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Theoretical-Experimental Model for the Design of Vortex Drop Shafts

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***Abstract*—This paper proposes two procedures for designing a vortex drop shaft, both of which are based on a recently set up mathematical model. This model is different from previous ones as it considers the possible existence of negative pressure at the drop shaft inlet, thus allowing the incongruence of the previous models to be overcome. This paper presents an analysis of the reliability of the model, within which the calculation parameters have been tested using experimental data from a number of working drop shafts.**

Keywords—drop shaft, vortex flow, mathematical model, experimental tests, hydraulic design.

I. Introduction

In surface flow processes, the phenomena accompanying changes in drop height are of great technical interest, especially when these develop in the context of large drops with considerable inlet flows and, consequently, extensive dynamic actions (i.e. outlet works of artificial reservoirs). If it is impossible or undesirable to subdivide the drop height into a number of steps, the cheapest and clearly the simplest solution appears to entail channelling the flow into a vertical shaft [1, 2, 3]. In this case, however, the free-falling stream can set off such phenomena as flow instability and pressure fluctuations which accompany irregularities in air entrainment inside the shaft. The occurrence of temporary blockages and flow pulsations can then lead to detrimental vibrations in the plant structure. A particular type of inlet which eliminates the aforesaid drawbacks is called a vortex flow inlet and was conceived by Drioli [4] for sub-critical streams several decades ago. The inlet configuration, schematically shown in Figure 1, comprises a volute consisting of four arcs of a circle with precise geometrical relations between the radii (R_1, R_2, R_3 and R_4 in the Figure 1) and bending centres located on the perimeter (typically in the four angles) of a square (with a side measurement d to be defined) whose barycentre coincides with the vertical shaft axis.

The configuration of the inlet chamber gives the stream a swirling motion which forces it to adhere to the internal surface already in the upper sections of the vertical drop shaft.

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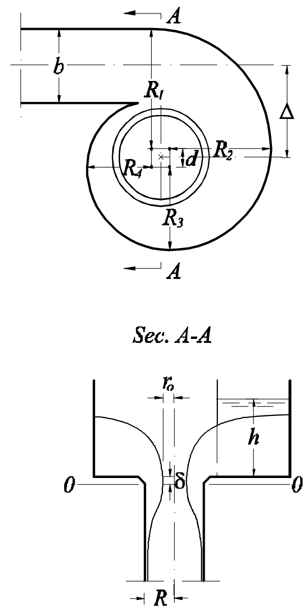

Figure 1: Configuration of the Drioli vortex-flow inlet.

The configuration of the inlet chamber gives the stream a swirling motion which forces it to adhere to the internal surface already in the upper sections of the vertical drop shaft.

Then, an almost circular space of radius r_o, which is called “the vortex core”, becomes available in the proximity of the axis, allowing the air to flow freely and thus eliminating flow irregularity. It should be noted that this remarkable property is maintained during operation even when the stream flow contains and/or transports various kinds of solid and other materials, thus making the device suitable, more generally, for rainfall drainage and discharge systems. After 1947, the interest provoked by the relatively simple structure of Drioli’s device led a number of researchers to investigate the hydrodynamic phenomena linked with vortex flow by means of theoretical and/or experimental methods, although not always obtaining analogous results [5-17]. It should also be remembered that the considerable practical applications of he free-falling problem led other groups of researchers to study inlet devices in shafts with geometries different from Drioli’s as well as investigating alternative configurations to the vertical conduit [18-21]. It must be pointed out that, overall, the vortex drop shaft not only achieves remarkable outflow stability but also ensures a notable energy dissipation along the entire length of the shaft and at its bottom: in a certain sense, this property makes the vortex drop shaft even more attractive from a technical point of view. The design and/or testing of a vortex drop shaft therefore involves three distinct but connected elements: i) the inlet chamber and its intake channel; ii) the vertical shaft; iii) the bottom bend and its outlet channel. From an energy point of view, the mathematical models employed when sizing a vortex drop shaft assume that the flow phenomena inside the inlet chamber result in a negligible energy loss, while a significant amount of energy is dissipated inside the drop shaft depending on the roughness of its inner wall, the flow rate, the shaft diameter and the drop height. Furthermore, the energy dissipated along the shaft and at the bottom bend are variably proportional: for high drops, the shaft generally accounts for most of the energy dissipated, while the opposite is true for small drops [10].

Sizing the vortex drop shaft is dependent on the determination of the experimental and/or theoretical parameters, which is not always a simple task and requires an assessment of their influence on the chosen calculation model in order to ensure that the achieved results are sufficiently reliable. An examination of the mathematical model set up in Naples [15, 16, 22, 23, 24] for the operation of the vortex inlet chamber conceived by Drioli will allow us to lay out some suggestions on sizing based on theoretical assessments and experimental data.

II. Theoretical-technical model

The School of Hydraulic Engineering of the University of Naples has always played a particularly active role in studying vortex flow. For about twenty years the sizing of a vortex drop shaft in order to define the outflow law has been based on a study by Viparelli [7]. Pica [13] later took up Viparelli's studies and proposed a praiseworthy sizing procedure practically based on a system of four equations. Hager [14] would later essentially reduce these to two dimensionless equations by means of a detailed analysis. In the same period, Ciaravino et al. [15, 16] proposed a model based on a series of equations common to models conceived by other authors [5-14] but differing from all the others in that it admits the existence of negative pressure values p in the 0-0 inlet section of the vertical shaft (Figure 1). The relations in common with other authors are: i) the Bernoulli equation; ii) the continuity equation; iii) the principle of maximum flow rate Q; iv) the Rankine vortex distribution of the tangential velocity component $Vt(r) = A/r$ (r is the distance from the shaft axis and A is the vortex characteristic constant); v) the flow rate momentum equation, which provides the expression of the constant $A=Q\Delta/bh$ (h is the head in proximity of the terminal section of the intake channel, of width b, and Δ is the distance between the channel axis and the shaft axis). The hypothesis of negative pressure in the shaft inlet section (which determines the sixth condition for the solution of the systems of equations) is derived from a thorough reading of Viparelli's experimental data [7] and from a series of subsequent experiments [15, 16, 22, 23, 24]. These negative values are justified by the bending of the trajectories in the vertical planes with a mean radius of r_{vm}. This bending is associated with pressures that are null in the vortex core and gradually decrease the further away from the shaft axis they are, contrasting the opposite effect of centrifugal force associated to the tangential velocity V_t. In the proposed model, the value of r_{vm} can be calculated using eqn (1):

$$r^* = \frac{r_{vm}}{R - r_o} = \frac{2}{(1-E)^3}\left(\sqrt{1-E} - E\ln\frac{1+\sqrt{1-E}}{\sqrt{E}}\right)^2 \qquad (1)$$

in which R is the shaft radius, r_o is the vortex core radius in the plane of section 0-0 and $E = (r_o/R)^2$.

Setting $p = 0$ for $r = r_o$, it is clear that, the pressure values are the result of two contributing components: i) one due to the bending of the trajectories in the horizontal plane, attributable to the tangential velocity component V_t, which corresponds to increasing pressure as r increases; ii) another due to bending in the vertical planes, which corresponds to pressure that decreases as the distance from the shaft axis increases. The former is given by eqn (2):

$$\frac{p'(r)}{\gamma} = \frac{A^2}{2g}\left(\frac{1}{r_0{}^2} - \frac{1}{r^2}\right) \qquad 2)$$

which at the wall ($r = R$) gives:

$$\frac{p'_R}{\gamma} = \frac{A^2}{2g}\left(\frac{1}{r_0{}^2} - \frac{1}{R^2}\right) \qquad (2')$$

the latter can be written as:

$$\frac{p''(r)}{\gamma} = \int_{r_0}^{r} -\frac{1}{\mathrm{g}}\frac{V_v{}^2}{r_v}\,\mathrm{dr} \qquad (3)$$

where V_v is the velocity component in the vertical plane and r_v is the bending radius of the trajectories in the same plane. Integration of eqn (3) up to the wall makes it possible to evaluate the entity of the contribution that the latter bending makes to the pressure on the shaft wall. Assuming a constant mean value V_{vm} of the vertical velocity component and an equivalent mean value r_{vm}, also constant, of the bending radius in the vertical planes, we can say that:

$$\frac{p''_R}{\gamma} = -\frac{1}{g}\frac{V_{vm}{}^2}{r_{vm}}(R - r_0) \qquad (4)$$

setting for V_{vm} the obvious eqn (5):

$$V_{vm} = \frac{Q}{\pi\left(R^2 - r_0{}^2\right)} \qquad ,(5)$$

the theoretical formulation of Ciaravino et Al. [15, 16] yields eqn (1).

From the values calculated for a number of drop shafts whose experimental data are available in the literature [7, 25] as well as from direct measurements [15,16, 22, 23, 24], the Authors have highlighted that the dimensionless value r^* of the bending radius in the vertical planes can be linearly correlated by means of the eqn (6):

$$r^* = m \cdot h^* + n \qquad (6)$$

with the likewise dimensionless parameter:

$$h^* = \sqrt{h(R + r_o)}\Big/R \qquad (7)$$

The values of m and n have been determined more recently [23, 24] by using the minimum squares method to linearly interpolate (Figure 2) the points obtained from the experimental data (h, Q) relative to ten reduced-scale models of shafts with different operating conditions but which can all be defined as having a strictly Drioli configuration. Specifically, shaft models 1-5 are from those used by Viparelli; 6-9 are reduced-scale models of plants built in Italy (shafts 6, 7 and 8) and France (shaft 9), for which the results of experimental investigations are reported by Drioli [25]; shaft 10 is the laboratory installation used in Naples to verify the proposed theoretical model. After processing the examined data, the following mean values (taken from the specific values of the individual shafts) are proposed for inlet chambers with a Drioli configuration [23, 24]: $m = 0.366$; $n = 0.910$.

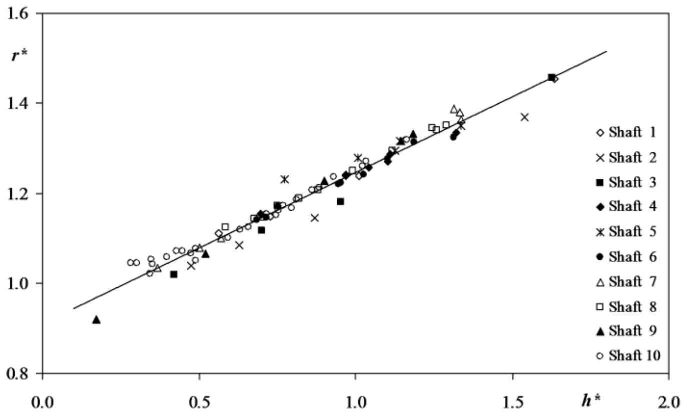

Figure 2: Values of $r^*(h^*)$ for the shafts under examination

The theoretical model, set up on the basis of the above mentioned considerations, has made it possible to propose a methodology for sizing vortex-flow inlets founded essentially on two equations:

$$\sqrt{1-E} - E\ln\frac{1+\sqrt{1-E}}{\sqrt{E}} = \sqrt{\left(m\sqrt{1-\sqrt{E}}\sqrt{\frac{h}{R}}+n\right)\frac{(1-E)^3}{2}} \quad (8)$$

$$\left[\frac{f^2}{E}\left(1+\frac{1}{2}\frac{b}{h}\frac{h}{R}\right)^2-1\right]\frac{Q^2}{2gh^5}=\varepsilon\left(\frac{b}{h}\right)^2 \quad (9)$$

in which:

$$f = \Delta/(R+b/2) \quad (10)$$

and

$$\varepsilon = (\delta + h)/h \quad (11)$$

where, in addition to the already mentioned symbols, δ is the length of the shaft inlet pipe (Figure 1).

These equations require the definition of the values not only of the above defined coefficients m and n but also of the f parameter (which also takes into account the size of the wall separating the intake channel from the drop shaft) and ε (which defines the size δ of the pipe that generally avoids detachment of the confined flow at the shaft inlet).

On the basis of the numerous operating vortex drop shafts examined, the parameters f and ε are seen to assume values in fairly limited ranges. After processing the examined data, the following values can be proposed: $f = 1.22$; $\varepsilon = 1.06$.

As for the E parameter (equal to the square of r_o/R, the ratio between the inlet radius ro of the vortex core and the shaft radius R), it is observed that a value close to 0.25 – corresponding to a r_o/R ratio of about 0.5 – can be considered, as suggested by Viparelli [7], a good compromise solution between the two opposed requirements of avoiding closure of the core vertex and oversizing the drop shaft.

III. Use of the model for design purposes

In addition to providing the outflow scale, the proposed model can also be used for design purposes. In this context it is both opportune and convenient to first report the dimensionless eqn (8) in graph form (Figure 3).

The two equations (eqn (8) depicted in Figure 3 and eqn (9)) make it possible to size a shaft using two different methods.

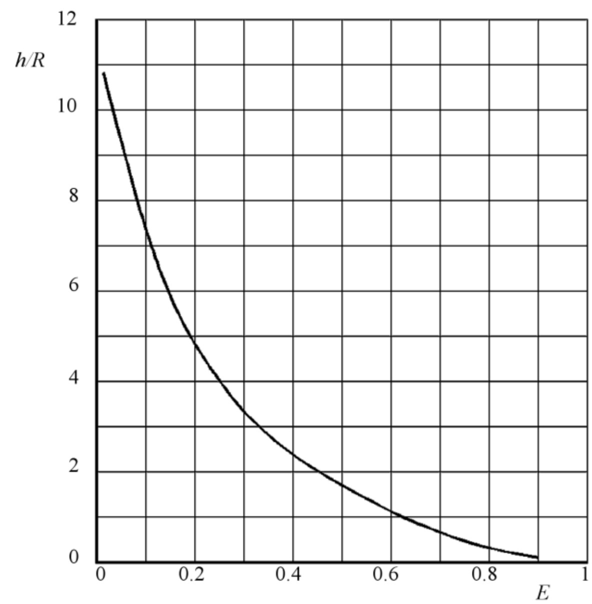

Figure 3: Graphical rappresentation of eqn (8).

3.1 First method

In order to ensure good shaft operation, the r_o/R ratio should be neither too small (otherwise the shaft may be subject to blockages) nor too large (otherwise the shaft will be oversized). In practice a r_o/R ratio of about 0.5 (corresponding to an E value of 0.25), once again according to Viparelli's observations [7], may be assumed as a good compromise between the two opposed requirements. By means of the graphical representation in Figure 3, the corresponding value of the *h/R* ratio is obtained. At this point the designer can choose the most appropriate *b/h* shape ratio of the inlet channel in order to obtain the dimensionless ratio $Q^2/2gh^5$ by means of eqn (9). Once this ratio has been obtained, and on the basis of the actual outflow value, it is finally possible to obtain the flow head *h* and then, by proceeding backwards on the basis of the *b/h* and *h/R* ratios, *b* and *R* respectively, with which the shaft will be properly sized. It goes without saying that this sizing method entails both the sizing of the shaft and the design of the final section of the intake channel, which must be such as to transport a flow *Q* with the b and h values fixed by the size of the shaft.

3.2 Second method

If the intake channel is pre-existing and *b* and *h* are fixed (and for an assigned design flow *Q*), it is necessary to proceed using the equations in the opposite direction. Specifically, when $Q^2/2gh^5$ and *b/h* are known, the set comprising the eqn (8) transposed in Figure 3 and eqn (9) constitute a system of two equations in the two unknowns *E* and *h/R*, which can thus be obtained. Moreover, the system does not allow a closed solution but requires successive attempts until the values of *E* and *h/b* are obtained. Obviously, as mentioned above, it must be ensured that the obtained value of *E* is neither too small so as to avoid blockages in the drop shaft nor too large so as to avoid excessive oversizing. In these cases it will also be necessary to make tentative variations in the value of *f* and ascertain the most convenient solution overall.

3.3 Relations between the radii R_1, R_2, R_3 and R_4

For both these methods, knowledge of the shaft radius *R*, of the intake channel axis width *b* and, therefore, of the distance Δ between the intake channel axis and the shaft axis makes it possible to set up a simple method for plotting the shape of the volute in the Drioli inlet chamber by determining the four necessary bending radii (Figure1):

$$R_1 = \Delta + b/2 - d/2 \quad (12.1)$$

$$R_2 = \Delta + b/2 - 3d/2 \quad (12.2)$$

$$R_3 = \Delta + b/2 - 5d/2 \quad (12.3)$$

$$R_1 = \Delta + b/2 - 7d/2 \quad (12.4)$$

where *d* is the side of a square, whose barycentre coincides with the shaft axis and whose four angles constitute the centres of the radii R_1, R_2, R_3 and R_4. On the basis of the vortex inlet chambers of models and operating prototypes [7, 23, 24, 25] it has been possible to determine that the value of the *d/b* ratio varies in a fairly small range, essentially between 0.27 and 0.33. Furthermore, it has been noted that the majority of these values (83.33%) vary in an even smaller range: in this range a mean value of 0.29 is acceptable. This estimate is based on the vortex inlet chambers of models and operating prototypes and takes into account the size of the wall separating the inlet channel from the vertical shaft (as for the evaluation of the *f* parameter in eqn (10))..

IV. Discussion and further observations

Despite the simplicity of the calculation methodology, the greater or lesser accuracy of the design solution is essentially dependent on the reliability of the estimation of parameters *f* and ε and of the numerical coefficients *m* and *n*.

Notwithstanding the validity of the proposed model, it should be noted that appreciable variations in the estimates of *m* and *n* (which, as mentioned above, are achieved from experimental data acquired and/or determined directly) are obtained from apparently insignificant differences in the design of the shaft geometry. Indeed, the above reported mean values of *m, n, f* and ε, obtained on the basis of inlet chambers with a strictly Drioli configuration [23, 24, 25], differ from those reported in [15, 16, 22], which were obtained on the basis of a larger number of vortex-flow inlets (models and operating prototypes) but which did not all have a Drioli type configuration: in the latter case, the process yields estimates equal to: m=0.322 , n=0.915, f=1.20 and ε =1.05.

The variation is small for *n*, *f* and ε and larger for *m*. These considerations nevertheless lead to the conclusion that, in the design of a Drioli shaft, it is worth using the mean values m=0.366, n=0.910, f=1.22 and ε=1.06. If, on the other hand, the design calls for a shaft that does not have a strictly Drioli configuration, then it might be more opportune to use the more general mean values proposed in [15,16, 22]: m=0.322 , n=0.915 , f=1.20 and ε =1.05.

Design reliability depends not only on the accuracy in determining the outflow scale but also and in particular on the estimate of the radius r_m of the minimum section of the vortex core, which is generally separate (and set at a higher level) from the ro radius (set at the shaft inlet).

Aeration of the vortex-flow therefore occurs through a smaller section than that defined by calculation, which means that any closing of the vortex flow will result in a lower level of safety.

Moreover, taking into account the lower filling level in the inlet section predicted by the model, it is worth assuming higher than expected r_o/R values in order to maintain the same level of safety. With a view to establishing a rule, the r_o, values calculated using the proposed model have been compared with the experimental values of r_m measured on the laboratory set-up previously referred to as shaft 10. Although this comparison confirms that r_o is generally larger than r_m, it does not make it possible to identify a correlation between them due to the scarcity of available data. It can be noted, however, that there is an increase in the relative deviation of r_o compared to r_m, which increases with the flow up to a maximum value of almost 18%.

The experimental data for shaft 10, which led to the assumption of an approximately 20% deviation between r_o and r_m, justify Viparelli's practical standard of setting r_o/R=0.5 for design purposes (corresponding to the proposed value E=0.25) because, with this position, it will be at most r_m/R=0.4 which corresponds to operating conditions with a still acceptable safety level.

V. Conclusions

In sizing hydraulic works, the ultimate goal of a calculation model lies in its ability to make sufficiently accurate predictions of the parameters regulating the phenomena that arise within the structure: basically, the model's reliability is measured by the adherence of theoretical results to real data.

The main parameters for a vortex drop shaft regard, on the one hand, the verification of the plant's outflow scale $Q(h)$ and, on the other, the determination of the size of r_o in the vortex core.

The above mentioned theoretical-experimental model has made it possible to propose a methodology for sizing vortex inlets that is essentially founded on the two eqns (8) and (9) examined above and on the four numerical parameters (m, n, f and ε) determined using experimental data taken both from reduced-scale models and from operating prototypes.

The results of the calculations carried out in order to verify this model on the basis of the above ten shafts point out that: i) the parameters f and ε assume values in fairly limited ranges; ii) the deviations between calculated and experimental values of the hydraulic head are generally small (with an overall mean quadratic deviation of 2.42) when referring to the coefficients m and n specific to the individual shafts, while they assume higher values (with an overall mean quadratic deviation of 7.30) when referring to the mean values of these coefficients.

The results of the above calculations constitute, on the one hand, a (not common) experimental confirmation of the model's validity and indicate, on the other hand, that greater congruence between theoretical predictions and real data may be achieved if reference is made to the m and n values of the chosen geometry. In the design sphere, given the current practical impossibility of knowing the relations between m and n and the shape parameters (regarding the real conformation assigned to the inlet chamber by the designer), it is necessary to adopt mean values for the coefficients. As these mean values give rise to more significant deviations, it is worth considering hydraulic head values up to 10% greater than the theoretical values in order to assess the safety level for the inlet channel.

It has also been pointed out that the 'actual' operation of drop shafts with specific m and n values is more favourable towards r_o than would appear in the design with mean m and n values. Indeed, with the exception of one case, which can presumably be labelled anomalous, for higher flows (i.e. in the most critical conditions also because of the closing of the vortex core) the r_o value calculated with mean m and n values is lower than that calculated with specific m and n values (seven out of ten shafts) or at least very close to it (two out of ten shafts). Consequently, designs performed using mean m and n values are precautionary.

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