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New analytical Solutions for a generalization of the Ito equations

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Abstract

In this work, we take into consideration the Ito equations.Truncated Painlevé expansion and the homogeneous balance method are extended to search for Backlund transformation for the Hirota-Satsuma equations.Moreover,the solutions obtained for these equations in this work are utilizing a new approach.

1 Introduction

it is well know that the nonlinear partial differential equations are widely used to describe many important phenomena in physics, biology, chemistry, etc.In the recent years, the effort are made to finding analytical and soliton solutions to nonlinear differential equations which could play worthwhile role in understanding various Mathematical and Physical phenomenon.

In this work, we are looking for analytical and soliton solutions for a generalization of the Ito equations:

$$u_t = v_x, \tag{1}$$
$$v_t = -2(v_{xxx} + 3uv_x) - 12ww_x, \tag{2}$$
$$w_t = w_{xxx} + 3uw_x. \tag{3}$$

The problem (1)-(3) play a crucial role in applied mathematics and physics and have many applications in Physics and Engineering. Eqs.(1)-(3) are reduced to a new complex KdV with $w = v^* and \ \ w = v$ in Wu et al [1]. These equation have the periodic wave solutions expressed by the Jacobi elliptic functions in Zhang et al [2]. By the improved F-expansion method, Fan et al [3], using Jacobi elliptic functions, Yu et al [4] have introduced the extended Jacobi elliptic function with rational expansion method to finding six families of new

Jacobi elliptic function solutions. Zang [5] has found some new types of exact solutions for the Eqs.(1)-(3), Ganji et al [6] has used the homotopy perturbation method (HPM) with initial conditions and the system have four kinds of the wave soliton solution om using an extended tanh-function method in Fan [7] and Zayed et al [8] have the solitary wave solution of system (1)-(3) in terms of $sn(x) - cn(x)$, Jacobi elliptic functions and $\sec_q - \tanh_q$ q-deformed hyperbolic functions.
Also,there are various methods for finding analytical and soliton solutions as Bäcklund transformations, tanh method and extended tanh method which has been invoking great interest in the recent years.
Hayany et al [9] and Woopyo et al [10] have shown that Bäcklund transformation exist for Burgers equation and general variable-coefficient KdV equations, respectively with some constraints.Kink-type solitonic solutions are found. Yan [11] showed two types of auto-Bäcklund transformations for a simplified model for reacting mixtures.

Malfliet [12] and Khater et al [13] consider travelling-wave solutions of coupled nonlinear evolution equations and reaction-diffusion equations by tanh method with initial conditions, respectively. Wazwaz [14-19] used another technique for tanh method without using initial conditions ,called the extended tanh method which transformes a system to a set of algebraic equations via symbolic software like Maple or Mathematica and is solved.

In this paper , we investigate the existence of a Bäcklund transformation by applying the truncated Painlevé expansion.A Symbolic computation methods are used to obtain an auto-Bäcklund transformation and certain soliton-typed explicit solutions.

2 Auto-Backlund transformations for system (1)-(3) and solitonic solutions

Let u, v and w be the solutions of nonlinear partial differential equations (1)-(3) of the form:-

$$\begin{aligned} u &= \phi^{\alpha_1} \sum_{j1=0}^{\infty} u_{j_1} \phi^{j_1}, \\ v &= \phi^{\alpha_2} \sum_{j1=0}^{\infty} v_{j_2} \phi^{j_2}, \\ w &= \phi^{\alpha_3} \sum_{j1=0}^{\infty} w_{j_3} \phi^{j3}. \end{aligned} \tag{4}$$

Where $u = u(x,t), v = v(x,t)$, $w = w(x,t)$,$\phi = \phi(x,t)$, $u_0 \neq 0$, $v_0 \neq 0$ and $w_0 \neq 0$.
To determine the values α_1, α_2 and α_3 for (4) ,Substituting Eq. (4) in the system (1)-(3) and balancing the highest order with highest order for nonlinear

terms, we get

$$\alpha_1 = \alpha_2 = \alpha_3 = -2. \tag{5}$$

$$\begin{aligned} u &= \phi^{-2} \sum_{j1=0}^{\infty} u_{j_1} \phi^{j_1}, \\ v &= \phi^{-2} \sum_{j1=0}^{\infty} v_{j_2} \phi^{j_2}, \\ w &= \phi^{-2} \sum_{j1=0}^{\infty} w_{j_3} \phi^{j3}. \end{aligned} \tag{6}$$

Therefore, the system (1)-(3), is said to be possess the Painlevé property and is conjectured to be integrable.

In order to find the solitonic solutions for Eqs.(1)-(3),we truncate the Painlevé expansion,

$$\begin{aligned} u(x,t) &= \phi^{-j_1}(x,t) \sum_{i=0}^{j_1} u_i(x,t)\phi^i(x,t), \\ v(x,t) &= \phi^{-j_2}(x,t) \sum_{i=0}^{j_2} v_i(x,t)\phi^i(x,t), \\ w(x,t) &= \phi^{-j_3}(x,t) \sum_{i=0}^{j_3} w_i(x,t)\phi^i(x,t). \end{aligned} \tag{7}$$

Balancing between power of the highest derivative and power of nonlonear terms in the system (1)-(3), we get:

$$j_1 = j_2 = j_3 = 2. \tag{8}$$

Consequently,Eq. (7) take the following form:

$$\begin{aligned} u(x,t) &= \phi^{-2}(x,t) \sum_{i=0}^{2} u_i(x,t)\phi^i(x,t), \\ u(x,t) &= \frac{u_0(x,t) + u_1(x,t)\phi(x,t) + u_2(x,t)\phi^2(x,t)}{\phi^2(x,t)} \end{aligned}$$

$$
\begin{aligned}
u(x,t) &= \frac{u_0(x,t)}{\phi^2(x,t)} + \frac{u_1(x,t)}{\phi(x,t)} + u_2(x,t), && (9)\\
v(x,t) &= \phi^{-2}(x,t)\sum_{i=0}^{2} v_i(x,t)\phi^i(x,t),\\
v(x,t) &= \frac{v_0(x,t) + v_1(x,t)\phi(x,t) + v_2(x,t)\phi^2(x,t)}{\phi^2(x,t)}\\
v(x,t) &= \frac{v_0(x,t)}{\phi^2(x,t)} + \frac{v_1(x,t)}{\phi(x,t)} + v_2(x,t), && (10)\\
w(x,t) &= \phi^{-2}(x,t)\sum_{i=0}^{2} w_i(x,t)\phi^i(x,t),\\
w(x,t) &= \frac{w_0(x,t) + w_1(x,t)\phi(x,t) + w_2(x,t)\phi^2(x,t)}{\phi^2(x,t)}\\
w(x,t) &= \frac{w_0(x,t)}{\phi^2(x,t)} + \frac{w_1(x,t)}{\phi(x,t)} + w_2(x,t). && (11)
\end{aligned}
$$

Using Eqs.(9)-(11), with the general assumption $\phi_x \neq 0$, in Eqs.(1)-(3) , we set the coefficients of like power of ϕ to zero,then we get the set of Painlevé-Backlund equations called Auto-Backlund transformations of the form:-

Eq. (1) gives :-

$$
\begin{aligned}
\phi^0 &: u_{2t} + 3u_2u_{2x} - 3w_2v_{2x} - 3v_2w_{2x} - \frac{1}{2}u_{2xxx} = 0 && (12)\\
\phi^{-1} &: u_{1t} + 3u_2u_{1x} + 3u_1u_{2x} - 3w_2v_{1x} - 3w_1v_{1x} - 3v_2w_{1x}\\
&\quad -3v_1w_{2x} - \frac{1}{2}u_{1xxx} = 0 && (13)
\end{aligned}
$$

$$
\begin{aligned}
\phi^{-2} &: -u_1\phi_t - 3u_1u_2\phi_x + 3v_2w_1\phi_x + 3v_1w_2\phi_x + u_{0t} + 3u_2u_{0x} + 3u_1u_{1x}\\
&\quad +3u_0u_{2x} + 3w_2v_{0x} - 3w_1v_{1x} - 3w_0v_{2x} - 3v_2w_{0x} - 3v_1w_{1x}\\
&\quad -3v_0w_{2x} + \frac{3}{2}u_{1x}\phi_{xx} + \frac{3}{2}\phi_x u_{1xx} + \frac{1}{2}u_1\phi_{xxx} - \frac{1}{2}u_{0xxx} = 0 && (14)\\
\phi^{-3} &: 2u_0\phi_t = -3u_1^2\phi_x - 6u_0u_2\phi_x + 6v_2w_0\phi_x + 6v_1w_1\phi_x + 6v_0w_2\phi_x\\
&\quad +3u_1u_{0x} + 3u_0u_{1x} - 3\phi_x^2u_{1x} - 3w_1v_0x - 3w_0v_{1x} - 3v_1w_{0x}\\
&\quad -3v_0w_{1x} + 3u_{0x}\phi_{xx} + 3\phi_x u_{0xx} + u_0\phi = 0 && (15)
\end{aligned}
$$

$$
\begin{aligned}
\phi^{-4} &: -9u_0u_1\phi_x + 9v_1w_0\phi_x + 9v_0w_1\phi_x + 3u_1\phi_x^3 + 3u_0u_{0x} - 9\phi_x^2u_{0x}\\
&\quad -3w_0v_{0x} - 3v_0w_{0x} - 9u_0\phi_x\phi_{xx} = 0 && (16)\\
\phi^{-5} &: -6u_0^2\phi_x + 12v_0w_0\phi_x + 12u_0\phi_x^3 = 0 && (17)
\end{aligned}
$$

Eq. (2) gives :-

$$\phi^0 \quad : \quad v_{2t} - 3u_2 v_{2x} + v_{2xxx} = 0 \tag{18}$$

$$\phi^{-1} \quad : \quad -3u_1 v_{2x} + v_{1t} - 3u_2 v_{1x} + v_{1xxx} = 0 \tag{19}$$

$$\begin{aligned} \phi^{-2} \quad : \quad & -3u_0 v_{2x} - v_1 \phi_t + 3u_2 v_1 \phi_x + v_{0t} - 3u_2 v_{0x} - 3u_1 v_{1x} \\ & -3v_{1x}\phi_{xx} - 3\phi_x v_{1xx} - v_1 \phi_{xxx} + v_{0xxx} = 0 \end{aligned} \tag{20}$$

$$\begin{aligned} \phi^{-3} \quad : \quad & -2v_0 \phi_t + 6u_2 v_0 \phi_x + 3u_1 v_1 \phi_x - 3u_1 v_{0x} - 3u_0 v_{1x} + 6\phi_x^2 v_{1x} \\ & +6v_1 \phi_x \phi_{xx} - 6v_{0x}\phi_{xx} - 6\phi_x v_{0xx} - 2v_0 \phi_{xxx} = 0 \end{aligned} \tag{21}$$

$$\begin{aligned} \phi^{-4} \quad : \quad & 6u_1 v_0 \phi_x + 3u_0 v_1 \phi_x - 6v_1 \phi_x^3 - 3u_0 v_{0x} + 18\phi_x^2 v_{0x} \\ & +18v_0 \phi_x \phi_{xx} = 0 \end{aligned} \tag{22}$$

$$\phi^{-5} \quad : \quad 6u_0 v_0 \phi_x - 24 v_0 \phi_x^3 = 0 \tag{23}$$

Eq. (3) gives :-

$$\phi^0 \quad : \quad w_{2t} - 3u_2 w_{2x} + w_{2xxx} = 0 \tag{24}$$

$$\phi^{-1} \quad : \quad -3u_1 w_{2x} + w_{1t} - 3u_2 w_{1x} + w_{1xxx} = 0 \tag{25}$$

$$\begin{aligned} \phi^{-2} \quad : \quad & -3u_0 w_{2x} - w_1 \phi_t + 3u_2 w_1 \phi_x + w_{0t} - 3u_2 w_{0x} - 3u_1 w_{1x} \\ & -3w_{1x}\phi_{xx} - 3\phi_x w_{1xx} - w_1 \phi_{xxx} + w_{0xxxx} = 0 \end{aligned} \tag{26}$$

$$\begin{aligned} \phi^{-3} \quad : \quad & -2w_0 \phi_t + 6u_2 w_0 \phi_x + 3u_1 w_1 \phi_x - 3u_1 w_{0x} - 3u_0 w_{1x} + 6\phi_x^2 w_{1x} \\ & +6w_1 \phi_x \phi_{xx} - 6w_{0x}\phi_{xx} - 6\phi_x w_{0xx} - 2w_0 \phi_{xxx} = 0 \end{aligned} \tag{27}$$

$$\begin{aligned} \phi^{-4} \quad : \quad & 6u_1 w_0 \phi_x + 3u_0 w_1 \phi_x - 6w_1 \phi_x^3 - 3u_0 w_{0x} + 18\phi_x^2 w_{0x} \\ & +18w_0 \phi_x \phi_{xx} = 0 \end{aligned} \tag{28}$$

$$\phi^{-5} \quad : \quad 6u_0 w_0 \phi_x - 24 w_0 \phi_x^3 = 0 \tag{29}$$

Solving equations (12)-(29) with respect to $u_0, u_1, u_2, v_0, v_1, v_2, w_0, w_1$ *and* w_2, then we obtain from equation (23) :

$$u_0(x,t) = 4\phi_x^2. \tag{30}$$

Substituting Eq.(30) into Eq.(17)and solving With respect to. $v_0(x,t)$, we get:-

$$v_0 w_0 = 4\phi_x^4 \quad \Longrightarrow \quad v_0(x,t) = \frac{4\phi_x^4}{w_0}, \tag{31}$$

where $w_0(x,t)$ is an arbitrary function different from zero.
Using Eqs.(30)and (31) into Eqs.(12)-(29) and solving Eqs.(16), (22) and (27) with respect to. u_1, v_1, w_1, we obtain:-

$$u_1(x,t) = -4\phi_{xx}, \tag{32}$$

$$v_1(x,t) = -\frac{4(-\phi_x^3 w_{0x} + 3w_0 \phi_x^2 \phi_{xx})}{w_0^2}, \tag{33}$$

$$w_1(x,t) = -\frac{\phi_x w_{0x} - w_0 \phi_{xx}}{\phi_x^2}. \tag{34}$$

Applying Eqs.(30)-(34) into Eq.(21) or (27) and solving with respect to $u_2(x,t)$, we get:-

$$u_2(x,t) = \frac{\phi_t\phi_x - 3\phi_{xx}^2 + 4\phi_x\phi_{xxx}}{3\phi_x^2}, \tag{35}$$

substituting from Eqs.(30)-(35) into Eq.(15) and solving with respect to v_2, w_2, we obtain:-

$$\begin{aligned} v_2(x,t) &= \frac{-4w_2\phi_x^4}{w_0^2} + \frac{1}{3w_0^3}(8w_0^2\phi_t\phi_x + 12\phi_x^2 w_{0x}^2 - 48w_0\phi_x\phi_{xx}w_{0x} \\ &\quad +42w_0^2\phi_{xx}^2 + 8w_0^2\phi_x\phi_{xxx}). \end{aligned} \tag{36}$$

From Eq.(13) and Eq.(30)-(36) we find that :

$$\begin{aligned} w_2(x,t) &= \frac{1}{12w_0\phi_x^4(-\phi_x w_{0x} + 2w_0\phi_{xx})}(-4w_0^2\phi_t\phi_x^2 w_{0x} - 6\phi_x^3 w_{0x}^3 + 2w_0^3\phi_x^2\phi_{xt} \\ &\quad +6w_0^3\phi_t\phi_x\phi_{xx} + 30w_0\phi_x^2 w_{0x}^2\phi_{xx} - 45w_0^2\phi_x w_{0x}\phi_{xx}^2 + 15w_0^3\phi_{xx}^3 \\ &\quad -4w_0^2\phi_x^2 w_{0x}\phi_{xxx} + 12w_0^3\phi_x\phi_{xx}\phi_{xxx} - w_0^3\phi_x^2\phi_{xxxx}) + R(t). \end{aligned} \tag{37}$$

to simplify, we put $R(t) = 0$.

Using Eq.(37), then Eq.(36) can be rewritten in the form:-

$$\begin{aligned} v_2(x,t) &= \frac{1}{3w_0^3}(8w_0^2\phi_t\phi_x + 12\phi_x^2 w_{0x}^2 - 48w_0\phi_x\phi_{xx}w_{0x} + 42w_0^2\phi_{xx}^2 + 8w_0^2 \\ &\quad \cdot\phi_x\phi_{xxx}) - \frac{1}{3w_0^3(-\phi_x w_{0x} + 2w_0\phi_{xx})}(-4w_0^2\phi_t\phi_x^2 w_{0x} - 6\phi_x^3 w_{0x}^3 \\ &\quad +2w_0^3\phi_x^2\phi_{xt} + 6w_0^3\phi_t\phi_x\phi_{xx} + 30w_0\phi_x^2 w_{0x}^2\phi_{xx} - 45w_0^2\phi_x w_{0x}\phi_{xx}^2 \\ &\quad +15w_0^3\phi_{xx}^3 - 4w_0^2\phi_x^2 w_{0x}\phi_{xxx} + 12w_0^3\phi_x\phi_{xx}\phi_{xxx} - w_0^3\phi_x^2\phi_{xxxx}) \end{aligned} \tag{38}$$

Substituting Eqs.(30)-(38) into Eqs.(9)-(11), we obtain the form of $u(x,t)$, $v(x,t)$ and $w(x,t)$

$$u(x,t) = \frac{4\phi_x^2}{\phi(x,t)^2} + \frac{-4\phi_{xx}}{\phi(x,t)} + \frac{\phi_t\phi_x - 3\phi_{xx}^2 + 4\phi_x\phi_{xxx}}{3\phi_x^2}, \tag{39}$$

$$\begin{aligned} v(x,t) &= \frac{4\phi_x^4}{w_0\phi(x,t)^2} + -\frac{4(-\phi_x^3 w_{0x} + 3w_0\phi_x^2\phi_{xx})}{w_0^2\phi(x,t)} + \frac{1}{3w_0^3}(8w_0^2\phi_t\phi_x \\ &\quad +12\phi_x^2 w_{0x}^2 - 48w_0\phi_x\phi_{xx}w_{0x} + 42w_0^2\phi_{xx}^2 + 8w_0^2\phi_x\phi_{xxx}) \end{aligned}$$

$$-\frac{1}{3w_0^3(-\phi_x w_{0x}+2w_0\phi_{xx})}(-4w_0^2\phi_t\phi_x^2 w_{0x}-6\phi_x^3 w_{0x}^3+2w_0^3\phi_x^2\phi_{xt}$$
$$+6w_0^3\phi_t\phi_x\phi_{xx}+30w_0\phi_x^2 w_{0x}^2\phi_{xx}-45w_0^2\phi_x w_{0x}\phi_{xx}^2+15w_0^3\phi_{xx}^3 \quad (40)$$
$$-4w_0^2\phi_x^2 w_{0x}\phi_{xxx}+12w_0^3\phi_x\phi_{xx}\phi_{xxx}-w_0^3\phi_x^2\phi_{xxxx}),$$
$$w(x,t) = \frac{w_0}{\phi(x,t)^2}+-\frac{\phi_x w_{0x}-w_0\phi_{xx}}{\phi_x^2\phi(x,t)}+\frac{1}{12w_0\phi_x^4(-\phi_x w_{0x}+2w_0\phi_{xx})}\cdot$$
$$(-4w_0^2\phi_t\phi_x^2 w_{0x}-6\phi_x^3 w_{0x}^3+2w_0^3\phi_x^2\phi_{xt}+6w_0^3\phi_t\phi_x\phi_{xx}+$$
$$30w_0\phi_x^2 w_{0x}^2\phi_{xx}-45w_0^2\phi_x w_{0x}\phi_{xx}^2+15w_0^3\phi_{xx}^3-4w_0^2\phi_x^2 w_{0x}\phi_{xxx}$$
$$+12w_0^3\phi_x\phi_{xx}\phi_{xxx}-w_0^3\phi_x^2\phi_{xxxx}). \quad (41)$$

with $w_0(x,t)$ is an arbitrary function ,where equations (12), (18), (19), (20), (24), (25) and (26) are the constraints equations for $\phi(x,t)$.
In the of the rest sections,we will find a family of the exact analytic solutions Eqs.(3.1)-(3.3) for two cases for $\phi(x,t)$:

special form for $\phi(x,t)$:

A trial solution is:

$$\phi(x,t)=1+Exp[A(t)x+B(t)] \quad (42)$$

Since $w_0(x,t)$ is an arbitrary function , we take $w_0(x,t)$ in the following form:

$$w_0(x,t)=Exp[C(t)x+S(t)] \quad (43)$$

substituting from Eqs.(42),(43) into the constraints equations (12), (18), (19), (20), (24), (25) and (26), after substitution to $u_0, v_0, u_1, v_1, w_1, u_2, v_2, w_2$ from Eqs.(30)-(38). Equating to zero the coefficients of like powers of x in every equations of constraints, we get a system of an ordinary equations for functions $A(t), B(t), C(t)$ *and* $S(t)$:-

$$47A(t)\frac{d}{dt}A(t)-48A(t)^2C(t)\frac{d}{dt}A(t)+12A(t)C(t)^2\frac{d}{dt}A(t)+8(\frac{d}{dt}A(t))\cdot$$
$$(\frac{d}{dt}B(t))-A(t)\frac{d^2}{dt^2}B(t) \ =0, \quad (44)$$
$$-8(\frac{d}{dt}A(t))^2+A(t)\frac{d^2}{dt^2}B(t) \ =0, \quad (45)$$

$$168A(t)^4C(t)\frac{d}{dt}C(t) - 78A(t)^3C(t)^2\frac{d}{dt}C(t) + 12A(t)^2C(t)^3\frac{d}{dt}C(t)$$
$$+268A(t)^5C(t)\frac{d}{dt}S(t) + 794A(t)^3C(t)^2\frac{d}{dt}A(t) - 21A(t)^2C(t)^3\frac{d}{dt}A(t) +$$
$$22A(t)C(t)_4\frac{d}{dt}A(t) - 124A(t)^5C(t)\frac{d}{dt}B(t) + 2A(t)C(t)\frac{d^2}{dt^2}A(t) -$$
$$16A(t)^2C(t)\frac{d^2}{dt^2}B(t) - 1264A(t)^4C(t)\frac{d}{dt}A(t) + 54A(t)^3C(t)^3\frac{d}{dt}S(t)$$
$$-6A(t)^2C(t)^4\frac{d}{dt}S(t) - 181A(t)^4C(t)^2\frac{d}{dt}S(t) + 28A(t)^4C(t)^2\frac{d}{dt}B(t)$$
$$+25A(t)^3C(t)^3\frac{d}{dt}B(t) - 14A(t)^2C(t)^4\frac{d}{dt}B(t) + 2A(t)C(t)^5\frac{d}{dt}B(t) +$$
$$16A(t)^2C(t)\frac{d}{dt}B(t)^2 - 16A(t)C(t)^2\frac{d}{dt}B(t)^2 + 4A(t)C(t)^2\frac{d^2}{dt^2}B(t)$$
$$-16A(t)^3(\frac{d}{dt}B(t))(\frac{d}{dt}S(t)) - 2A(t)(\frac{d}{dt}A(t))(\frac{d}{dt}C(t)) - 571A(t)^4C(t)^5$$
$$+2C(t)(\frac{d}{dt}A(t))^2 - 4071A(t)^6C(t)^3 + 16A(t)^3\frac{d^2}{dt^2}B(t) - 120A(t)^5\frac{d}{dt}C(t)$$
$$+96A(t)^6\frac{d}{dt}B(t) - 4A(t)^2\frac{d^2}{dt^2}A(t) - 2A(t)C(t)(\frac{d}{dt}A(t))(\frac{d}{dt}S(t)) + 90A(t)^3C(t)^6$$
$$2C(t)^2(\frac{d}{dt}A(t))(\frac{d}{dt}B(t)) + 4A(t)^2(\frac{d}{dt}A(t))(\frac{d}{dt}S(t)) - 148A(t)^6\frac{d}{dt}S(t) +$$
$$4C(t)^3(\frac{d}{dt}B(t))^2 - 6A(t)^2C(t)^7 - 3236A(t)^8C(t) + 744A(t)^5\frac{d}{dt}A(t) +$$
$$+16A(t)^2C(t)(\frac{d}{dt}B(t))(\frac{d}{dt}S(t)) - 4A(t)C(t)(\frac{d}{dt}A(t))(\frac{d}{dt}B(t)) + 888A(t)^9$$
$$+1984A(t)^5C(t)^4 - 4A(t)C(t)^2(\frac{d}{dt}B(t))(\frac{d}{dt}S(t) + 4922A(t)^7C(t)^2 = 0, \qquad (46)$$
$$48A(t)^5\frac{d}{dt}A(t) - 38A(t)^4C(t)\frac{d}{dt}A(t) - 5A(t)^3C(t)^2\frac{d}{dt}A(t) +$$
$$10A(t)^2C(t)^3\frac{d}{dt}A(t) - 2A(t)C(t)^4\frac{d}{dt}A(t) - 2C(t)(\frac{d}{dt}A(t))^2 +$$
$$16A(t)C(t)(\frac{d}{dt}A(t))(\frac{d}{dt}B(t)) - 8C(t)^2(\frac{d}{dt}A(t))(\frac{d}{dt}B(t)) -$$

$$74A(t)^5\frac{d}{dt}C(t) + 97A(t)^4C(t)\frac{d}{dt}C(t) - 42A(t)^3C(t)^2\frac{d}{dt}C(t) +$$
$$6A(t)^2C(t)^3\frac{d}{dt}C(t) + 2A(t)(\frac{d}{dt}A(t))(\frac{d}{dt}C(t)) - 8A(t)^2(\frac{d}{dt}B(t))(\frac{d}{dt}C(t))$$
$$+4A(t)C(t)(\frac{d}{dt}B(t))(\frac{d}{dt}C(t)) - 8A(t)^2(\frac{d}{dt}A(t))(\frac{d}{dt}S(t)) +$$
$$4A(t)C(t)(\frac{d}{dt}A(t))(\frac{d}{dt}S(t)) + 8A(t)^2\frac{d^2}{dt^2}A(t) - 4A(t)C(t)\frac{d}{dt}A(t) = 0, (47)$$

$$-C(t)\frac{d}{dt}A(t) + A(t)\frac{d}{dt}C(t) = 0, \tag{48}$$

$$2A(t)^5 + 5A(t)^4C(t) - 11A(t)^3C(t)^2 + 6A(t)^2C(t)^3 - A(t)C(t)^4$$
$$-10A(t)\frac{d}{dt}A(t) + 3C(t)\frac{d}{dt}A(t) + 8A(t)^2\frac{d}{dt}B(t) - 7A(t)C(t)\frac{d}{dt}B(t)$$
$$+C(t)^2\frac{d}{dt}B(t) + A(t)\frac{d}{dt}C(t) + 3A(t)^2\frac{d}{dt}S(t) - A(t)C(t)\frac{d}{dt}S(t) = 0, \tag{49}$$

$$8A(t)^2\frac{d}{dt}A(t) - 7A(t)C(t)\frac{d}{dt}A(t) + C(t)^2\frac{d}{dt}A(t) + 3A(t)^2\frac{d}{dt}C(t) -$$
$$A(t)C(t)\frac{d}{dt}C(t)0, \tag{50}$$

$$14A(t)^4 - 23A(t)^3C(t) + 12A(t)^2C(t)^2 - 2A(t)C(t)^3 - 2\frac{d}{dt}A(t) +$$
$$8A(t)\frac{d}{dt}B(t) - 5C(t)\frac{d}{dt}B(t) + A(t)\frac{d}{dt}S(t) = 0 \tag{51}$$

$$8A(t)\frac{d}{dt}A(t) - 5C(t)\frac{d}{dt}A(t) + A(t)\frac{d}{dt}C(t) = 0 \tag{52}$$

$$-120A(t)^4C(t)\frac{d}{dt}C(t) + 66A(t)^3C(t)^2\frac{d}{dt}C(t) - 12A(t)^2C(t)^3\frac{d}{dt}C(t)$$
$$+124A(t)^5C(t)\frac{d}{dt}S(t) - 118A(t)^3C(t)^2\frac{d}{dt}A(t) + 14A(t)^2C(t)^3\frac{d}{dt}A(t)$$
$$+2A(t)C(t)^4\frac{d}{dt}A(t) - 220A(t)^5C(t)\frac{d}{dt}B(t) + 2A(t)C(t)\frac{d^2}{dt^2}A(t)$$
$$+16A(t)^2C(t)\frac{d^2}{dt^2}B(t) + 240A(t)^4C(t)\frac{d}{dt}A(t) + 42A(t)^3C(t)^3\frac{d}{dt}S(t) -$$
$$6A(t)^2C(t)^4\frac{d}{dt}S(t) + 64A(t)^2(\frac{d}{dt}A(t))(\frac{d}{dt}B(t)) - 109A(t)^4C(t)^2\frac{d}{dt}S(t)$$

$$+172A(t)^4C(t)^2\frac{d}{dt}B(t) - 47A(t)^3C(t)^3\frac{d}{dt}B(t) - 2A(t)^2C(t)^4\frac{d}{dt}B(t)$$
$$+2A(t)C(t)^5\frac{d}{dt}B(t) + 16A(t)^2C(t)\frac{d}{dt}B(t)^2 - 16A(t)C(t)^2\frac{d}{dt}B(t)^2 -$$
$$4A(t)C(t)2\frac{d^2}{dt^2}B(t) - 16A(t)^3(\frac{d}{dt}B(t))(\frac{d}{dt}S(t)) - 2A(t)(\frac{d}{dt}A(t))(\frac{d}{dt}C(t))$$
$$-16A(t)^3\frac{d^2}{dt^2}B(t) - 427A(t)^4C(t)^5 - 6C(t)(\frac{d}{dt}A(t))^2 - 2247A(t)^6C(t)^3$$
$$+72A(t)^5\frac{d}{dt}C(t) + 96A(t)^6\frac{d}{dt}B(t) + 16A(t)(\frac{d}{dt}A(t))^2 + 2A(t)C(t)\times$$

$$
\begin{aligned}
&(\frac{d}{dt}A(t))(\frac{d}{dt}S(t)) + 14C(t)^2(\frac{d}{dt}A(t))(\frac{d}{dt}B(t)) - 52A(t)^6\frac{d}{dt}S(t) - 4A(t)^2 \times \\
&(\frac{d}{dt}A(t))(\frac{d}{dt}S(t)) + 4C(t)^3(\frac{d}{dt}B(t))^2 - 6A(t)^2C(t)^7 - 1316A(t)^8C(t) \\
&-152A(t)^5\frac{d}{dt}A(t) + 78A(t)^3C(t)^6 + 16A(t)^2C(t)(\frac{d}{dt}B(t))(\frac{d}{dt}S(t)) \\
&-60A(t)C(t)(\frac{d}{dt}A(t))(\frac{d}{dt}B(t)) + 312A(t)^9 + 1276A(t)^5C(t)^4 \\
&-4A(t)^2\frac{d^2}{dt^2}A(t) - 4A(t)C(t)^2(\frac{d}{dt}B(t))(\frac{d}{dt}S(t)) + 2330A(t)^7C(t)^2 \ = 0, \qquad (53)
\end{aligned}
$$

$$
\begin{aligned}
&48A(t)^5\frac{d}{dt}A(t) - 86A(t)^4C(t)\frac{d}{dt}A(t) + 43A(t)^3C(t)^2\frac{d}{dt}A(t) \\
&-2A(t)^2C(t)^3\frac{d}{dt}A(t) - 2A(t)C(t)^4\frac{d}{dt}A(t) + 32A(t)(\frac{d}{dt}A(t))^2 - \\
&14C(t)(\frac{d}{dt}A(t))^2 + 16A(t)C(t)(\frac{d}{dt}A(t))(\frac{d}{dt}B(t)) - 48A(t)^5\frac{d}{dt}A(t) \\
&-86A(t)^4C(t)\frac{d}{dt}A(t) + 43A(t)^3C(t)^2\frac{d}{dt}A(t) - 2A(t)^2C(t)^3\frac{d}{dt}A(t) \\
&-2A(t)C(t)^4\frac{d}{dt}A(t) + 32A(t)(\frac{d}{dt}A(t))^2 - 14C(t)(\frac{d}{dt}A(t))^2 \\
&(\frac{d}{dt}A(t))(\frac{d}{dt}B(t)) - 8C(t)^2(\frac{d}{dt}A(t))(\frac{d}{dt}B(t)) - 26A(t)^5\frac{d}{dt}C(t) \\
&+49A(t)^4C(t)\frac{d}{dt}C(t) - 30A(t)^3C(t)^2\frac{d}{dt}C(t) + 6A(t)^2C(t)^3\frac{d}{dt}C(t) \\
&-2A(t)(\frac{d}{dt}A(t))(\frac{d}{dt}C(t)) - 8A(t)^2(\frac{d}{dt}B(t))(\frac{d}{dt}C(t)) + 4A(t)C(t) \times \\
&(\frac{d}{dt}B(t))(\frac{d}{dt}C(t)) - 8A(t)^2(\frac{d}{dt}A(t))(\frac{d}{dt}S(t)) + 4A(t)C(t)(\frac{d}{dt}A(t)) \times \\
&(\frac{d}{dt}S(t)) - 8A(t)^2\frac{d^2}{dt^2}A(t) + 4A(t)C(t)\frac{d^2}{dt2}A(t) = 0, \qquad (54)
\end{aligned}
$$

$$
\begin{aligned}
&26A(t)^5 - 51A(t)^4C(t) + 35A(t)^3C(t)^2 - 10A(t)^2C(t)^3 + A(t)C(t)^4 \\
&+16A(t)C(t) - 2A(t)\frac{d}{dt}A(t) - C(t)\frac{d}{dt}A(t) + 8A(t)^2\frac{d}{dt}B(t) \\
&-3A(t)C(t)\frac{d}{dt}B(t) - C(t)^2\frac{d}{dt}B(t) + A(t)\frac{d}{dt}C(t) \\
&-A(t)^2\frac{d}{dt}S(t) + A(t)C(t)\frac{d}{dt}S(t) = 0, \qquad (55) \\
&8A(t)^2\frac{d}{dt}A(t) - 3A(t)C(t)\frac{d}{dt}A(t) - C(t)^2\frac{d}{dt}A(t) - A(t)^2\frac{d}{dt}C(t) \\
&+A(t)C(t)\frac{d}{dt}C(t) = 0. \qquad (56)
\end{aligned}
$$

By using symbolic software Maple, solving an ordinary equations (3.44)-(3.56), with respect to $A(t), B(t), C(t)$ *and* $S(t)$, we obtain:-

$$\begin{aligned} A(t) &= k_4, \\ B(t) &= \frac{1}{4}[4k_2 - (3k_3^2k_4 - 12k_3k_4^2 + 10k_4^3)t], \\ C(t) &= k_3, \\ S(t) &= \frac{1}{4}[4k_1 - (7k_3^3 - 36k_3^2k_4 + 54k_3k_4^2 - 24k_4^3)t] \end{aligned} \quad (57)$$

Where k_1, k_2, k_3 *and* k_4 are arbitrary constants.
From Eq.(57) into Eqs.(42)-(43), we get the form for $\phi(x,t), w_0(x,t)$

$$\phi(x,t) = 1 + \exp[k_4x + \frac{1}{4}(4k_2 - (3k_3^2k_4 - 12k_3k_4^2 + 10k_4^3)t)] \quad (58)$$

$$w_0(x,t) = \exp[k_3x + \frac{1}{4}(4k_1 - (7k_3^3 - 36k_3^2k_4 + 54k_3k_4^2 - 24k_4^3)t)] \quad (59)$$

Substituting from Eqs(58),(59) into Eqs.(39)-(41) and Combining all terms, we find a family of the analytical solutions of Eqs.(1)-(3) as follows,

$$u(x,t) = -k_4^2 \sec h^2(\frac{T}{2}) + \frac{1}{4}(-k_3^2 + 4k_3k_4 - 2k_4^2),$$

$$\begin{aligned} v(x,t) &= 4k_4^4\frac{e^{4T}}{e^M(1+e^T)^2} + (k3 - 3k4)k_4^3\frac{e^{3T}}{e^M(1+e^T)} + (k_3 - k_4)^2k_4^2\frac{e^T}{e^M}, \\ w(x,t) &= \frac{e^M}{(1+e^T)^2} + (\frac{k_4 - k_3}{k_4})(\frac{e^T}{e^T(1+e^T)}) + \frac{(k_4 - k_3)^2}{4k_4^2}(\frac{e^M}{e^{2T}}). \end{aligned} \quad (60)$$

Where $T = A(t)x + B(t)$ and $M = C(t)x + S(t)$
.

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