International Journal of Civil and Structural Engineering– IJCSE
Volume 3 : Issue 1 *[ISSN : 2372-3971]*
Publication Date: 18 April, 2016

Effect Of Supports Position On The Immediate Deflection For Reinforced Concrete Beam

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Abstract—Deflection computations for reinforced concrete beams form an important part in the design procedure of reinforced concrete structures. This study is divided into two parts; the first part studies the sensitivity of models to the experimental load-deflection from previous work, and the second part studies the effect of vertical eccentricity according to the supports position on beam deflection behavior. The analysis is performed using a three-dimensional model to determine the deflections in beams. A finite element analysis program is used considering nonlinear material behavior. The finite element model is chosen to conduct a parametric study in order to investigate the effect of vertical eccentricity according to the supports position on beam behavior. The study indicates that values of deflection depend on the position of supports (at below the beam or at neutral axis of the beam).

*Keywords—b*eam, concrete, FEM, deflection, nonlinear.

I. Introduction

Large deflection in beams may cause serviceability problems and may lead to failure. There are several methods that can be used for predicting beam behavior. The methods include Double Integration Method, Moment Area Method, Virtual Work Method, Super Imposed Method, Coupled Beam Method, Energy Method and Castigliano Theorem [1]. These methods can be considered as analytical solution. In analytical solution, it is assumed that beam supports were located at the beam neutral axis. On the effect of wide support, there are several composite slab experimental tests that use pour stop or end stop at the edge of the slab or beam. The pour stop at the outer side of the support may provide some stiffness to bending. While in fixed end condition, beam is rigidly connected to supports such as columns and therefore its stiffness increases.

Traditionally, analytical methods assume beams to be supported at their neutral axes. In these methods, eccentricity between beam support and beam neutral axis is neglected. However, in most bending tests, beam specimens are supported at the bottom face. This produces a vertical eccentricity between beam support and beam neutral axis. The objective of this paper is to determine the effect of eccentricity between beam support and neutral axis on immediate deflection of reinforced concrete beams. The study is limited to beams with uniformly distributed loads.

Some building codes require the control of deflection of flexural members as part of the serviceability requirements of the structure. It is well known, however, that the deflection computations involve the evaluation of a number of geometrical and material properties. Of particular interest is the flexural rigidity, EI, of the partially cracked member since its value changes as cracking propagates under loads. A simplified empirical model is used to predict the effective value of the beams moment of inertia to be used in the deflection computations which was originally proposed by Branson [2] in form

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \qquad (1)$$

Where, $M_a \geq M_{cr}$; otherwise $I_e = I_g$. This equation has been adopted by the ACI-318 Building Code [2] since 1971.

To calculate beam deflections a standard fundamental formula is used to determine deflections base on beam curvature. This is given by the expression:

$$Curvature = \frac{1}{R} = \frac{M}{EI} = -\frac{d^2 y}{dx^2} \qquad (2)$$

Where

R = the radius of the shape of the curved beam at a distance x from the origin, normally taken at the left or right hand end of the beam

E = Young's modulus of the material from which the beam is fabricated.

M = the bending moment at the section, distance x from the origin

y = the vertical deflection at the section distance x from the origin.

In the above formula E and I are normally constant values whilst y, x, and M are variables. M can be expressed in terms of distance x and hence integrating the above expression twice will enable the deflection v to be calculated [1].

II. Literature Review

A researcher [4] studied the effect of eccentricity at beam support on beam stiffness. A beam reacts to loading through bending action. Therefore, beam bending stiffness can be inversely proportional to deflection. Theoretically, beam stiffness is governed by span length, elastic modulus, moment of inertia and support type. In the analytical analysis, beams are assumed simply supported or fixed supported. However, based on real cases and lab experiments there are other factors that are not included in the theoretical equation but affect the beam stiffness. Factors such as eccentricity between beam neutral axis and beam support (vertical eccentricity), pour stop stiffness in composite beam/slab effect and column size effect were analyzed in this study. The effects were studied using plane

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stress element finite. Pour stop stiffness were modeled using spring element. From the analysis, vertical eccentricity does not give significant effect to beam stiffness and it can be neglected. Beam deflection is independent on column deflection when column width is three times bigger than beam depth. The models are 2-D Finite Element in plane stress condition, using linear elastic materials. Analysis are performed to examine; The effect of vertical eccentricity at support, The effect of restraining the beam (ends at supports) on the beam stiffness. The effect of column beam size to stiffness. There are three main conclusion can be drawn from this study. For the study on the effect of vertical eccentricity to beam stiffness, it can be concluded that the difference between beam supported at their neutral axis and beam supported at the bottom face is very small. Hence, vertical eccentricity at beam support can be neglected. However, as beam depth increase, the difference between result from finite element analysis and analytical solution also increased.

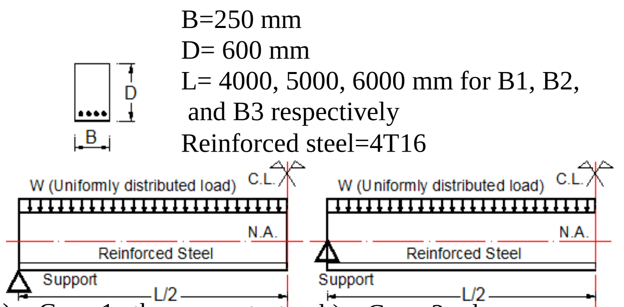
III. Finite Element Models

For this study, finite element analyses, which were performed using the ANSYS software [5], were used to investigate the behavior of the beams reinforced with high-strength steel as a flexural reinforcement, with different beam sizes and different loading values.

Three pairs of reinforced concrete beams with different lengths and boundary conditions were modeled without web reinforcement. The loading arrangement, geometrical properties and reinforcement distribution of the analyzed beams are shown in Fig. 1. All beams are subjected to uniformly distributed load acting at mid-span. Only the longitudinal reinforcement has been considered.

A. Element Types

A solid element, SOLID65, is used to model the concrete in ANSYS. The solid element has eight nodes with three degrees of freedom at each node, translations in the nodal x, y, and z directions. The element is capable of plastic deformation, and cracking in three orthogonal directions. A LINK8 element is used to model the steel reinforcement. Two nodes are required for this element. At each node, degrees of freedom are identical to those for the SOLID65. The element is also capable of plastic deformation.



a) Case 1: the support at the base.

b) Case 2: the support at the neutral axis.

Figure 1. Reinforced concrete beam models.

B. Material Properties

Concrete: SOLID65 elements are capable of predicting the nonlinear behavior of concrete materials using a smeared crack approach. The smeared crack approach has been adopted widely in the recent decades. Concrete is a quasi-brittle material and has very different behaviors in compression and tension. The tensile strength of concrete is typically 8-15% of the compressive strength. The ultimate concrete compressive and tensile strengths for each beam model were calculated by Eqs. (3& 4) [3] respectively.

$$f_c' = \left(\frac{E_c}{4700}\right)^2 \tag{3}$$

$$f_r = 0.623\sqrt{f_c'} \tag{4}$$

where:

E_c = elastic modulus of concrete, MPa

f_c' = ultimate compressive strength, MPa

f_r = ultimate tensile strength, MPa

Next, Eqs. (5& 6) are used along with Eq. (7) to construct the uniaxial compressive stress-strain curve for concrete in this study.

$$f = \frac{E_c \varepsilon}{1 + \left(\frac{\varepsilon}{\varepsilon_0}\right)^2} \tag{5}$$

$$\varepsilon_0 = \frac{2 f_c'}{E_c} \tag{6}$$

$$E_c = \frac{f}{\varepsilon} \tag{7}$$

where:

f = stress at any strain, MPa

ε = strain at stress f

E_c = concrete elastic modulus, MPa

ε_0 = strain at the ultimate compressive strength f_c'

In tension, the stress-strain curve for concrete is assumed to be linearly elastic up to the ultimate tensile strength. After this point, the concrete cracks and the strength decreases to zero. Fig. 2 shows the simplified uniaxial stress-strain relationship that is used in this study.

Poisson’s ratio for concrete is assumed to be 0.2 and is used for all beams. The value of a shear transfer coefficient, representing conditions of the crack face, used in many studies of reinforced concrete structures varied between 0.05 and 0.25. The shear transfer coefficient used in this study is equal to 0.2.

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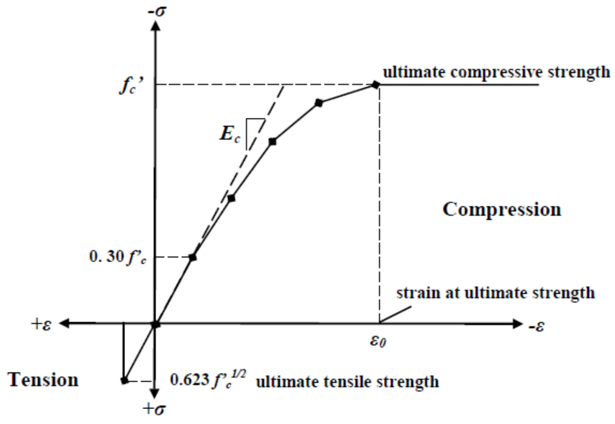


Figure 2. Simplified uniaxial stress-strain curve for concrete [5].

Steel Reinforcement: Steel reinforcement in the beams was modeled with typical steel reinforcing bars. Elastic modulus and yield stress (f_y = 410 Mpa) for the steel reinforcement used in this FEM study follow the design material properties used for the previous experimental investigation [6]. The steel for the finite element models is assumed to be an elastic-perfectly plastic material and identical in tension and compression. A Poisson's ratio of 0.3 is used for the steel reinforcement. Fig. 3 shows the stress-strain relationship used in this study. Material properties for the concrete and steel reinforcement are summarized in Table I.

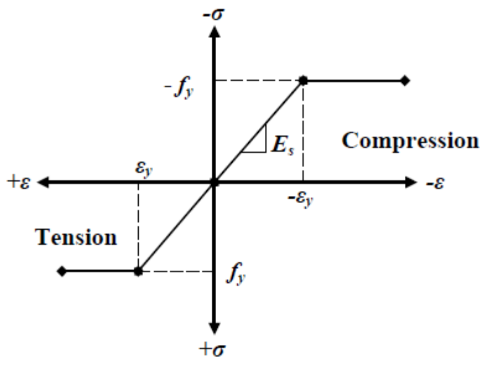


Figure 3. Stress-strain curve for steel reinforcement.

TABLE I. SUMMARY OF MATERIAL PROPERTIES FOR REINFORCED CONCRETE [5].

Concrete				Steel Rebar		
E_c (GPa)	f'_c (MPa)	f_r (MPa)	υ	E_s (GPa)	F_y (MPa)	υ
22	20	2.4	0.2	200	410	0.3

C. *Modeling Methodology*

By taking advantage of the symmetry of the beams, a half of the full beam is used for modeling with proper boundary conditions. This approach reduces computational time and computer disk space requirements significantly. The steel reinforcement is simplified in the model by ignoring the stirrups and top reinforcement. Ideally, the bond strength between the concrete and steel reinforcement should be considered. However, in this study, perfect bond between materials is assumed. Fig. 4 shows the mesh of finite element model.

D. *Sensitivity Of Finite Element Model*

The goal of the comparison of the finite element model results with the experimental results is to ensure that the elements, material properties, real constants and convergence criteria are adequate to express the behavior response of the member. The results obtained by the numerical finite element model for the beam is compared with the experimental results described in previous research [6].

Deflections are measured at mid-span at the center of the bottom face of the beams. Fig. 5 shows the load-deflection plots for the simple beams. In general, the load deflection plots for the beams from the finite element analyses agree quite well with the experimental data. The finite element load-deflection plots in the linear range are the same values of the experimental plots, but the beam experienced cracks during the experiment under a load 25% less than predicted by the FEM. After first cracking, the deflection values of the finite element models were higher than that of the experimental beams by about 8%. That indicates that, the ANSYS model predicted the load and deflection at various stages, namely, at cracking and at ultimate quite accurately.

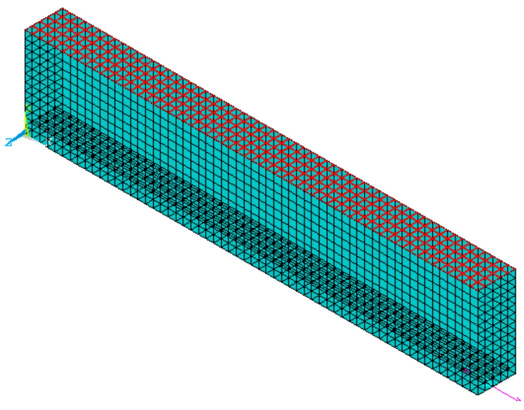


Figure 4. Finite elements mesh.

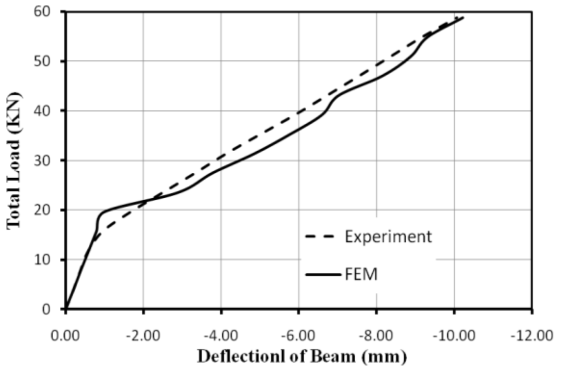


Figure 5. Load-Deflection Plot.

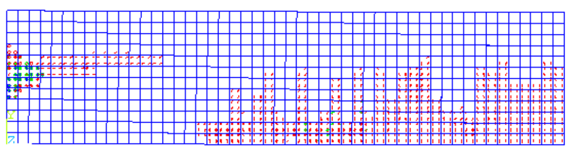
IV. Results And Discussions

A. Crack Patterns

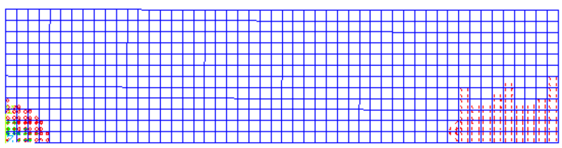
Fig. 6 shows that the cracks predicted using the different finite element models. The ANSYS model predicted cracking of concrete at the ultimate load, which was indicated by large deformation at the node. ANSYS program displays circles at locations of cracking or crushing in concrete elements. Cracking is shown with a circle outline in the plane of the crack, and crushing is shown with an octahedron outline. The crack pattern in case a support at natural axis gives cracks and their lengths are more than the second case (support at bottom of beam).

B. Load Deflection Curves

The mid-span deflection values for three pairs of beams with different lengths and equal sections are calculated and plotted as shown in Figs.7- 9. The six beams are divided into three groups each group consists of a pair of beams the first beam is supported at bottom of beam while the second is supported at the neutral axis. All beams appeared to display linear behavior to the cracking load point and from that point to first yield of the steel reinforcement. After yielding of the reinforcement began, a large increase in deflection was noticed, while the applied load changed little, this behavior continued until failure was happened.



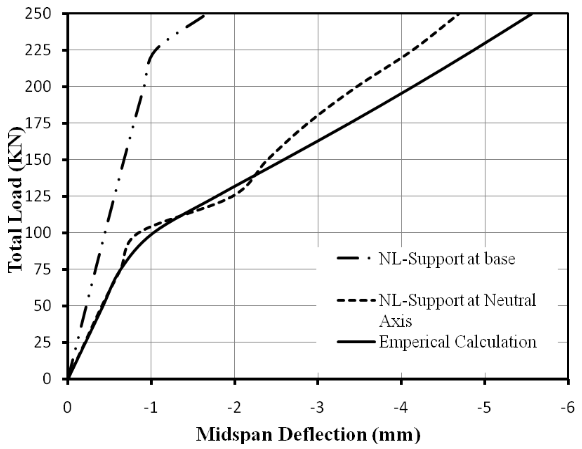
a. Crack pattern in case of support at neutral axis.



b. Crack pattern in case of support at bottom.

Figure 6. Crack Patterns for the Beam at the Same Load.

The values of deflection at the beam with support at neutral axis are in good agreement with the mathematical calculations. The values of the deflection based on mathematical calculations are higher than the deflection values resulted from the FEM model with support at bottom by about 80% after cracking of the beam. The difference in deflection value may be explained by the fact that the support place at the bottom of beam provides a higher level of restraint and the connection becomes more rigid.



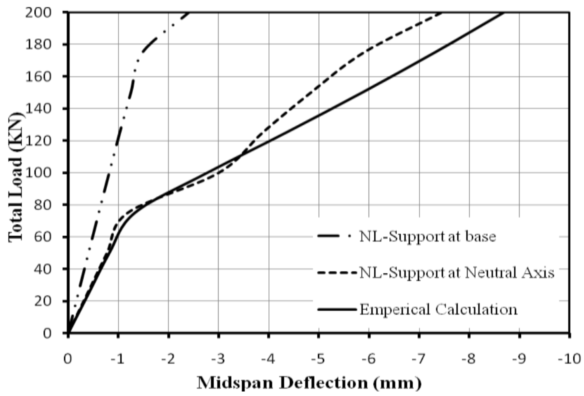


Figure 7. Load-Deflection Curve for Beam B1.

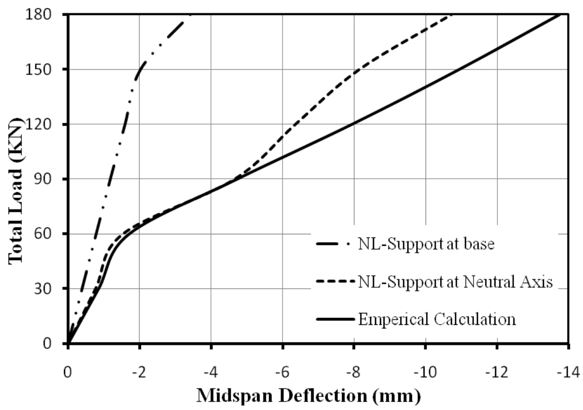


Figure 8. Load-Deflection Curve for Beam B2.

Figure 9. Load-Deflection Curve for Beam B3.

V. CONCLUSIONS

From the previous results, namely, loads, deflections using the model the following conclusions can be drawn

- The presented finite element model is capable of producing results in good agreement with previous published test results and it can be confidently used in design and analysis situations.
- The value of deflection based on the actual simulation (support at the bottom of the beam) is lower than the value from mathematical calculation by 75-80%.
- Most of the previous beam test (hinge-roller beam) are not simulating the actual condition, where the beams in reality are hinged or partially fixed at both ends.

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This research mainly focus on the changing in the deflection, where that it depends on the position of supports (at below the beam or at neutral axis of the beam).