TDOA/AOA Hybrid Position Estimate Solution using Newton Method

[Kyunghyun Lee, Jungkeun Oh, and Kwanho You]

Abstract— **The position determination method has been used widely in location based industry. Precise determination of a mobile position is one of the significant problems in geolocation. Among geolocation techniques, both time difference of arrival (TDOA) and angle of arrival (AOA) are the most widely used methods for estimation of mobile position. In this paper, TDOA/AOA hybrid position estimation formula is proposed to express a mobile position. This hybrid scheme improves the estimation performance by increasing reliable measurement data set. Also, we suggest Newton method based optimization algorithm to obtain a solution that minimizes determination error. Some simulations confirm that the proposed Newton method based optimization algorithm improves the accuracy for acquiring a mobile position estimate.**

Keywords— **position estimation, time difference of arrival, angle of arrival, Newton method, optimization, measurement error compensation.**

I. Introduction

The mobile position estimation problem has received enormous interests in observation industry such as target tracking, guidance, wireless communication and many types of mobile application over the past few years. In order to implement a mobile position determination system, there are several approaches including the time of arrival (TOA) and received signal strength (RSS). Recently, the position determination systems based on TDOA and AOA method have received significant attention. In the TDOA method, the time difference data of signal arrival between each base station and reference base station is measured. In two dimensional coordinates, each TDOA measurement defines a hyperbolic curve with a constant range difference between two base stations. The estimated position using TDOA method can be obtained by the intersection of more than two hyperbolas. In real case, however, this hyperbolic equation does not derive one unique solution since the measurement data contains some noises. In the AOA method, the multiple antenna array in base station measures the arrival angle of transmitted signal from a mobile. Using measured angle data, the position determination equation can be formulated. In this paper, we suggest TDOA/AOA hybrid method in order to determine the position of a mobile [1].

In both TDOA and AOA based localization system, the measurement noise is one of the major factors that cause the uncertainty of estimation. In order to compensate measurement error, Gustafsson [2] suggested both a Monte Carlo based method for positioning and a gradient search algorithm using a nonlinear least squares framework. Peng [3] proposed a distributed AOA based localization and orientation approach for wireless sensor networks under the assumption that all unknown sensors are capable of detecting angles of the incident signal from the neighboring nodes. Chiang [4] proposed Kalman filter based tracking algorithm for wireless location systems for improving the precision of estimation.

In this paper, we suggest Newton method based optimization algorithm for minimizing the error between the real position and the estimation of a mobile. Newton method is an iterative method to approximate roots of equation. Several iterations using derivative data of an objective function are progressed until the solution that minimizes the estimation error is determined. The rest of this paper is organized as follows. In section II, a mobile position formulation using TDOA and AOA data is analyzed. Section III demonstrates the objective function that expresses the estimation error about a mobile position. Also, the Newton method based iteration process is applied to the objective function in this section. The effectiveness of our proposed algorithm is confirmed through some simulations in section IV. Finally, conclusions are drawn in section V.

II. Mobile Position Formulation

In this section, the position determination formula that is composed with the TDOA and AOA data is derived for optimization problem. Among various hybrid geolocation methods, the TDOA and AOA measurements used scheme is the most efficient one. In order to apply Newton method to mobile geolocation formula, at least three base stations are needed in two dimensional coordinates. In section II, the TDOA and AOA based geolocation equations are formulated separately. In the end of this section, these two formulas are combined into the final position determination equation.

A. *TDOA based geolocation*

When the TDOA data is available, the geolocation formula of a mobile can be derived. For formulation, let $m = [x, y]^T$ be

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an unknown mobile position that we try to estimate and let $b_i = [x_i, y_i]^T$, $i = 1, 2, \cdots, M$ be a known position of based station. The TDOA measurements can be obtained as follows

$$d_i = \|m - b_i\|,\\ d_{i1} = ct_{i1} = c(t_i - t_1), \tag{1}$$

where d_i denotes the distance from a mobile to i-th receiver and c means the speed of signal propagation. d_{i1} and t_{i1} are the measured range difference data and TDOA data between i-th receiver and the first receiver, respectively.

In real condition, the measured TDOA data contains environmental noise due to measurement error and nonline-of-sight (NLOS) problem. On the assumption that the TDOA measurements do not contain environmental noise, we can obtain the following set of linear equations

$$(x - x_1)(x_i - x_1) + (y - y_1)(y_i - y_1) + d_{i1}d_1\\ = \frac{1}{2}\left[(x_i - x_1)^2 + (y_i - y_1)^2 - d_{i1}^2\right]. \tag{2}$$

The unknown position of a mobile can be obtained by rewriting equation (2) as the following equations

$$A_{TDOA}u = b_{TDOA} \tag{3}$$

with

$$A_{TDOA} = \begin{bmatrix} x_2 - x_1 & y_2 - y_1 & d_{21} \\ \vdots & \vdots & \vdots \\ x_M - x_1 & y_M - y_1 & d_{M1} \end{bmatrix},\\ b_{TDOA} = \frac{1}{2}\begin{bmatrix} (x_2 - x_1)^2 + (y_2 - y_1)^2 - d_{21}^2 \\ \vdots \\ (x_M - x_1)^2 + (y_M - y_1)^2 - d_{M1}^2 \end{bmatrix}. \tag{4}$$

The parameter vector $u = [x - x_1 \quad y - y_1 \quad d_1]^T$ in equation (3) is the solution of formulated TDOA based geolocation equation that includes the position of a mobile. The environmental noise in TDOA data causes a difference between the estimation position and the true position of a mobile.

B. *AOA based geolocation*

The AOA data of the propagated signal from the mobile at the i-th base station, denoted by θ_i, can be formulated with the position of a mobile and a base station as follows

$$\tan\theta_i = \frac{\sin\theta_i}{\cos\theta_i}\\ = \frac{y - y_i}{x - x_i}, \quad i = 1, 2, \cdots, M. \tag{5}$$

The equation (5) can be rewritten by rearranging as a following set of equations

$$x\sin\theta_i - y\cos\theta_i = x_i\sin\theta_i - y_i\cos\theta_i. \tag{6}$$

The matrix form of equation (6) is obtained as follows

$$G_{AOA}m = h_{AOA} \tag{7}$$

with

$$G_{AOA} = \begin{bmatrix} \sin\theta_1 & -\cos\theta_1 \\ \vdots & \vdots \\ \sin\theta_M & -\cos\theta_M \end{bmatrix}\\ h_{AOA} = \begin{bmatrix} x_1\sin\theta_1 - y_1\cos\theta_1 \\ \vdots \\ x_M\sin\theta_M - y_M\cos\theta_M \end{bmatrix}. \tag{8}$$

The vector m is the solution that can be obtained by solving the AOA based formulation. Also, the measurement AOA data θ_i at i-th base station contains the environmental noise in real case. It causes the error in the matrix G_{AOA} and h_{AOA}. These differences between a measurement and a real data lead to an inaccuracy of a mobile position estimation.

C. *Final position estimation formula*

The combination of different kind of the measurement data can improve the position estimation performance and reduce the number of base stations due to the increase of data set. In order to combine TDOA and AOA formula, it is necessary that the AOA solution vector m needs to be transformed into the TDOA solution parameter u. Also, it is necessary that the matrix G_{AOA} and h_{AOA} change according to the dimensions of A_{TDOA} and b_{TDOA}, respectively.

Combining equation (3) and (7) into a single matrix form denotes as

$$Au = b \tag{9}$$

where

$$A = \begin{bmatrix} A_{TDOA} \\ A_{AOA} \end{bmatrix},\ b = \begin{bmatrix} b_{TDOA} \\ b_{AOA} \end{bmatrix}$$

$$b_{AOA} = \begin{bmatrix} 0 \\ (x_2 - x_1)\sin\theta_2 - (y_2 - y_1)\cos\theta_2 \\ \vdots \\ (x_M - x_1)\sin\theta_M - (y_M - y_1)\cos\theta_M \end{bmatrix}. \tag{10}$$

In equation (10), the matrix A_{AOA} denotes $[G_{AOA} \quad 0_M]$. Combining TDOA and AOA formula, $(2M-1)$ measurement data set can be obtained for estimation of a mobile position at M base stations. In order to solve this TDOA/AOA geolocation problem denoted by equation (9), we propose the Newton method based optimization algorithm in section III.

III. Newton Method based Optimization Algorithm

In order to obtain the optimized solution of a mobile position, the error between the estimated position and the real position needs to be minimized. In this section, we derive the objective function related to the error of estimation and apply Newton method to this optimization formula. Among optimization algorithms, Newton method is a generalized process to find the roots of a differential equation.

In equation (9), the set of linear equations is inconsistent due to the environmental noisy elements in matrix A and vector b. By solving equation (9) via standard LS, the objective function for minimizing error can be obtained as follows

$$f = (A\hat{u} - b)^T (A\hat{u} - b) \tag{11}$$

where $\hat{u} = \left[\hat{x} - x_1, \hat{y} - y_1, \hat{d}_1\right]^T$ is an optimization variable vector. Since the design variable $\hat{u}$ is a (3×1) vector in two dimensional coordinates, the gradient and the Hessian matrix are utilized to solve an optimization problem through Newton method. The gradient vector $J(\hat{u})$ and the Hessian matrix $H(\hat{u})$ can be acquired as follows

$$\begin{aligned} J(\hat{u}) &= \begin{bmatrix} \frac{\partial f}{\partial \hat{x}} & \frac{\partial f}{\partial \hat{y}} & \frac{\partial f}{\partial \hat{d}_1} \end{bmatrix}^T \\ &= A^T (A\hat{u} - b) \\ H(\hat{u}) &= \begin{bmatrix} \frac{\partial^2 f}{\partial \hat{x}^2} & \frac{\partial^2 f}{\partial \hat{x}\partial \hat{y}} & \frac{\partial^2 f}{\partial \hat{x}\partial \hat{d}_1} \\ \frac{\partial^2 f}{\partial \hat{y}\partial \hat{x}} & \frac{\partial^2 f}{\partial \hat{y}^2} & \frac{\partial^2 f}{\partial \hat{y}\partial \hat{d}_1} \\ \frac{\partial^2 f}{\partial \hat{d}_1\partial \hat{x}} & \frac{\partial^2 f}{\partial \hat{d}_1\partial \hat{y}} & \frac{\partial^2 f}{\partial \hat{d}_1^2} \end{bmatrix} = A^T A. \end{aligned} \tag{12}$$

Using the gradient and Hessian matrix in equation (12), Newton based optimization solution of the objective function denoted by equation (11) is derived as

$$\hat{u}_{i+1} = \hat{u}_i + \alpha\left[H(\hat{u}_i)\right]^{-1} J(\hat{u}_i) \tag{13}$$

with the step size $\alpha \in (0,1]$. By iteration process according to equation (13), the error between the true position and the estimated position is decreased gradually.

Then the procedure of the proposed Newton method based algorithm is as follows:

1) Set initial condition of $\hat{u}_i$. If $f(\hat{u}_i)$ of equation (11) is 0, the iteration stops.
2) Compute the gradient $J(\hat{u}_i)$ and the Hessian matrix $H(\hat{u}_i)$. Acquire $\hat{u}_{i+1}$ using equation (13).
3) If $\varepsilon = \left|f(\hat{u}_i) - f(\hat{u}_{i+1})\right|$ is smaller than the threshold that we choose experimentally, stop iteration. Otherwise, repeat $(i+1)$-th iteration through step 2) and 3).

IV. Simulation Results

In this section, some simulations confirm the validity of a proposed TDOA/AOA hybrid geolocation algorithm using Newton method. In our simulations, the position of a mobile is estimated by using TDOA and AOA data from five receivers. The positions of each receiver are (0, 0), (0, 100), (100, 0), (100, 100), and (50, 100) km, respectively. The simulated trajectory has 321 time samples with one second interval. We suppose that a mobile advances with constant speed in a straight line. In real environment, TDOA and AOA measurements contain some environmental errors. We assume that the environmental errors of TDOA and AOA data follow the Gaussian distribution with a variance 0.1 and 0.01, respectively. Also the means of TDOA and AOA measurement noises are 1 and 0.1, respectively. In order to

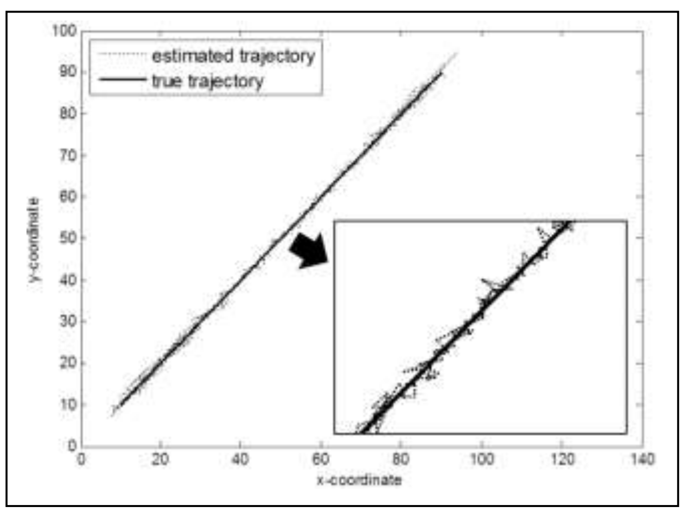

Figure 1. Estimated trajectory of a mobile

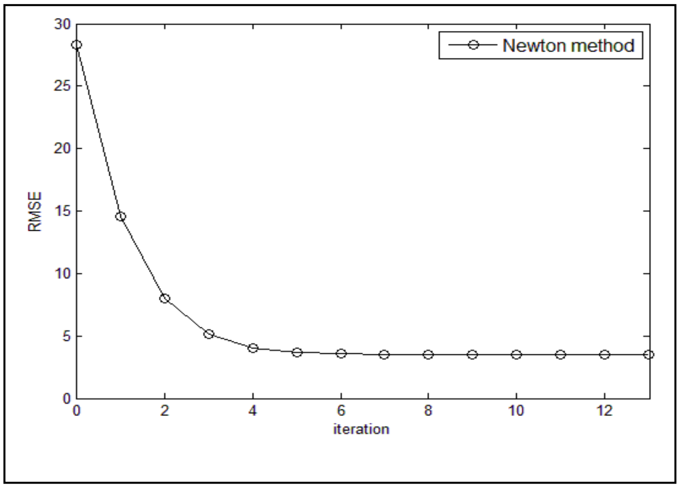

Figure 2. RMSE change for different iteratios.

simplify the computation, the signal propagation c is supposed as 1. We set that the initial estimated position $\hat{u}_0$ and step size α are $[50,\ 50,\ 70.71]^T$ and 0.5, respectively.

Fig. 1 describes the estimated trajectory of a mobile using Newton method. The dotted line is the estimated trajectory and the solid line means the true trajectory of a mobile. As shown in Fig. 1, the measurement noise compensated trajectory using the proposed Newton method based optimization algorithm is almost the same with a true trajectory. In Fig. 2, the root mean square error (RMSE) of estimated mobile position is demonstrated for the increased number of iteration when the true position is (70, 70) km. The RMSE of estimation converges on minimized error that is computed by optimized solution of an objective function, denoted by equation (11).

V. Conclusion

In this paper, we suggested the Newton method based optimization algorithm for minimizing the error between the true position and the estimation of a mobile. In order to formulate the objective function for minimizing the estimation error, the TDOA and AOA measurement data were utilized. The position estimation inaccuracy that is one of the most critical problems in geolocation was caused by the environmental errors in TDOA and AOA measurements. Newton method was applied to the objective function for compensation of environmental error. As the iteration process was progressed, the closed solution that minimizes estimation error was obtained. The effectiveness of our proposed algorithm was confirmed through some simulations.

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