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Generalized Static Analysis of Non-Planar Coupled Shear Walls with both Stiffening Beams and Stepwise Cross-Sectional Changes

[Engin EMSEN, Orhan AKSOGAN]

***Abstract*—In the present work, the static analysis of non-planar coupled shear walls with any number of stiffening beams has been considered. The properties of the connecting beams and the cross-sectional area of the piers are also assumed to be varying stepwise in the vertical direction. The analysis is based on the Continuous Connection Method (CCM), in conjunction with Vlasov's theory of thin-walled beams. A computer program has been prepared and using this computer program an asymmetrical example has been solved and compared with the solutions found by the frame method and a perfect match has been observed.**

***Keywords*—static analysis, non-planar, coupled shear wall, stiffening beam, cross-sectional change, continuous connection method.**

I. Introduction

In this paper, the static analysis of a non-planar coupled shear wall, resting on a rigid foundation, is studied. Employing a time saving computation procedure, the presented method is suitable for the pre-design and dimensioning of coupled shear wall structures. When the height restrictions prevent connecting beams from fulfilling their tasks of reducing the maximum bending moments at the bottom and the maximum lateral displacements at the top, beams with high moments of inertia, called "stiffening beams", are placed at certain heights to make up for this deficiency. Stiffening of coupled shear walls decreases the lateral displacements, thus, rendering an increase in the height of the building possible [3,4]. The analysis considers coupled shear walls with any number of stiffening beams. The properties of the connecting beams and the cross-sectional area of the piers are also assumed to be varying stepwise in the vertical direction [5].

The analysis is based on the Continuous Connection Method (CCM), in conjunction with Vlasov's theory [1] of thin-walled beams, following an approach similar to the one used by Tso and Biswas [2] to solve the static problem of non-planar coupled shear walls. In the CCM, the discrete connecting beams are replaced by an equivalent continuous system of laminae. No study has been seen by the author concerning the static analysis of non-planar coupled shear walls with both stiffening beams and stepwise cross-sectional changes in the literature.

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The present formulation is implemented with a Fortran computer program. Using this computer program an asymmetrical example has been solved and compared with the solutions found by the SAP2000 [8] analysis program using the frame method [6,7] and a perfect match has been observed.

II. Analysis

To explain this procedure, the top, bottom and each height at which there is a stiffening beam and/or change of wall cross-section will be called "ends" and the part between any two consecutive ends will be called a "region". A non-planar coupled shear wall and its plan for one region are given in Figures 1 with global axes OX, OY and OZ, the origin being at the mid-point of the clear span in the base plane.

The axial force (T_i) in the piers is determined from the differential equation which is obtained by using the compatibility and the equilibrium equations. All relevant quantities of the problem are determined employing their expressions in terms of the axial force [4,5]. Then, employing the equilibrium equations, the corresponding displacements are obtained.

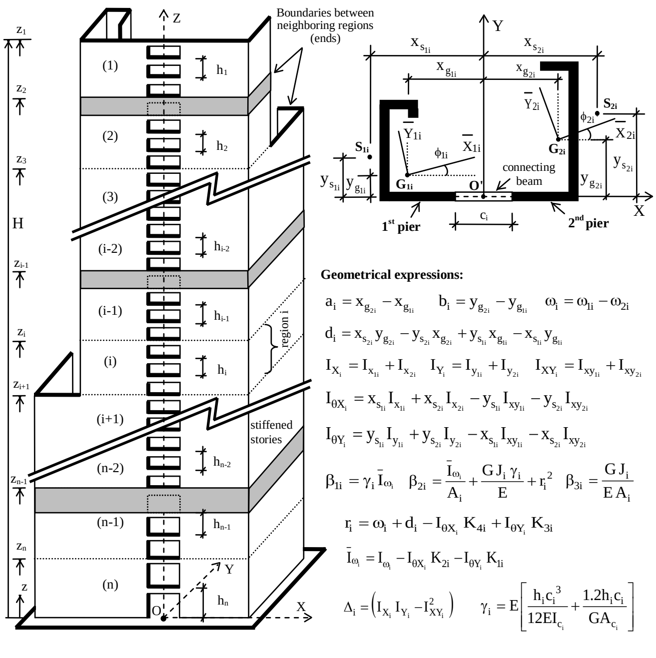

Figure 1. Non-planar coupled shear wall and a representative cross-section.

The basic assumptions of the CCM for non-planar coupled shear walls can be summarized as: The geometric and material properties are constant throughout each region i along the height. The discrete set of connecting beams with bending stiffness EI_{ci} in region i are replaced by an equivalent continuous connecting medium of flexural rigidity EI_{ci}/h_i per unit length in the vertical direction. The outline of a transverse section of the coupled shear wall at a floor level remains unchanged in plan (due to the rigid diaphragm assumption). The discrete shear forces in the connecting beams in region i are replaced by an equivalent continuous shear flow function q_i, per unit length in the vertical direction along the mid-points of the connecting laminae. The torsional stiffness of the connecting beams is neglected. The walls and beams are assumed to be linearly elastic. Bernoulli-Navier hypothesis is assumed to be valid for the connecting beams. The St. Venant twisting moment term ($GJ_i\theta_i'$) is neglected in the torsional equilibrium equation.

While obtaining the compatibility equations, all connecting laminae are cut through their mid-points, O', which are the points of zero moment. The vertical displacement due to bending can be obtained as the product of the slope at the section considered and the distance of point O' from the respective neutral axis. In addition, vertical displacement arises, also, due to the twisting of the piers, and is equal to the value of the twist at the section considered, times the sectorial area, ω, at point O'. For the compatibility of displacements, the relative vertical displacements of the cut ends must be equal to zero. The terms of the compatibility equation are the contributions of the bending of the piers about the principal axes, the twisting of the piers, the axial deformation of the piers, the bending deformation in the laminae and the shearing deformation in the piers. Differentiating the compatibility equation with respect to z the following equation is obtained (i = 1,2,...,n):

$$u_i'' a_i + v_i'' b_i + \theta_i''(\omega_i + d_i) - \frac{1}{E}\left(\frac{1}{A_{1i}} + \frac{1}{A_{2i}}\right)T_i + \frac{T_i''}{E}\gamma_i = 0 \qquad (1)$$

The internal moments, along with the couple produced by the axial force, T_i, balance the external bending moments M_{EX_i} and M_{EY_i}. The equilibrium of the moments about the X and Y axes yields

$$M_{EX_i} = EI_{XY_i}u_i'' + EI_{X_i}v_i'' + EI_{\theta X_i}\theta_i'' + T_i b_i \qquad (2)$$

$$M_{EY_i} = EI_{Y_i}u_i'' + EI_{XY_i}v_i'' - EI_{\theta Y_i}\theta_i'' + T_i a_i \qquad (3)$$

In order to obtain the bimoment and the twisting moment equilibrium equations, the coupled shear wall will be cut through by a horizontal plane such that an upper part is isolated from the lower part of the structure. Equating the external bimoment, B_{E_i}, and the external twisting moment, M_{Et_i}, to the internal resisting bimoments, the bimoment and the twisting moment equilibrium equations for all regions of the structure can be written as follows:

$$B_{E_i} = EI_{\theta Y_i}u_i'' - EI_{\theta X_i}v_i'' - EI_{\omega_i}\theta_i'' - (\omega_i + d_i)T_i \qquad (4)$$

$$M_{Et_i} = EI_{\theta Y_i}u_i''' - EI_{\theta X_i}v_i''' + GJ_i\theta_i' - EI_{\omega_i}\theta_i''' - (\omega_i + d_i)T_i' \qquad (5)$$

Using the compatibility equation (1) and the equilibrium equations (2-5), the 4n unknowns of the problem, namely u_i, v_i, θ_i, and T_i, can be found under the applied loadings M_{EX_i}, M_{EY_i}, B_{E_i}, and M_{Et_i}. The elimination of u_i, v_i and θ_i from (1-5) yields the following differential equation for T_i:

$$\begin{aligned}(\beta_{1i})T_i'''' - (\beta_{2i})T_i'' + (\beta_{3i})T_i &= -M_{EY_i}''\left(\bar{I}_{\omega_i}K_{3i} + K_{1i}r_i\right) \\ -M_{EX_i}''\left(\bar{I}_{\omega_i}K_{4i} - K_{2i}r_i\right) &+ \frac{GJ_i}{E}\left(M_{EY_i}K_{3i} + M_{EX_i}K_{4i}\right) + M_{Et_i}' r_i \qquad (6)\end{aligned}$$

Thus, the governing differential equation of the analysis of non-planar coupled shear walls is found as (6). This equation is written for each region separately. Solving the resulting differential equation, T_i is found as follows:

$$\begin{aligned} T_i = {} & D_{1i}\,\mathrm{Sinh}[\alpha_{1i} z] + D_{2i}\,\mathrm{Cosh}[\alpha_{1i} z] \\ & + D_{3i}\,\mathrm{Sinh}[\alpha_{2i} z] + D_{4i}\,\mathrm{Cosh}[\alpha_{2i} z] \\ & + \frac{1}{2\beta_{3i}^2 E}\Big\{\ 2\beta_{2i}GJ_i\left(K_{3i}W_X + K_{4i}W_Y\right) \\ & + \beta_{3i}\ \big[GJ_i(H-z)\big(2K_{3i}P_X + 2K_{4i}P_Y + HK_{3i}W_X \\ & + HK_{4i}W_Y - (K_{3i}W_X + K_{4i}W_Y)z\big) \\ & - 2E\big(\bar{I}_{\omega_i}K_{3i}W_X - d_{WY}r_iW_X + K_{1i}r_iW_X \\ & + \bar{I}_{\omega_i}K_{4i}W_Y + d_{WX}r_iW_Y - K_{2i}r_iW_Y\big)\big]\Big\} \qquad (7)\end{aligned}$$

in which

$$\alpha_{1i} = \sqrt{\left(\frac{\beta_{2i} - \sqrt{\beta_{2i}^2 - 4\beta_{1i}\beta_{3i}}}{2\beta_{1i}}\right)} \quad \alpha_{2i} = \sqrt{\left(\frac{\beta_{2i} + \sqrt{\beta_{2i}^2 - 4\beta_{1i}\beta_{3i}}}{2\beta_{1i}}\right)} \qquad (8)$$

$$K_{1i} = \frac{\left(I_{X_i}I_{\theta Y_i} + I_{XY_i}I_{\theta X_i}\right)}{\Delta_i} \quad K_{2i} = \frac{\left(I_{XY_i}I_{\theta Y_i} + I_{Y_i}I_{\theta X_i}\right)}{\Delta_i}$$

$$K_{3i} = \frac{\left(a_iI_{X_i} - b_iI_{XY_i}\right)}{\Delta_i} \quad K_{4i} = -\frac{\left(a_iI_{XY_i} - b_iI_{Y_i}\right)}{\Delta_i} \qquad (9)$$

To determine the integration constants D_{1i} to D_{4i}, the boundary conditions at the top, bottom and between each pair of consecutive regions are employed. Then, the general solutions for the displacements u_i, v_i and θ_i can be found using (2,3,5).

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III. Numerical Results

In this study, the non-planar coupled shear wall structures are analyzed using Macleod's frame method [6], also, for comparison purposes. In Macleod's method, the planar wall units are modelled as column members in wide column analogy. Macleod's 3-D wide-column-frame analogy for core wall analysis has been improved over the time in the literature [7]. In this study, as a modification to Macleod's method, additional rigid beam members are placed between the storey levels to improve the continuity of the connection between the wall units. This modification was observed to improve the results by various comparisons with the CCM.

In order to verify the present method, several examples were solved both by the present method (CCM) and by the frame method using the SAP2000 structural analysis program [8]. As an example, the static analysis of a coupled shear wall with and without stiffening beams was carried out under the effect of the external loads given in Figure 2. The piers of the example coupled shear wall have stepwise cross-sectional changes along the height, also.

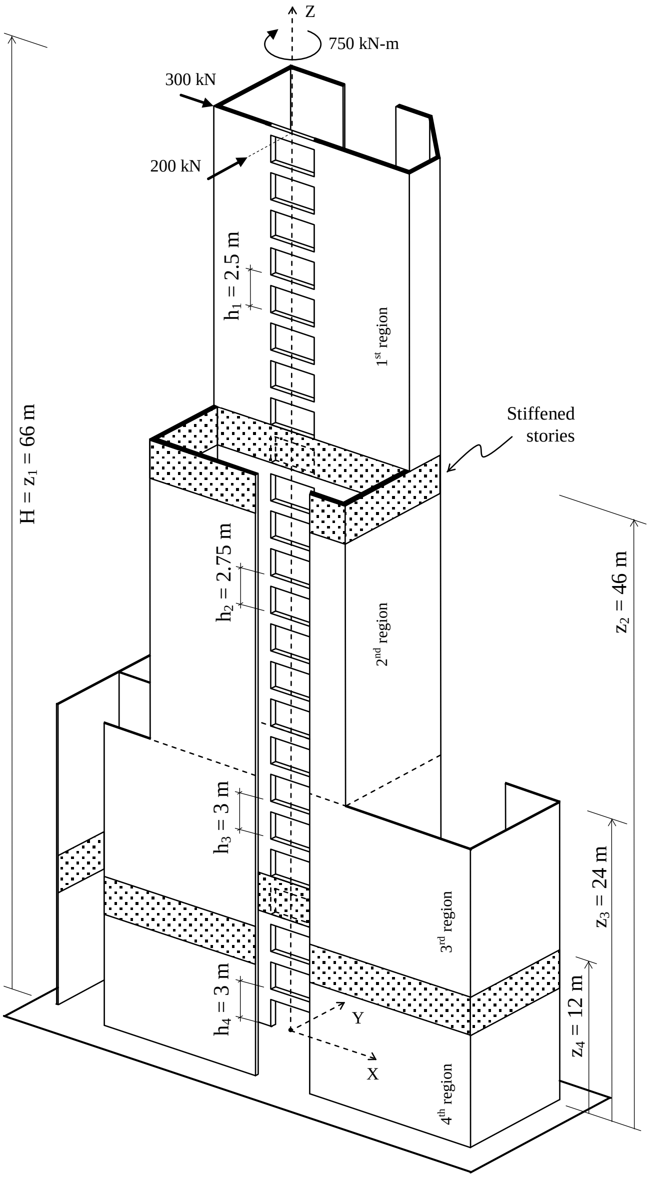

Figure 2. Non-planar non-symmetrical structure.

The total height of the non-symmetrical 24 storey structure is 66 m. The storey heights, the thicknesses of the piers and the connecting beams are shown in Figures 2-3. The height of the connecting beams is 0.4 m and the elasticity and shear moduli are $E = 2.85\times10^6$ kN/m^2 and $G = 1.06\times10^6$ kN/m^2, respectively. The geometrical properties of the piers of the example structure are presented in TABLE I. Stiffening beam of 3.0 m height is placed at the level of the fourth storey and another one with 2.75 m height at the level of the sixteenth storey, as seen in Figure 2.

The lateral displacements and the rotations at the midpoint of the connecting beams are compared for the unstiffened and stiffened cases, in Figure 4. Figure 5-7 present the variation of axial forces and the total shear wall bending moments along the height.

TABLE I. Geometrical Properties of the Piers

$\mathbf{A_{ji}}$: Cross sectional area of the jth pier in region i	A_{11}	2.400 m^2	A_{21}	2.700 m^2
	A_{12}	4.500 m^2	A_{22}	4.050 m^2
	A_{13}	7.500 m^2	A_{23}	6.250 m^2
$\mathbf{G_{Xji}}$, $\mathbf{G_{Yji}}$: Coordinates of centroid of the jth pier in region i, referring to global axes, X and Y, respectively.	G_{X11}	-2.500 m	G_{X21}	2.917 m
	G_{Y11}	2.000 m	G_{Y21}	1.722 m
	G_{X12}	-2.200 m	G_{X22}	3.194 m
	G_{Y12}	-0.033 m	G_{Y22}	0.482 m
	G_{X13}	-3.693 m	G_{X23}	4.566 m
	G_{Y13}	0.127 m	G_{Y23}	-0.024 m
$\mathbf{I_{Xji}}$, $\mathbf{I_{Yji}}$: Moments of inertia of the jth pier in region i w.r.t. axes passing through the centroid parallel to global axes.	I_{X11}	6.409 m^4	I_{X21}	7.004 m^4
	I_{Y11}	1.009 m^4	I_{Y21}	2.464 m^4
	I_{X12}	29.513 m^4	I_{X22}	20.827 m^4
	I_{Y12}	5.136 m^4	I_{Y22}	3.349 m^4
	I_{X13}	45.470 m^4	I_{X23}	32.899 m^4
	I_{Y13}	35.349 m^4	I_{Y23}	27.979 m^4
$\mathbf{I_{XYji}}$: Product of inertia of the jth pier in region i w.r.t. axes passing through the centroid parallel to global axes.	I_{XY11}	0.000 m^4	I_{XY21}	-0.885 m^4
	I_{XY12}	-4.680 m^4	I_{XY22}	-3.339 m^4
	I_{XY13}	-10.541 m^4	I_{XY23}	-9.225 m^4
$\mathbf{J_{ji}}$: St. Venant torsional constant (moment of inertia) of the jth pier in region i.	J_{11}	0.072 m^4	J_{21}	0.081 m^4
	J_{12}	0.135 m^4	J_{22}	0.122 m^4
	J_{13}	0.175 m^4	J_{23}	0.151 m^4
$\mathbf{S_{Xji}}$, $\mathbf{S_{Yji}}$: Coordinates of shear center of the jth pier in region i, referring to global axes, X and Y, respectively.	S_{X11}	-3.748 m	S_{X21}	5.005 m
	S_{Y11}	2.000 m	S_{Y21}	1.023 m
	S_{X12}	-3.790 m	S_{X22}	4.684 m
	S_{Y12}	-1.648 m	S_{Y22}	0.593 m
	S_{X13}	-4.627 m	S_{X23}	6.182 m
	S_{Y13}	-0.852 m	S_{Y23}	-4.664 m
$\mathbf{I_{\omega ji}}$: Sectorial moment of inertia of the jth pier in region i.	$I_{\omega 11}$	2.800 m^6	$I_{\omega 21}$	2.890 m^6
	$I_{\omega 12}$	20.657 m^6	$I_{\omega 22}$	7.246 m^6
	$I_{\omega 13}$	152.169 m^6	$I_{\omega 23}$	164.834 m^6
$\boldsymbol{\omega_{ji}}$: Sectorial area of the jth pier in region i at point O.	ω_{11}	4.504 m^2	ω_{21}	-2.670 m^2
	ω_{12}	-4.666 m^2	ω_{22}	-2.259 m^2
	ω_{13}	-2.393 m^2	ω_{23}	14.328 m^2

IV. Conclusion

In this study, the static analysis of non-planar coupled shear walls with both any number of stiffening beams and stepwise cross-sectional changes is carried out. As an example, the non-planar non-symmetrical 24 storey coupled shear wall structure is considered. The results obtained are compared with those of frame method and a good agreement is observed for the stiffened and unstiffened cases. As seen in the figures, the stiffening of coupled shear walls causes a decrease in the maximum displacements at the top and the maximum bending moments at the bottom of a building. Thus, by using such stiffening beams the heights of buildings can be increased more.

The method proposed in this study has two main advantages. First, the data preparation is much easier compared to the frame method. Second, modeling and computation time needed is much shorter compared to the other methods for non-planar coupled shear walls. Hence, the method presented is very useful for pre-design and dimensioning purposes while determining the geometry of non-planar coupled shear wall structures. Furthermore, through this study, the static behavior of the non-planar coupled shear walls is determined analytically.

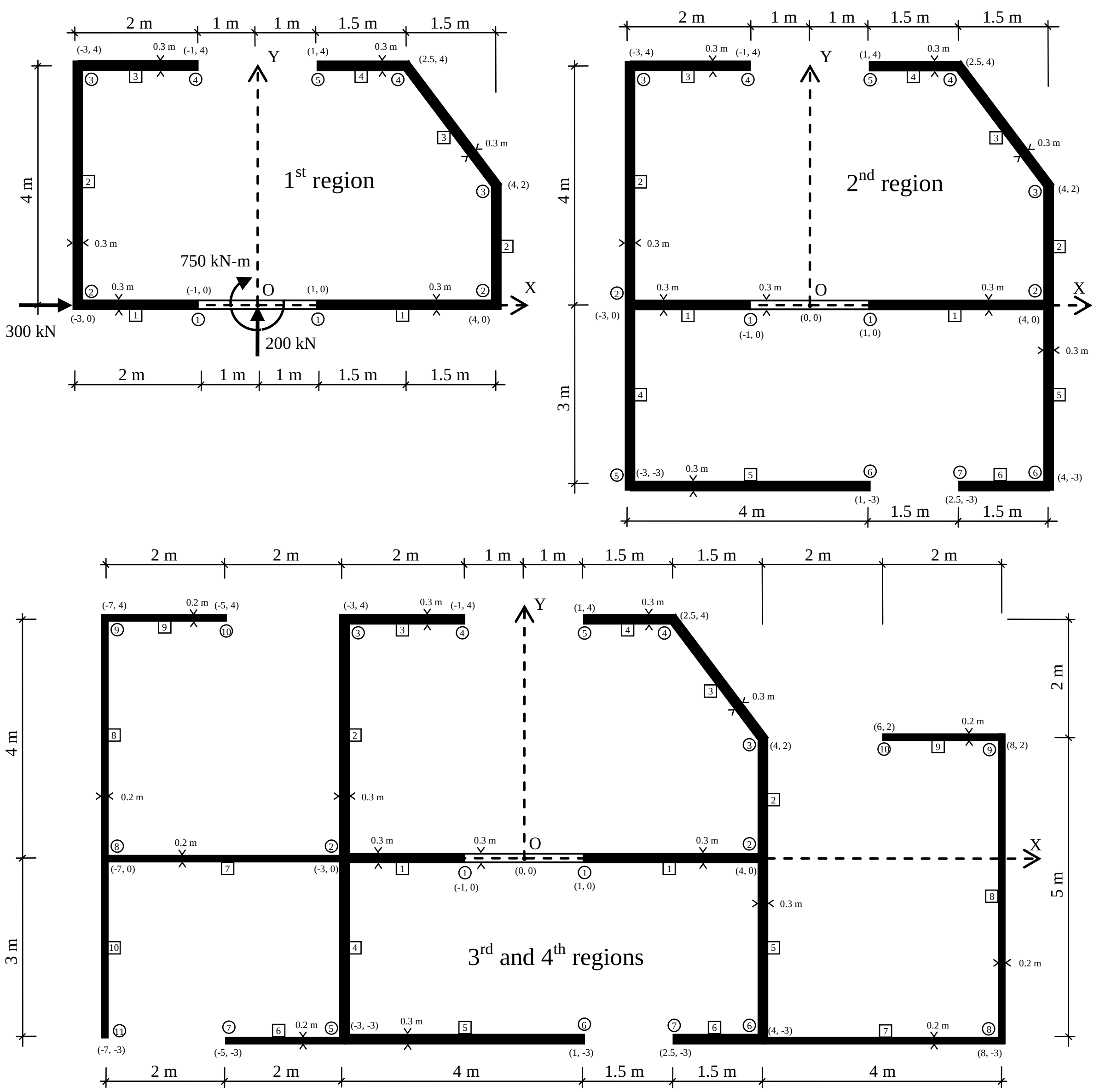

Figure 3. Cross-sectional view of the regions of the example structure.

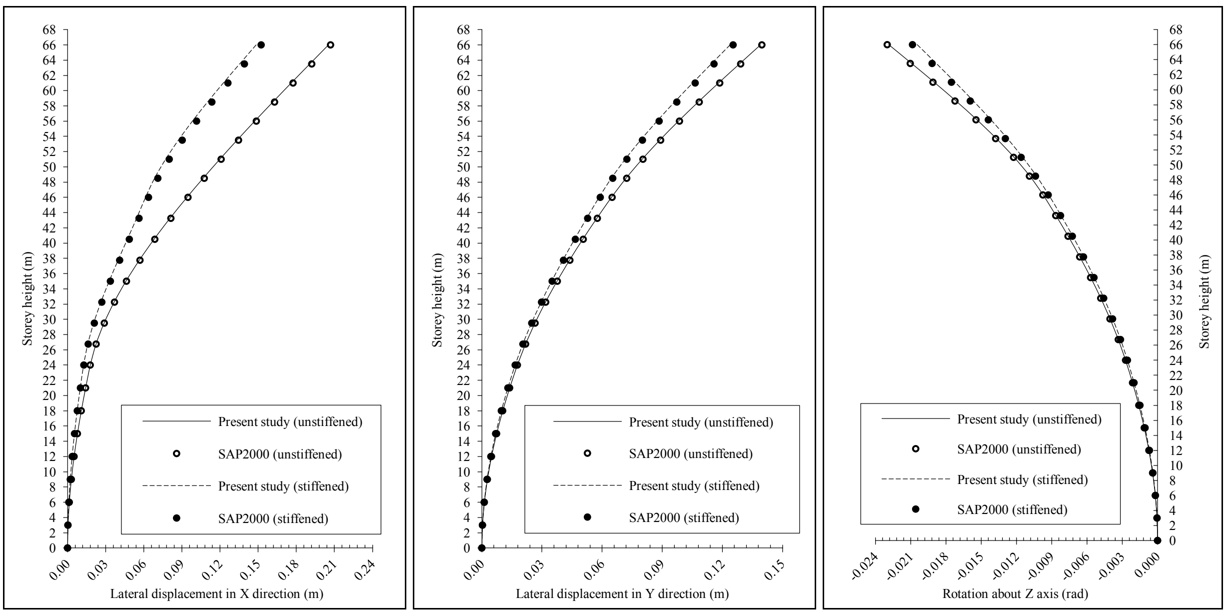

Figure 4. The lateral displacements and rotations at the midpoint of the connecting beams.

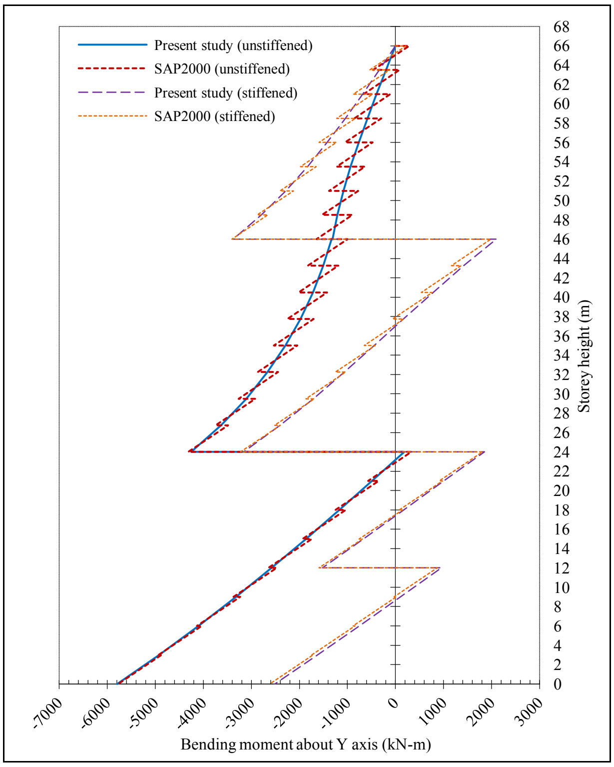

Figure 5. The total shear wall bending moment about Y axis along the height.

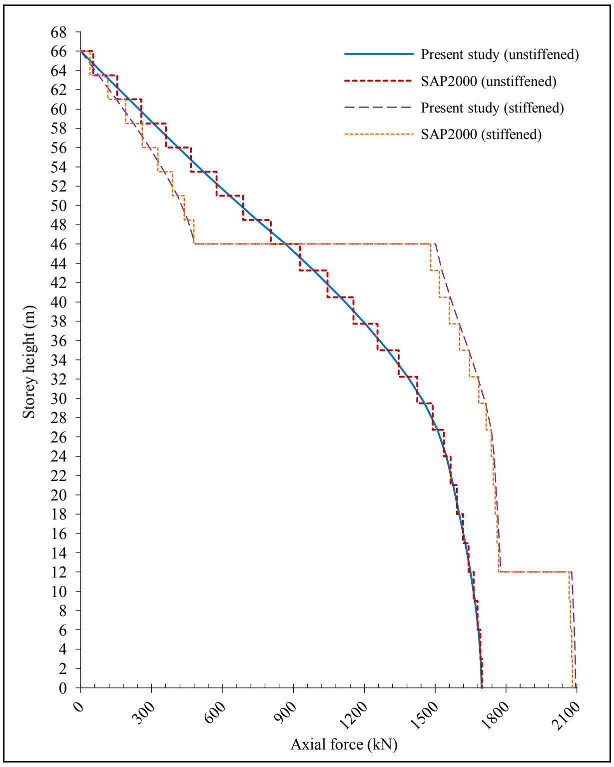

Figure 7. The axial forces along the height.

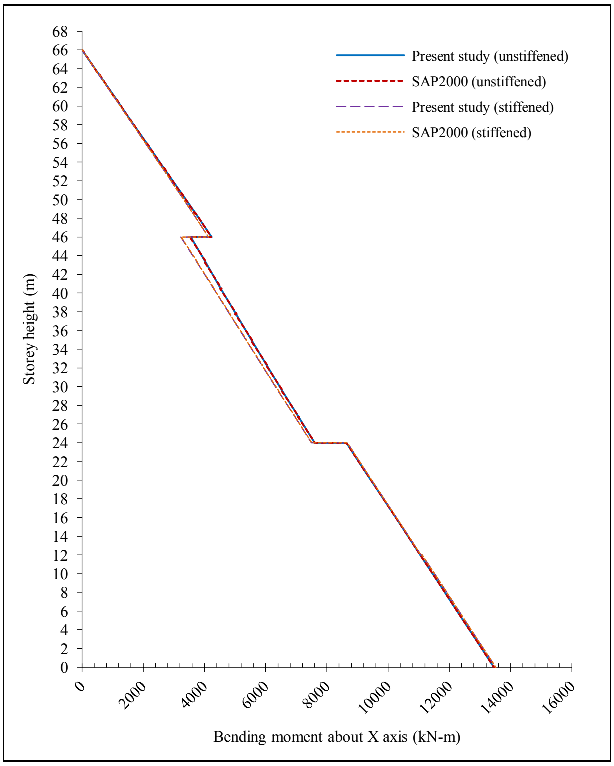

Figure 6. The total shear wall bending moment about X axis along the height.

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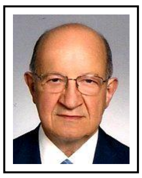

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