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Atmospheric Perturbations and Longitudinal Motion Analysis of an Aircraft Using SIMULINK

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Abstract— **This paper encompasses the longitudinal dynamics of a Beechcraft King Air 350 aircraft with atmospheric perturbation. Longitudinal simulations with and without integrator states are also discussed. The Linear Quadratic Regulator (LQR) technique showed robustness and it is possible to correct the error in most of the situations. Matlab-Simulink is used as a tool for simulation work.**

***Keywords*—— Longitudinal dynamics, Aircraft, Atmospheric Perturbations, Simulink**

I. Introduction

This project presents a study on dynamic systems and its control techniques. The vehicle used in this project was the *Beechcraft King Air 350* aircraft shown in figure (1). The objective of this work is to design a controller which offers good disturbance rejection and tracking properties. Beechcraft King Air 350 data sheet specifications such as all the flight and actuators specifications, the inertia and wing data's and the aerodynamics derivatives are given below. The following specifications are used for MATLAB/SIMULINK for simulations. Beech 350:Flight condition; h=9753m, M=0.58, gg0 = 0.00deg, aa0=1.29deg, u0=338.1kt, flapa=0 deg **Throttle:** tho=98%, Teng = 0.91s; demax =+ 16/-15 deg, dmax=18deg, drmax=30deg **Inertial data**.:- m=6177Kg, Ix=66942kgm^2 Iy= 67819kgm^2, Iz=94947kgm^2 ,Ixz=3300kgm^2. **Wingdata**:S=28.8m^2 b=17.424m,c=1.651m, aamax=C=15.18 deg. **Aerodynamic coefficients in SI unites**):Xu= - 0.0161, Xw=0.0165, Zu= - 0.1134, Zw=-1.0541,Zwp =-0.0035, Zq = -1.4994, mu=0.0000, mw= -0.0729, mq= -0.2096,mwp= -0.0004, ybb= -17.2335, lbb= -4.7286, nbb=5.0008, yp=0.0000, lp=-8981, np= - 0.0517,yr = 0.5216, l r=0.3549, nr=-2085, xde=0.000, zde=-11.719,mde= -5.846, xdf= - -2.412, zdf=-20.276, mdf= - 0.453,xdt-1.335, mdt=-0.019, Lda=8.006, Nda=0.000, ydr = -2.750, Ldr = -0.733, Ndr = -1.717, $u_0\alpha_0 = 3.9161$, g = 9.8 $Mu + M_{\dot{\omega}} Z_u = 0.1134$, $M_\omega + M_{\dot{\omega}} Z_\omega = 1.0541$, $Mq + M_{\dot{\omega}} U_0 = 173.9337$ $M_{\delta E} + M_{\dot{w}} Z_{\delta E} = 5.8553$, $M_{\delta T} + M_{\dot{w}} Z_{\delta T} = 0.019$

The paper is organized as follows, in section II, a review of the control theory used in this work is presented. Section III introduces the aircraft control theory and main concepts of this work. Longitudinal motion simulations are shown in section IV.

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Fig 1: Beechcraft King Air 350

Conclusions, the results of SIMULINK simulations and the designed controller actions are shown in section V.

II. Control Theory

The systems dynamics can then be expressed by the following general expression for a non-autonomous (time-varying), nonlinear system of order *n*, with *m* inputs and *p* outputs:

$$\dot{x} = F(x,u,t) \quad (1)$$

$$y = h(x,u,t) \quad (2)$$

The state *x* is, in *n*-dimensional space. The time instant *t* is a scalar. The control *u* is a *m*-dimensional space The output *y* is a *p*-dimensional space. Obviously *x*, *y* and probably *u* are functions of time, but the variable *t* is explicitly used to represent the time-dependency of certain system parameters that affect the dynamics (such as mass, temperature or general disturbances); if the systems parameters remain unchanged, then it is said to be autonomous, and *t* does not appear on the equations. For this work, the variable t will not appear on the equations, under the assumption that even if those parameters change, those changes are slow enough that their effect on the dynamics will be minimal. Usually the output of a system doesn't vary instantly with its control signal, being only a function of the state vector (possibly including a selection of some state variables): $y = h(x)$ (3) However, the simplest class to analyze comprehends the *LTI* (Linear and Time-Invariant) systems, with no dependency on time and only linear relations on the dynamics, so that they can be represented by the equation

$$\dot{x} = Ax + Bu \quad (4)$$

$$y = Cx + Du \quad (5)$$

Where *A*, *B*, *C* and *D* are constant matrices, their dimensions being respectively: *n*×*n*, *n*×*m*, *p*×*n* and *p*×*m*. This type of dynamical systems has been thoroughly studied and its properties permit an easy understanding of the system, as well as practical ways to design a controller given the desired performance [1]. The systems that we analyzed describes the motion equations of the aircraft (longitudinal dynamics) with wings level and rectilinear flight. Those equations are linearized around an equilibrium point in absents of perturbations or noise [2].

III. Aircraft Dynamics

The mathematical model of longitudinal dynamics with and without atmospheric perturbations, are explained in this section.

A. *Longitudinal dynamics model*

The system matrix and input matrix for the longitudinal motion of aircraft (4) in state space with usual notations are given by.

$$A=\begin{bmatrix} X_u & X_w & -U_0\alpha_0 & -g\cos\theta_0 & 0 \\ Z_u & Z_w & U_0 & -g\sin\theta_0 & 0 \\ M_u+M_{\dot{w}}Z_u & M_w+M_{\dot{w}}Z_w & M_q+M_{\dot{w}}U_0 & -M_{\dot{w}}g\sin\theta_0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & U_0 & 0 \end{bmatrix} \quad (6)$$

$$B=\begin{bmatrix} X_{\delta E} & X_{\delta T} \\ Z_{\delta E} & Z_{\delta T} \\ M_{\delta E}+M_{\dot{w}}Z_{\delta E} & M_{\delta T}+M_{\dot{w}}Z_{\delta T} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (7)$$

With $x=\begin{bmatrix} u & w & q & \theta & h \end{bmatrix}^T$ and $u=\begin{bmatrix} \delta_E & \delta_T \end{bmatrix}^T$

This equations are subject to change since these are related to without integrator states approach. Another approach will be needed to satisfy the control objectives. So, the second approach used (with integrator states) has the following formulation: A=

$$\begin{bmatrix} X_U & X_w & -U_0\alpha_0 & -g\cos\theta_0 & 0 & 0 & 0 \\ Z_u & Z_w & U_0 & -g\sin\theta_0 & 0 & 0 & 0 \\ M_u+M_{\dot{w}}Z_u & M_w+M_{\dot{w}}Z_w & M_q+M_{\dot{w}}U_0 & -M_{\dot{w}}g\sin\theta_0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & U_0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (8)$$

and input matrix:

$$B=\begin{bmatrix} X_{\delta E} & X_{\delta T} \\ X_{\delta E} & Z_{\delta T} \\ M_{\delta E}+M_{\dot{w}}Z_{\delta E} & M_{\delta T}+M_{\dot{w}}Z_{\delta T} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (9)$$

with $x=\begin{bmatrix} u & w & q & \theta & h & \int udu & \int hdh \end{bmatrix}^T$ and $u=\begin{bmatrix} \delta_E & \delta_T \end{bmatrix}^T$

B. *Atmospheric perturbations*

In this work a mathematical model is used to simulate atmospheric perturbations. The mathematical model shown in equation (6) and (7) modified [1] and [3] and the final form of the state space system is given in (10) and (11)

$$\dot{x}=Ax+Bu+E\,d \quad (10)$$

$$y=Cx+Du+F\,n \quad (11)$$

In which the vector *d* is related to the non- controlled disturbances and the vector *n* is related to the existing noise in the system's measurements. The matrixes *E* and *F* are still constants, due to the assumptions made, which tells that the system is linearized around an equilibrium point in steady-state flight. In case of an aircraft system, the alterations provided by the introduction of the air speed changes in the aerodynamic forces expressions origins the appearance of an additional term in the state equations, already mentioned and showed in (11). With the term corresponds to the input of the atmospheric perturbations in the aircraft motion dynamics for the longitudinal motion, the new state-space system, is as follows: In a particular case of this work, *F* matrix entrances are all zeros (noise in measurements signals negligible) and the *E* matrix is implemented directly from the *A* matrix and suffers the influence of the inputs provided by the Dryden model to atmospheric perturbations.

$$\begin{bmatrix} \dot{u} \\ \dot{\omega} \\ \dot{q} \\ \dot{\theta} \\ \dot{h} \end{bmatrix}=\begin{bmatrix} X_u & X_\omega & -u_0\alpha_0 & -g\cos\theta_0 & 0 \\ Z_u & Z_\omega & u_0 & -g\sin\theta_0 & 0 \\ M'_u & M_\omega' & M_q' & M_0' & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & u_0 & 0 \end{bmatrix}\begin{bmatrix} u \\ w \\ q \\ \theta \\ h \end{bmatrix}+\begin{bmatrix} X_{\delta E} & X_{\delta T} \\ Z_{\delta E} & Z_{\delta T} \\ M_{\delta E}' & M_\omega' \\ 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \delta_E \\ \delta_T \end{bmatrix}$$

$$+\begin{bmatrix} -X_u & -X_\omega & 0 \\ -Z_u & -Z_\omega & 0 \\ -M'_u & -M'_\omega & -M'_q \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} u_g \\ \omega_g \\ q_g \end{bmatrix} \quad (12)$$

The aircraft speed and the perturbations changes itself, in the space and time, obligates the consideration of continuous distributions of perturbations, based on statistical analysis. This statistical analysis provides the power spectrum of the perturbation and an approximate model of its dynamics (it is possible then to implement this model in simulations). In aeronautics one of the

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models used is the Dryden model, which has the following spectrum,

$$\phi_u = \sigma_u^2 \frac{2L_u}{\pi} \frac{1}{(1+L_u^2\Omega^2)^2} \quad (13)$$

Where L is the scale length (altitude dependent) and σ is a constant that describes the perturbation intensity.

IV. Simulations

This section presents longitudinal motion simulations without integrator states with integrator states and with gust disturbances.

A. *Longitudinal simulations*

For longitudinal motion the controller type used is the LQR. Feedback control of system outputs is a useful tool to obtain the needed corrections in the system behavior. It is necessary to define a method for predicting the dimensioning of the controller characteristics and, by inherency, the feedback system .Modern control theory is a tool with those characteristics that is obtained through a *performance index function* in which its minimization origins only one optimal controller. The performance index function usually has the form $J = \int_{t_1}^{t_2} L(x,u,t)\ dt$ (14)

The optimization with the dynamics mentioned in (6) brings one solution u^0 which minimizes the value of J. Between several possibilities for the *performance index function* there is the usual case of the null reference and semi-infinity time interval. This case corresponds to the linear quadratic regulator: $J = \frac{1}{2}\int_0^\infty (x'Qx + u'Ru)\ dt$ (15)

where the matrixes Q and R are quadratic, symmetrical, Q is semi-definite positive and R is positive definite. This solution gives the optimal solution in its linear form: $u^0 = -Kx$ (16)

We can say that minimizing the *performance index function* with the system dynamics restriction (refereed in eq. (6) results from the resolution of the of Riccati equation (no time depended in this case):

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \quad (17)$$

This equation when solved for P (constant matrix, symmetrical, positive semi-definite) origins the feedback gains matrix: $K = -R^{-1}B^T P$ (18) This feedback matrix K can be computed in the feedback loop, or in the servo loop. In the first case we guarantee the stability of the system. In the second case we can control the error between the value of the state variables and its references

B. *Without integrator states*

In this case it was used the longitudinal state space linear model obtained in (8). The system matrixes used have the follow numerical values shown in section I and the corresponding open loop poles are

Eigenvalue	Damping	Freq. (rad/s)
$-0.00839 \pm 0.0787i$	0.106	0.0792
0.00	−1.00	0.00
$-0.666 \pm 3.53i$	0.186	3.59

Knowing the physics of characteristics (frequency and damping) of the aircraft, it is possible to distinguish the two modes of the longitudinal motion (phugoid and short period) [3,4,5]. This can be possible due to the difference between both natural frequencies and short period natural frequency a slower one. Analysis of these elements verifies the open loop model stability. The aircraft structure is the conventional one (two pair of conjugate complex poles). The vehicle is class II type ($weight < 30000kg$) and flights in B category phase flight (non terminal flight phase, with moderate maneuvers and trajectory control). This characteristics are obtained through [6] and [7]. Finally, it is possible to achieve the following criterions: Phugoid: $\xi = 0.106 > 0.04$ - Level I. Short period: $\xi = 0.186 \pm 0.1 < \xi$ - Level III Modes separation: $\frac{\varpi_{ph}}{\varpi_{sp}} = 0.022 < 0.1$ - Modes well separated. Despite this acceptable characteristics, this work implemented a controller for the puropose of reference u and h tracking. Here we have used the LQR with the following characteristics:

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}; \quad R = \begin{bmatrix} 1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

Which provides the gain matrix K:

$$K = \begin{bmatrix} -0.0825 & 1.2913 & -10.6988 & -409.3245 & -2.2332 \\ 3.1812 & -0.5582 & 0.9582 & 106.9098 & 0.358 \end{bmatrix}$$

C. With Integrator states

The state space formulation has the following numerical values, according to (9) and (10): The poles at the origin are related to the altitude equation and the integrator states (h, $\int$udu and $\int$hdh equations, respectively).

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$$A=\begin{bmatrix}-0.0161 & 0.0165 & -3.9161 & -9.8075 & 0 & 0 & 0\\ -0.1134 & -1.0541 & 173.9337 & -0.2209 & 0 & 0 & 0\\ 0.0000 & -0.0725 & -0.2792 & 0.0001 & 0 & 0 & 0\\ 0 & 0 & 1.000 & 0 & 0 & 0 & 0\\ 0 & -1.000 & 0 & 173.9337 & 0 & 0 & 0\\ 1.0000 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1.0000 & 0 & 0\end{bmatrix};$$

$$B=\begin{bmatrix}0 & 1.3350\\ -11.719 & 0\\ -5.8553 & -0.0190\\ 0 & 0\\ 0 & 0\\ 0 & 0\\ 0 & 0\end{bmatrix}$$

The phugoid mode has a lower damping value due to the weak aerodynamic drag. The lift-over-drag ratio (*L/D*) is then high, this situation benefits the controller action and the profile mission objectives of the aircraft. It is observed that this system is marginally stable. To correct this problem an LQR controller was implemented. The LQR (linear quadratic regulator) ensures the augmentation of stabilization and system control through a modern control solution with a feedback of all system states, to both system inputs (elevator and propulsion force). As mentioned before, this implementation is based on the theory of section III. The LQR used has the following characteristics

$$Q=\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 5 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 15 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix};\ R=\begin{bmatrix}1 & 0\\ 0 & 0.1\end{bmatrix}$$

Which provides the gain matrix K:

$$K=\begin{bmatrix}-0.0649 & 1.7751 & -11.7371 & -498.2541 & -3.2723 & 0.2012 & -0.9987\\ 5.3257 & -0.3590 & 0.4857 & 77.0862 & 0.5509 & 12.2309 & 0.1643\end{bmatrix}$$

Consequently, the closed loop system poles are:

Eigenvalue	Damping	freq.(rad/s)
−0.445	1.00	0.445
−0.997 ± 1.02*i*	0.699	1.43
−3.55 ± 1.93*i*	0.879	4.04
−23.4 ± 21.8*i*	0.732	32

It is important to make some references related to the choice of the *Q* and *R* matrixes of the LQR implementation. For the first matrix the weights were chosen according to the importance of each state variable. This is the reason for the entrances related to the altitude and altitude integration having a bigger value than the other entries. For the *R* matrix, according to [1] it is possible to conclude that the *weight value* related to the elevator input has one more value order (as 10 has one more value order than 1, for example) comparing with the *weight value* of the propulsion input. SIMULINK diagram for Longitudinal dynamics with and without integrators are designed. Also SIMULINK models of Longitudinal dynamics with atmospheric perturbations are also developed.

D. *Atmospheric perturbations*

This section shows the simulations based on equations (10) and (12) the new state space system has the following numerical values:

$$A=\begin{bmatrix}-0.0161 & 0.0165 & -3.9161 & -9.8075 & 0\\ -0.1134 & -1.0541 & 173.9337 & -0.2209 & 0\\ 0 & -0.0725 & -0.2792 & 0.0001 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & -1 & 0 & 173.9337 & 0\end{bmatrix}$$

$$B=\begin{bmatrix}0 & 1.335\\ -11.719 & 0\\ -5.8553 & -0.0190\\ 0 & 0\\ 0 & 0\end{bmatrix}\quad E=\begin{bmatrix}0.0161 & -0.0165 & 0\\ 0.1134 & 1.0541 & 0\\ -0.000 & -0.0725 & 0.2792\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

V. Conclusions

This section gives the results obtained, based on state space system without integrator states u reference and h reference are shown in figures 2 and 3. From figure 4 and 5 respectively u tracking error and h tracking error without integrators. it is observed that the steady state error is non zero. Hence it is to be reformulated according to the theory explained in the previous section with integrator states. The results obtained, based on new state space system with integrator states are shown in figures 6 and 7. The results show zero steady state error, so it is concluded that the controller works perfectly in its objectives (the reference tracking). At the same time, the spikes observed are due to the changes in *u* and *h* references. The other variables of the system gave good results too. The pitch rate (q), for example, becomes zero due to the controller action (as necessary); the ascend velocity (w) and pitch angle (θ) stabilize over fixed values, not showing oscillations beside the ones produced by *u* and *h* references changes, which can be proved by figures 8 and 9.

Here the simulation is carried out in the presence of atmospheric perturbation, results show that the controller still works perfectly in its objectives. Figures 10 and 11 depict the error between the variable output and its reference. Comparing with the last results (figures 8 and 9), these figures show the presence of tracking error all over the simulation time. This is due to the motion turbulence existence provided to the system implemented with atmospheric perturbations. As given in figure 10, the tracking is still a reality, with the controller always working

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to get a null tracking error. The existence of bigger spikes values is with turbulence factors. The new achievements by the controller in this new system conditions are the ones showed. Finally figure 12 with the two signals (in this example altitude h and reference altitude h_{ref}) confirm the achievement of objectives of the controller action

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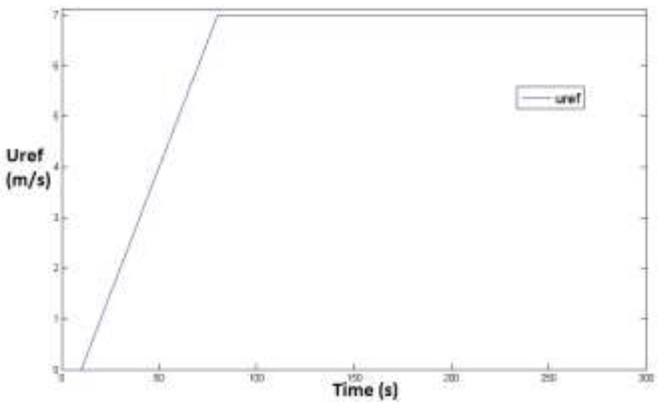

Fig 2 u reference

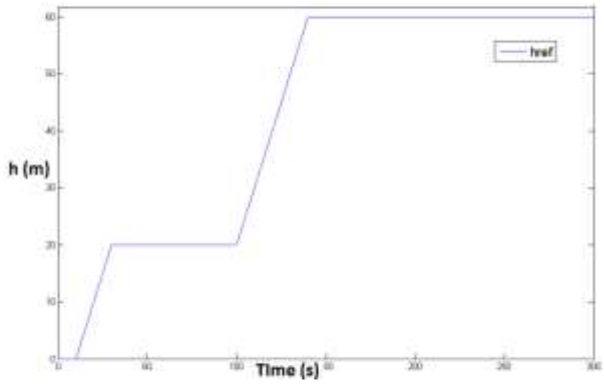

Fig 3 h reference

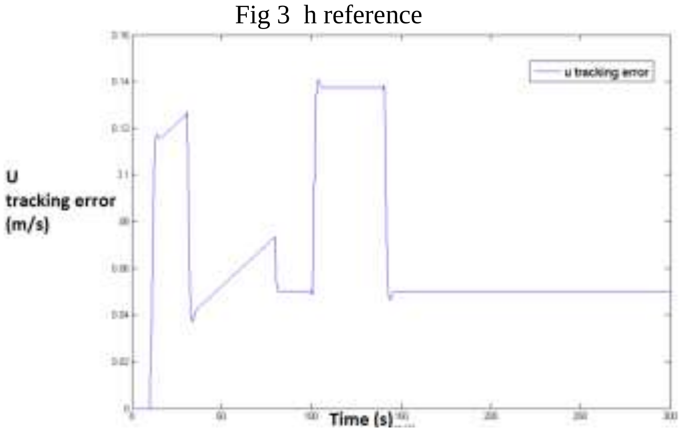

Fig 4: *u* tracking error - without integrator

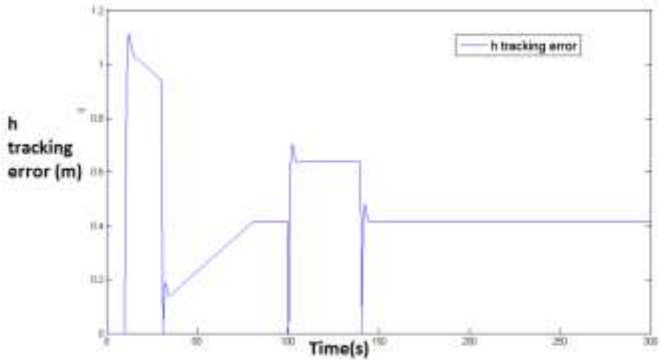

Figure 5: *h* tracking error - without integrators

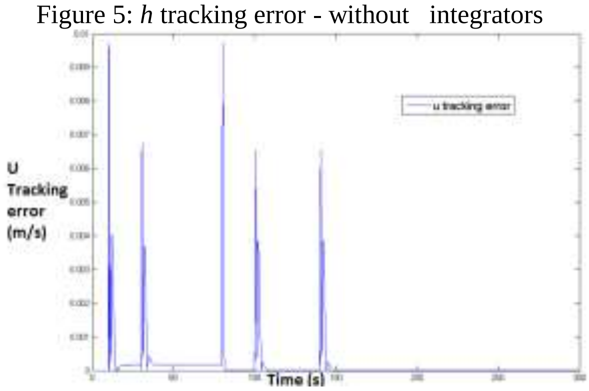

Fig 6: *u* tracking - with integrator

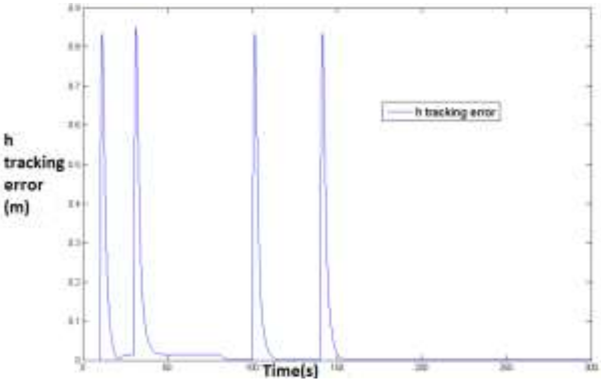

Fig7: *h* tracking error with integrators

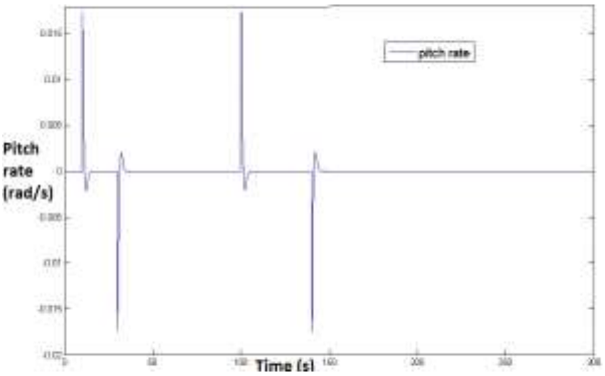

Fig 8: *q* pitch rate - with integrator

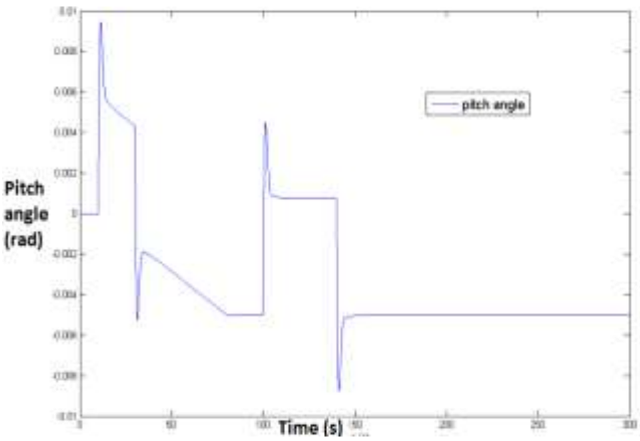

Fig 9: θ pitch angle - with integrator

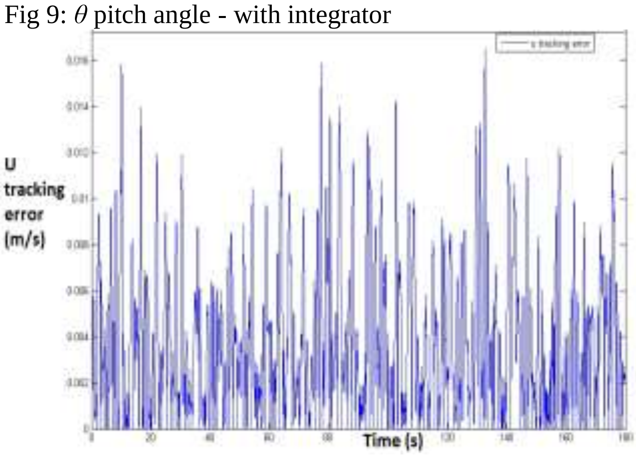

Fig10: *u* tracking error - with atmospheric perturbations

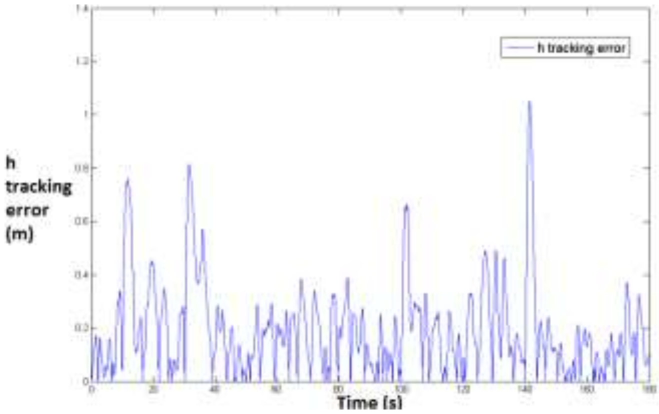

Fig 11: *h* tracking error - with atmospheric perturbations

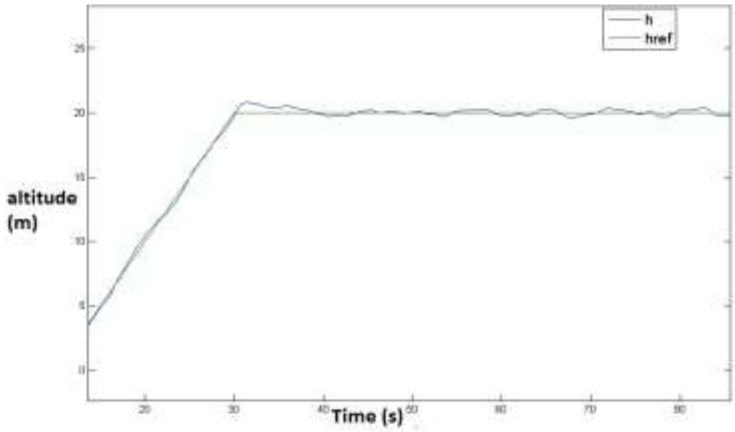

Fig12 h tracking (detailed) with atmospheric perturbations

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