lobe roll-off ratio characteristic of Ultraspherical window and Kaiser window's main-lobe width and ripple ratio. 3000 data for simulation of ANN were used. The Ultraspherical window's parameters, $\mu$, $x_\mu$, Kaiser's parameter $\alpha k$ and filter length N were used as inputs to the ANN model. The output layer is coefficients of new window containing together amplitude response of Kaiser and Ultraspherical windows. The obtained results show that the improved method with ANN has better characteristic than Kaiser and Ultraspherical windows. Hence, the neural computing technique could be employed successfully in modeling of window having useful spectral parameters.

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