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Analytical model of overlapped FRP bars embedded in Concrete under high temperature

[Hizia Bellakehal, Ali Zaidi]

Abstract*—Fiber reinforced polymer (FRP) bars have become progressively more used in civil engineering because of their excellent properties with respect to steel bars, mainly, their corrosion resistance. The designers generally adopt over-reinforced sections to avoid a sudden failure mode of FRP bars. Therefore, the overlap of the bars is omnipresent. However, under high temperature, the area of overlap may be regarded as a zone of radial stresses concentration generated by pressure caused by the thermal expansion of both bars. The aim of this study is to develop an analytical model allows to investigate the thermal stress and strain distributions in FRP bars and in concrete of beams reinforced with two overlapped FRP bars. By varying the ratio of concrete cover thickness to FRP bar diameter (c/d_b) from 1 to 3.2 and the temperature variation from 0°C to 60°C. The analytical model is based on the linear elasticity. Analytical results are compared with those obtained from numerical model using ADINA finite elements software. The results predicted from the proposed model are in good agreement with numerical results.

***Keywords*—Analytical model, Numerical simulation, Temperature, FRP bars, concrete cover thickness, Transverse stresses.**

I. Introduction

During the 1970s, composites of Fiber Reinforced Polymers (FRP) were introduced in civil engineering constructions sector. They become as one of the most effective solutions to the steel corrosion problem because of their high resistance to the corrosion and their distinguished mechanical and physical properties (EL-Zaroug et al. 2007). Due to the lack of ductility and high deformability of concrete sections reinforced with FRP bars compared to those reinforced with steel bars, all concrete sections reinforced with FRP bars must be designed so that the rupture of the section will take place by crushing of concrete in compression (CAN/CSA–S806–2012). This means that the concrete sections must be over-reinforced. To satisfy this requirement, the overlap of bars is generally omnipresent in FRP bars-reinforced concrete sections. Under high temperatures, the region of overlapped bars may be regarded as a zone of tensile stresses concentration generated by the radial pressure caused by the transverse thermal expansion of the overlapped bars. It should be noted that the main drawback of FRP bar is the lack of thermal compatibility between the concrete and FRP bars, particularly in the transverse direction. These tensile stresses may cause splitting cracks within concrete and, eventually, degradation of the member stiffness.

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As a consequence of the appearance of the first micro-cracking in concrete, important thermal strains take place when the thermal stress in the concrete around the glass FRP (GFRP) bars, in different locations, reaches its tensile strength (f_t). These induced thermal cracks cause loss of the bond between GFRP bar and the surrounding concrete and, eventually, the failure of the concrete cover if the confining action of concrete is not sufficient (Masmoudi et al. 2005, Galati et al. 2006, EL-Zaroug et al. 2007, Zaidi et al. 2008, Zaidi et al. 2015). The aim of this study is to develop an analytical model based on linear elasticity theory, to investigate the effect of vertical overlap FRP bars anchored in the concrete on the transverse tensile stresses and strains of prismatic reinforced concrete beams under high temperatures. These stresses permit to evaluate the temperature variation producing the first crack in concrete (ΔT_{cr}) at FRP bar/concrete interface. Also, to evaluate the temperature variation producing the total failure of concrete cover (ΔT_{sp}) as a function of concrete cover thickness to FRP bar diameter ratio (c/d_b). The analytical results are compared with those obtained by numerical simulation using ADINA nonlinear analysis software.

II. Numerical Model

The numerical results were obtained by Bellakehal et al. (2015). The numerical model is established using the finite element method, using ADINA software to study the effect of vertical overlap of two GFRP bars on the behavior of reinforced concrete beams under high temperature. This study analyzes the transverse thermal stresses in the concrete cover and FRP bars by varying the temperature variation (ΔT) from 0 to +60°C, and the concrete cover thickness to FRP bar diameter ratio (c/d_b). Also, it allows to determine the temperature variation (ΔT_{cr}) producing the first crack in the concrete at the interface of FRP bar/concrete in the interaction zone of both FRP bars, and the temperature variation producing the failure of concrete cover (ΔT_{sp}) as a function of the ratio (c/d_b). All beams have a constant length of 380 mm and a variable transverse section (b' x h). Details of beams are presented in Table 1 and Fig. 1. The selected concrete covers were chosen to obtain a wide range of concrete cover thickness to FRP bar diameter (c/d_b) ratio ranging from 1.0 to 3.2.

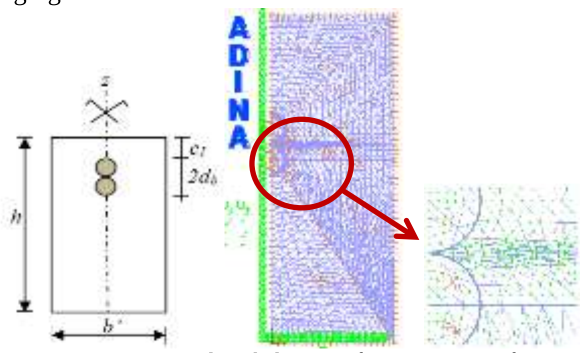


Figure 1. Geometry and mesh diagram of a cross section of concrete beam reinforced with two GFRP bars.

Table 1: Geometrical and mechanical properties of beams.

Beam designation	Width b' (mm)	Height h (mm)	Bar diameter d_b (mm)	Concrete cover c (mm)	c/d_b	f_{fu} (MPa)	E_l (GPa)	ε_{lu} (%)
P.#10.20*	76	100	9.5	20	2.1	627	42	1.8
P.#10.25	76	100	9.5	25	2.6	627	42	1.8
P.#10.30	76	100	9.5	30	3.2	627	42	1.8
P.#13.20	76	100	12.7	20	1.6	617	42	1.5
P.#13.25	76	100	12.7	25	2.0	617	42	1.5
P.#13.30	76	100	12.7	30	2.5	617	42	1.5
P.#16.20	76	100	15.9	20	1.3	535	42	1.4
P.#16.25	76	100	15.9	25	1.6	535	42	1.4
P.#16.30	76	100	15.9	30	1.9	535	42	1.4
P.#19.20	100	125	19.1	20	1.0	600	40	1.5
P.#19.25	100	125	19.1	25	1.3	600	40	1.5
P.#19.30	100	125	19.1	30	1.6	600	40	1.5
P.#25.25	100	150	25.4	25	1.0	N/a	N/a	N/a
P.#25.30	100	150	25.4	30	1.2	N/a	N/a	N/a
P.#25.35	100	150	25.4	35	1.4	N/a	N/a	N/a

In view of the fact that the axial strains of the beams are constant, beams were modeled by means of two dimensional plane stress elements, using triangular finite elements of six nodes (Fig. 1). As shown in Fig. 1, the study was carried out only for the half cross section of beams since the beams are symmetric with respect to z-z axis. The temperature variation was increased from 0 to 60°C with an increment of 5°C. The mechanical and physical properties of concrete such compression strength f_{c28}, tensile strength f_{ct28}, Young's modulus E_c, the coefficient of thermal expansion α_c, concrete density γ_c, and Poisson's ratio ν_c are respectively equal to 40 MPa, 4.1 MPa, 28 GPa, 1.16. 10^{-5}/°C, 2.4 g/cm^3 and 0.17. The Poisson's ratio in the longitudinal direction ν_{lt}, Poisson's ratio in the transverse direction ν_{tt}, and the transverse modulus of elasticity E_t, of GFRP bars are, respectively, equal to 0.28, 0.38 and 7.1 GPa. The other mechanical properties such as ultimate tensile strength f_{fu}, the longitudinal modulus of elasticity E_l, and the ultimate tensile strain ε_{lu}, are shown in Table 1.

III. Analytical Model

The difference between coefficients of thermal expansion of FRP bars and concrete produces a radial pressure P at *FRP bars/concrete* interface of concrete beams reinforced with FRP bars under thermal loads. The analytical model is based on the linear elasticity theory (Timoshenko et al. 1970) for a hollow cylinder under internal hydrostatic pressure *P*, by considering the in-plane stresses state (Aiello et al. 2001, Zaidi et al. 2008). The radial stress (σ_ρ) and the circumferential stress (σ_t) in a concrete element situated at a radius (ρ) from the center of the concrete cylinder due to the radial pressure (P) are given by (Fig. 2):

$$\sigma_\rho = P\,(1 - b^2/\rho^2)\,/\,(r^2 - 1), \quad (1)$$

$$\sigma_t = P\,(1 + b^2/\rho^2)\,/\,(r^2 - 1). \quad (2)$$

Where:

$$P = (\alpha_{ft} - \alpha_c)\,\Delta T/[(1/E_c)(\beta + \nu_c) + (1/E_t)(1 - \nu_{tt})], \quad (3)$$

Where : $\beta = (r^2 + 1)/\,(r^2 - 1)$;

$r = b/a$; and $b = c + a$

a: radius of FRP bar, *c*: thickness of concrete cover, α_c: coefficient of thermal expansion of concrete, α_{ft}: transverse coefficient of thermal expansion of FRP bar, E_c: modulus of elasticity of concrete, E_t: modulus of elasticity of FRP bar in transverse direction, ν_c: Poisson's ratio of concrete, ν_{tt}: Poisson's ratio of FRP bar in transverse direction.

The maximum circumferential stress at the concrete/bar interface ($\rho = a$) due to the radial pressure, obtained from the equation 2, is given by:

$$\sigma_{t,max} = \beta P. \quad (4)$$

The transverse tensile thermal strains in FRP bar ε_{ft} at FRP bar/concrete interface and the transverse tensile thermal strains of concrete ε_{ct} at the external surface of concrete cover of slabs, due to the radial pressure P and the temperature variation ΔT, are given by the following equations :

$$\varepsilon_{ft}\,(a) = \alpha_{ft}\,\Delta T - P\,(1 - \nu_{tt})\,/\,E_t\,, \quad (5)$$

$$\varepsilon_{ct}\,(b) = \alpha_c\,\Delta T + 2P/\,(r^2 - 1)E_t. \quad (6)$$

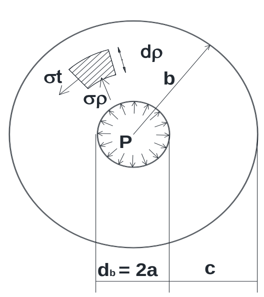
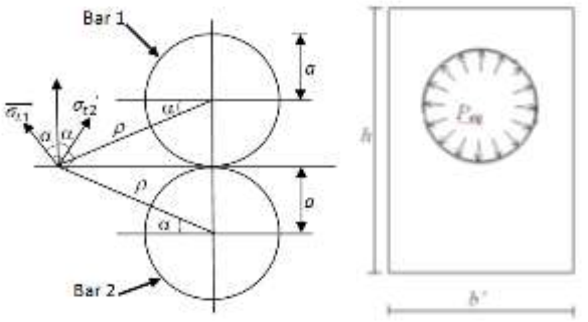


Figure 2. Concrete cylinder concentrically reinforced with FRP bar.



(a) **(b)**

Figure 3: Prismatic concrete beam reinforced with two FRP bars: **(a)** Schema of the overlapped bars stresses. **(b)** The cross section replacing the overlapped bars section by one equivalent FRP bar section.

In the case of prismatic concrete beams reinforced with two overlapped FRP bars (Fig. 3a), the concrete situated in the interaction line of bars is submit to the dual action of radial pressure of both FRP bars. To consider this effect, the cross section of beam is simulated by concrete section reinforced by one equivalent FRP bar, whose cross section equal two times the section of the considered FRP bar, and which generated the equivalent radial pressure, $P_{eq} = 2P$ (Fig. 3b). In this case, the equivalent radial pressure is given by :

$$P_{eq} = 2(\alpha_{ft} - \alpha_c)\,\Delta T/[(1/E_c)(\beta_{eq} + v_c) + (1/E_t)(1 - v_{tt})], \quad (7)$$

Where: $\beta_{eq} = (r^2_{eq} + 1)/\,(r^2_{eq} - 1)$;
$r_{eq} = (2c + d_{eq})/\,d_{eq}$; $d_{eq} = \sqrt{2}\, d_b$

The maximum circumferential tensile stress at the interface FRP bar/concrete of the equivalent section (which represent that at the interaction zone) due to the radial pressure P_{eq}, is given by :

$$\sigma_{tmax,eq} = \beta_{eq}\, P_{eq}\,. \quad (8)$$

The transverse tensile thermal strains in FRP bar $\mathcal{E}_{ft}$ at FRP bar/concrete interface and the transverse tensile thermal strains of concrete $\mathcal{E}_{ct}$ at the external surface of concrete cover of slabs, due to the radial pressure P_{eq} and the temperature variation ΔT, are given by the following equations :

$$\mathcal{E}_{ft}\,(a) = \alpha_{ft}\,\Delta T - P_{eq}\,(1 - v_{tt})\,/\,E_t, \quad (9)$$

$$\mathcal{E}_{ct}\,(b) = \alpha_c\,\Delta T + 2\,P_{eq}\;/\,(r^2 - 1)E_t. \quad (10)$$

The first radial crack appears in the concrete at the interface FRP bar/concrete when the circumferential stress reaches the tensile strength of the concrete ($\sigma_{t\ max,eq} = f_{ct}$). From (7) and (8) we can get (11), which represent the temperature variation producing the first radial crack in concrete for beams reinforced with overlapped bars:

$$\Delta T_{cr} = f_{ct}[(1/E_c) + \{(v_c/E_c) + (1/E_t)(1 - v_{tt})\}/\beta]/2(\alpha_{ft} - \alpha_c) \quad (11)$$

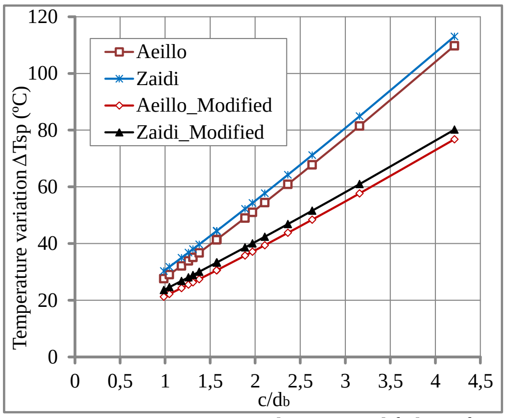


Figure 4. Temperature variations producing a total failure of concrete cover of overlapped and non-overlapped bars sections.

Aiello (2001) was proposed an analytical model to predict the temperature variation producing the failure of concrete cover ΔT_{sp}, for concrete cylinder axially reinforced with FRP bar, this model is given by (12). In the case of concrete beams reinforced with two overlapped bars, Aiello's model was modified to estimate the effect of overlapping bars on the temperature variation producing a total failure of concrete cover, this model is given by (13):

$$\Delta T_{sp} = \frac{0.3\ f_{ct}\ r}{\alpha_{ft} - \alpha_c}\left[\frac{1}{E_c}\left[\ln(0.48\ r) + 1.6 + v_c\right] + \frac{1}{E_t}(1 - v_{tt})\right] \quad (12)$$

$$\Delta T_{sp,eq} = \frac{0.3\ f_{ct}\ r_{eq}}{\alpha_t - \alpha_c}\left[\frac{1}{E_c}\left[\ln(0.48\ r_{eq}) + 1.6 + v_c\right] + \frac{1}{E_t}(1 - v_{tt})\right] \quad (13)$$

Another model was proposed by Zaidi (2008) to predict the temperature variation producing the failure of concrete cover ΔT_{sp}, for prismatic concrete beams reinforced with FRP bars not overlapped, this model is given by (14). Also to predict the temperature variation producing the failure of concrete cover of beams reinforced with two overlapped bars, zaidi's model was modified as follow (15):

$$\Delta T_{sp} = \frac{0.23 f_{ct}\ r}{(\alpha_t - \alpha_c)}\left[\frac{1}{E_c}\left[\ln(0.72\ r) + 3.2 + v_c\right] + \frac{1}{E_t}(1 - v_{tt})\right] \quad (14)$$

$$\Delta T_{sp,eq} = \frac{0.23 f_{ct}\ r_{eq}}{(\alpha_t - \alpha_c)}\left[\frac{1}{E_c}\left[\ln(0.72\ r_{eq}) + 3.2 + v_c\right] + \frac{1}{E_t}(1 - v_{tt})\right] \quad (15)$$

Fig. 4 exhibits the temperature variation producing a total failure of concrete cover (ΔT_{sp}) as a function of concrete cover thickness to FRP bar diameter ratio (c/d_b). This figure compared ΔT_{sp} of sections reinforced with non-overlapped bars, predicted by Aiello's and Zaidi's model, and that of sections reinforced by two overlapped bars predicted by the modified Aiello's and Zaidi's model. It can be seen that ΔT_{sp} of non-overlapped bars section is higher than that of the overlapped bars section. This is due to the effect of the dual action of radial pressure exerted by both FRP bars. This effect increases with the increase of c/d_b ratio. It can be observed also, that Aiello's model, for overlapped and non-overlapped bars section, underestimates ΔT_{sp} by 3 to 9% compared to that predicted by Zaidi's model for c/d_b ratio varying from 4.0 to 1.0, respectively. This is explained by the fact that Aiello's model is based on the behavior of concrete cylinder axially reinforced with FRP bar in which the concrete confinement is constant. However,

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Zaidi's model is based on the behavior of FRP bars-reinforced concrete beams in which concrete cover thickness is not constant, which gives a biggest concrete confinement.

IV. Comparison of Analytical and Numerical Results

A. *Temperature variation producing the first crack*

Fig. 5 shows a comparison between analytical and numerical results, in terms of the temperature variation producing the first radial crack in concrete (ΔT_{cr}) at FRP bar/concrete interface as a function of concrete cover thickness to FRP bar diameter ratio (c/d_b). It can be observed that the first cracks, for concrete beams reinforced by two overlapped bars, appear at a temperature variation of 11°C ± 1°C (i.e. at a temperature of 31°C since the reference temperature is 20°C). As shown in Fig. 6, the first cracks appear in bars interaction zone (Fig. 6a), then in the outer zone at FRP bar/concrete interface for a temperature variation (ΔT_{cr}) around 25°C (Fig. 6b).

From fig. 5, it can be observed that $\boldsymbol{\Delta T_{cr}}$ obtained from the linear elastic analysis are very close to those predicted from the finite element analysis. Consequently, it can be concluded that the approach supposing the radial pressure in the interaction zone of overlapped bars equal two times the radial pressure generated by one bar in FRP bar/concrete interface; and that supposing the radial pressure in the interaction zone of overlapped bars equal the radial pressure generated by one bar having a cross section equal that of both overlapped bars, is a significant approach. Nevertheless, it can be seen, that $\boldsymbol{\Delta T_{cr}}$ given by the numerical model are almost constant. This is due to the fact that the thermal load is applied on the whole surface of the cross section. So the thermal strains at FRP bar/concrete interface are independent to the concrete cover thickness. It should be noted that the analytical formula of $\boldsymbol{\Delta T_{cr}}$ (11) increase with the increase of $[(r^2 - 1)/(r^2 + 1)]$ ratio and c/d_b ratio.

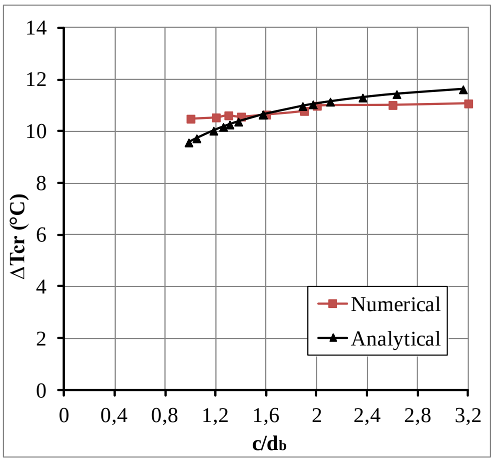

Figure 5: Temperature variation producing the first radial crack in concrete at FRP bar/concrete interface of beams reinforced with two overlapped bars.

(a) Time increment 3. **(b)** : Time increment 5.

Figure 6: Temperature producing first radial cracks in concrete at FRP bar/concrete interface for the beam P.#13.20: **(a)** in bars interaction zone, **(b)** in FRP bar/concrete interface zone.

B. *Temperature variation producing the failure of concrete cover*

Fig. 7 present a comparison between the temperature variations producing the total failure of concrete cover (ΔT_{sp}) predicted from the numerical model, for concrete beams reinforced with two overlapped GFRP bars, and those obtained from the modified equations of Aiello and Zaidi. It can be seen that the theoretical values of ΔT_{sp} increase with increasing c/d_b ratio. Furthermore, the nonlinear numerical results are higher than those predicted by the analytical models. This difference varies between 30 to 40% with a variation of c/d_b ratio from 1.9 to 1.0, respectively. This gap is due to the fact that the numerical model takes into account the contribution of no cracked elements. This is primarily due to the fact that the finite element analysis distributes the stresses more evenly over the concrete cover causing lower circumferential stresses within concrete. Consequently, it can be concluded that the analytical model underestimate the values of ΔT_{sp}. It can be seen that the numerical results doesn't overstep 60°C for ratios $c/d_b \geq 2$, because the total failure of concrete cover was not observed for these beams as illustrated in Fig. 8b.

Fig. 8 illustrates cracks propagation in concrete cover of beams P.#13.20 and P.#13.30. Fig. 8a shows that the beam P.#13.20 had a complete cracking of concrete cover at a temperature variation ΔT_{sp} = 55°C. However, for beam the P.#13.30, the complete failure of concrete cover was not observed until ΔT = 60°C (Fig. 8b).

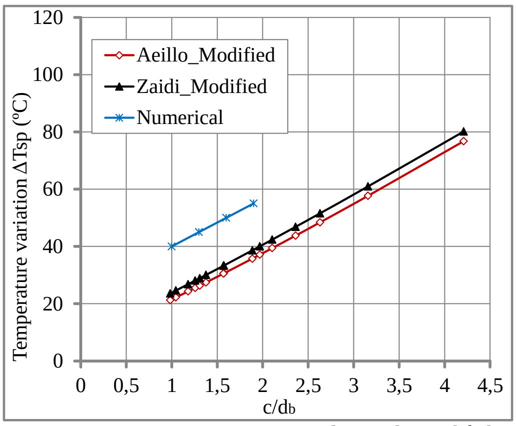

Figure 7: Temperature variations producing the total failure of concrete cover (ΔT_{sp}) for concrete beams reinforced with two overlapped bars.

(a) c/d_b =1.6 : ΔT_{sp} = 55°C, **(b)** c/d_b = 2.5 : ΔT = 60°C.

Figure 8: Cracks propagation in concrete cover: **(a)** Total Cracking of concrete cover of beam P.#13.20 **(b)** Partial cracking of beam P.#13.30.

C. Transverse thermal stresses

Fig. 9 presents a typical comparison of analytical and numerical curves in terms of transverse tensile stresses at FRP bar/concrete interface in bars interaction zone, for prismatic concrete beams reinforced by two GFRP bars overlapped vertically, subjected to high temperatures, and having a ratio of concrete cover thickness to bar diameter (c/d_b) equal to 2.6 and 1.4. It can be seen that the curves obtained by the analytical model based on the theory of elasticity (8) vary linearly. However, those obtained by the nonlinear numerical model exhibit a peak whose corresponds to the tensile strength of concrete (f_{ct} = 4.1MPa) and the thermal load producing the first radial cracks in concrete (≈11°C). After this peak the tensile stresses of concrete decrease progressively with temperature increasing due to crack development. So, the numerical and analytical results are in good agreement before occurrence of cracks. However, after the cracking of concrete the results of both models show a considerable gap due to the crack appearance which has not been considered in the analytical model. This is attributed to the inclusion of nonlinear concrete material behavior in the finite element analysis. From these figures it is also observed that the first crack occurred in the concrete at FRP bar/concrete interface when the circumferential tensile stress of concrete reaches the tensile strength of concrete (f_{ct}) whose value is 4.1 MPa. Note that there is no remarkable effect of c/d_b ratio on the temperature variation ΔT_{cr}.

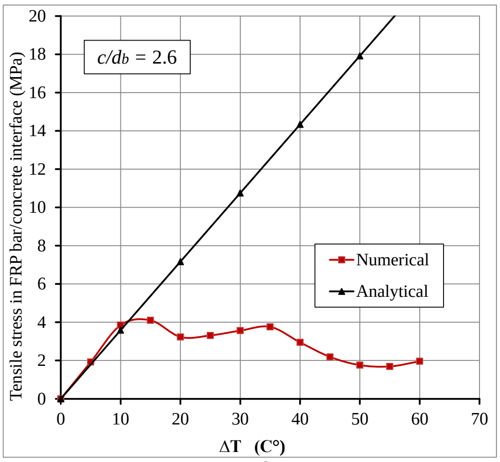

Figure 9: Transverse tensile stress in concrete at FRP bar/concrete interface of beam for beam P.#25.10, having c/d_b=2.6.

CONCLUSIONS

- The proposed analytical model is in good agreement with the nonlinear numerical results obtained from ADINA software, in terms of temperature producing a first radial cracks, thermal tensile stresses and transverse thermal strains. It can be concluded that approach adopted by the analytical model is significant. However, beyond cracks formation the numerical results are widely higher. This is attributed to the inclusion of nonlinear concrete material behavior in the finite element analysis, contrary to the analytical model which is based on the elastic linear theory. Furthermore, the analytical model underestimates the temperature producing the total failure of concrete cover.
- The bars interaction zone develops within concrete high tensile stresses generated by the dual pressure exerted by both GFRP bars under high temperature.
- According to the analytical and numerical models, the first cracks appear at FRP bar/concrete interface in bars interaction zone at a temperature (T_{cr}) around 31°C. Then they appear in the outer zone at FRP bar/concrete interface at a temperature (T_{cr}) around 45°C.
- The reduction of c/d_b ratio reduces slightly the temperature producing the first cracks (T_{cr}), but affect significantly the temperature produces a complete failure of concrete cover (T_{sp}).

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