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STABILITY OF NEURAL NETWORKS

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Abstract— We develop a new approach to the stability analysis of Hopfield-type neural networks with time varying delays in the presence of impulses. With the new approach, we improve and generalize some previous works of other researchers. We study stability of equilibrium points of impulsive systems which are either a generalization of those existing or new. This paper deals with the problem of delay-dependent stability criterion of delay-difference system with multiple delays of Hopfield neural networks. Based on quadratic Lyapunov functional approach and free-weighting matrix approach, some linear matrix inequality criteria are found to guarantee delay-dependent asymptotical stability of these systems.

Keywords— Hopfield neural networks; Time-varying Delay; Stability; Quadratic Lyapunov functional approach.

I. INTRODUCTION

Hopfield neural networks have been extensively studied in past years and found many applications in different areas such as pattern recognition, associative memory and combinatorial optimization. Such applications heavily depend on the dynamical behaviors. Thus, the analysis of the dynamical behaviors is necessary step for practical design of neural networks. A neural network is a network that performs computational tasks such as associative memory, pattern recognition, optimization, model identification, signal processing, etc. on a given pattern via interaction between a numbers of interconnected units characterized by simple functions. From the mathematical point of view, an artificial neural network corresponds to a nonlinear transformation of some inputs into certain outputs. Many types of neural networks have been proposed and studied in the literature and the Hopfield-type network has become an important one due to its potential for applications in various fields of daily life. The model proposed by Hopfield, also known as Hopfield's graded response neural network is based on analog circuit consisting of capacitors, resistors and amplifiers. Among the most popular models in the literature of artificial neural networks (see, e.g., [1–7]) is the continuous time model described by a system of ordinary differential equations. The multiple models are still multiple linear models with different parameters, and neural network is used only to compensate for the modeling error of linear model. In this case, the nonlinear system should not be very complex, and too big modeling error between the system and linear model is forbidden. Dynamical systems are often broadly classified into two categories: continuous time systems or discrete time systems. Recently there has been introduced a somewhat new category of dynamical systems which is neither purely continuous time nor purely discrete time ones; these are called dynamical systems with impulses (see for instance [8-11] and references therein). Stability conditions for various types of stability of neural networks problems such as complete stability, asymptotic stability, absolute stability and exponential stability have been studied extensively. One should underline the fact that stability properties of a neural network basically depend on the intended problems. For example in the solution of optimization problems, the neural network must be designed to have only one equilibrium point and this equilibrium point is globally stable. See more details in [12-15] and references given therein.

In this paper, we consider delay-difference system with multiple delays of Hopfield neural networks of the form

$$u(k+1) = -C_i u(k) + \sum_{i=1}^{m} B_i S(u(k-h_i)) + f\,, \qquad (1)$$

where $u(k) \in \Omega \subseteq \mathbf{R}^n$ is the neuron state vector, $0 \le h_1 \le \ldots \le h_m$, $C_i = diag\{a_{1i}, \ldots, a_{ni}\}$, $c_i \ge 0$, $i = 1, 2, ..., n$ is the constant relaxation matrix, $B_i, i = 1, 2, ..., m$ are $n \times n$ constant weight matrices, $f = (f_1, \ldots, f_n) \in \mathbf{R}^n$ is the constant external input vector and $S(z) = [s_1(z_1), \ldots, s_n(z_n)]^T$ with $s_i \in C^1\left[\mathbf{R}, (-1,1)\right]$ where s_i is the neuron activations and monotonically increasing for each $i = 1, 2, ..., n$.

The asymptotic stability of the zero solution of delay-difference system with multiple delays of Hopfield neural networks has been developed during the past several years. Much less is known regarding the asymptotic stability of the zero solution of the control discrete-time system of neural networks. Therefore, the purpose of this paper is to establish sufficient condition for the asymptotic stability of the zero solution of (1) in terms of certain matrix inequalities.

II. PRELIMINARIES

The following notations will be used throughout the paper. $\mathbf{R}^+$ denotes the set of all non-negative real numbers; $\mathbf{Z}^+$ denotes the set of all non-negative integers; $\mathbf{R}^n$ denotes the

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n-finite-dimensional Euclidean space with the Euclidean norm $\|.\|$ and the scalar product between x and y is defined by $x^T y$; $\mathbf{R}^{n\times m}$ denotes the set of all $(n\times m)$-matrices; and A^T denotes the transpose of the matrix A; Matrix $Q\in\mathbf{R}^{n\times n}$ is positive semidefinite $(Q\geq 0)$ if $x^T Qx\geq 0$, for all $x\in\mathbf{R}^n$. If $x^T Qx>0 (x^T Qx<0$, resp.) for any $x\neq 0$, then Q is positive (negative, resp.) definite and denoted by $Q>0, (Q<0,$ resp.). It is easy to verify that $Q>0$, $(Q<0,$ resp.) iff $\exists\beta>0$: $x^T Qx\geq\beta\|x\|^2, \forall x\in\mathbf{R}^n$, ($\exists\beta>0$: $x^T Qx\leq-\beta\|x\|^2, \forall x\in\mathbf{R}^n$, resp.) .

Lemma 1 The zero solution of difference system is asymptotic stability if there exists a positive definite function $V(x):\mathbf{R}^n\to\mathbf{R}^+$ such that

$$\exists\beta>0:\Delta V(x(k))=V(x(k+1))-V(x(k))\leq-\beta\|x(k)\|^2,$$

along the solution of the system. In case the above condition holds for all $x(k)\in V_\delta$, we say that the zero solution is locally asymptotically stable.

Lemma 2 [15] For any constant symmetric matrix $M\in\mathbf{R}^{n\times n}$, $M=M^T>0$, scalar $s\in\mathbf{Z}^+/\{0\}$, vector function $W:[0,s]\to\mathbf{R}^n$, we have

$$s\sum_{i=0}^{s-1}(w^T(i)Mw(i))\geq\left(\sum_{i=0}^{s-1}w(i)\right)^T M\left(\sum_{i=0}^{s-1}w(i)\right).$$

We present the following technical fact and lemmas, which will be used in the proof of our main result.

III. Main Results

In this section, we consider the sufficient condition for asymptotic stability of the zero solution u^* of (1) in terms of certain matrix inequalities. Without loss of generality, we can assume that $u^*=0, S(0)=0$ and f =0 (for otherwise, we let $x=u-u^*$ and define $S(x)=S(x+u^*)-S(u^*))$.

The new form of (1) is now given by

$$u(k+1)=-C_i u(k)+\sum_{i=1}^{m}B_i S(u(k-h_i)) \qquad (2)$$

Throughout this paper we assume the neuron activations $s_i(x_i)$, $i=1,2,\ldots,n$ is bounded and monotonically nondecreasing on $\mathbf{R}$, and $s_i(x_i)$ is Lipschitz continuous, that is, there exist constant $l_i>0$, $i=1,2,\ldots,n$ such that

$$|s_i(v_1)-s_i(v_2)|\leq l_i|v_1-v_2|, \quad \forall v_1,v_2\in\mathbf{R}. \qquad (3)$$

By condition (3), $s_i(x_i)$ satisfy

$$|s_i(x_i)|\leq l_i|x_i|, \qquad i=1,2,...,n. \qquad (4)$$

Theorem 1 The zero solution of the delay-difference system (2) is asymptotically stable if there exist the symmetric positive definite matrices $P_i(k)$, $G_i(k)$, $W_i(k)$, and $L_i=diag[l_{1i},\ldots,l_{ni}]>0$, $i=1,2,...,m$ satisfying the following matrix inequalities:

$$\psi_i=\begin{bmatrix}
(0,0) & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0\\
0 & (1,1) & (1,2) & \cdots & (1,m_i) & 0 & 0 & 0 & \cdots & 0\\
0 & (2,1) & (2,2) & \cdots & (2,m_i) & 0 & 0 & 0 & \cdots & 0\\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\
0 & (m_i,1) & (m_i,2) & \cdots & (m_i,m_i) & 0 & 0 & 0 & \cdots & 0\\
0 & 0 & 0 & \cdots & 0 & (m_i+1,m_i+1) & 0 & 0 & \cdots & 0\\
0 & 0 & 0 & 0 & 0 & 0 & (m_i+2,m_i+2) & 0 & \cdots & 0\\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & 0 & \ddots & \ddots & \vdots\\
0 & 0 & 0 & 0 & 0 & 0 & \vdots & \ddots & \ddots & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & (2m_i,2m_i)
\end{bmatrix}<0, \qquad (5)$$

where

$$(0,0)=C_i^T P_i(k)C_i-P_i(k)+\sum_{i=1}^{m}\left(h_i G_i(k)+W_i(k)\right),$$
$$(1,1)=L_i B_{1i}^T P_i(k)B_{1i}L_i+\varepsilon_{1i}^{-1}L_i L_i-W_{1i}(k),$$
$$(1,2)=L_i B_{1i}^T P_i(k)B_{2i}L_i+\varepsilon_{1i}^{-1}L_i L_i,$$
$$(1,m)=L_i B_{1i}^T P_i(k)B_{mi}L_i+\varepsilon_{1i}^{-1}L_i L_i,$$
$$(2,1)=L_i B_{2i}^T P_i(k)B_{1i}L_i+\varepsilon_{1i}^{-1}L_i L_i,$$
$$(2,2)=L_i B_{2i}^T P_i(k)B_{2i}L_i+\varepsilon_{1i}^{-1}L_i L_i-W_{2i}(k),$$
$$(2,m)=L_i B_{2i}^T P_i(k)B_{mi}L_i+\varepsilon_{1i}^{-1}L_i L_i,$$
$$(m,1)=L_i B_{mi}^T P_i(k)B_{1i}L_i+\varepsilon_{1i}^{-1}L_i L_i,$$
$$(m,2)=L_i B_{mi}^T P_i(k)B_{2i}L_i+\varepsilon_{1i}^{-1}L_i L_i$$
$$(m_i,m_i)=L_i B_{mi}^T P_i(k)B_{mi}L_i+\varepsilon_{1i}^{-1}L_i L_i-W_{mi}(k),$$
$$(m_i+1,m_i+1)=-h_{1i}G_{1i}(k),$$

$(m_i+2, m_i+2) = -h_{2i}G_{2i}(k)$,
$(2m_i, 2m_i) = -h_{mi}G_{mi}(k)$.

Proof Consider the Lyapunov function

$V(y(k)) = V_1(y(k)) + V_2(y(k)) + V_3(y(k))$, where

$$V_1(y(k)) = x^T(k)P_i(k)x(k),$$

$$V_2(y(k)) = \sum_{i=1}^{m}\sum_{j=k-h_i+1}^{k}(h-k+i)x^T(j)G_i(k)x(j),$$

$$V_3(y(k)) = \sum_{i=1}^{m}\sum_{j=k-h_i+1}^{k}x^T(j)W_i(k)x(j),$$

$P_i(k), G_i(k), W_i(k)$, $i = 1,2,...,m$ being symmetric positive definite solutions of (5) and

$$y(k) = [x(k), x(k-h_1), \ldots, x(k-h_m)].$$

Then difference of $V(y(k))$ along trajectory of solution of (2) is given by

$$\Delta V(y(k)) = \Delta V_1(y(k)) + \Delta V_2(y(k)) + \Delta V_3(y(k)).$$

From the above inequality it follows that:

$$\Delta V \le x^T(k)[C_iP_i(k)C_i - P_i + \sum_{i=1}^{m}h_iG_i(k) + W_i(k)$$
$$+\varepsilon_{1i}\sum_{i=1}^{m}\sum_{j=1}^{m}C_iP_i(k)B_iB_j^TP_i(k)C_i + \varepsilon_i^{-1}L_iL_i]x(k)$$
$$+\sum_{i=1}^{m}\sum_{j=1}^{m}x^T(k-h_i)[LB_i^TP(k)B_jL + \varepsilon_1^{-1}LL]x(k-h_j)$$
$$-\sum_{i=1}^{m}x^T(k-h_i)W_i(k)x(k-h_i)$$
$$-\sum_{i=1}^{m}\left(\frac{1}{h_i}\sum_{j=k-h_i+1}^{k}x(j)\right)^T(h_iG_i(k))\left(\frac{1}{h_i}\sum_{j=k-h_i+1}^{k}x(j)\right)$$
$$= y^T(k)\psi_i y(k),$$

By the condition (5), ΔV is negative definite, namely there is a number $\beta > 0$ such that $\Delta V(y(k)) \le -\beta\|y(k)\|^2$, and hence, the asymptotic stability of the system immediately follows from **Lemma 1**. This completes the proof. □

Remark 1 Theorem 1 gives a sufficient condition for the asymptotic stability of delay-difference system (2) via matrix inequalities. These conditions are described in terms of certain diagonal matrix inequalities, which can be realized by using the linear matrix inequality algorithm proposed in [4]. But Hu and Wang [9] these conditions are described in terms of certain symmetric matrix inequalities, which can be realized by using the Schur complement lemma and linear matrix inequality algorithm proposed in [4].

IV. NUMERICAL SIMULATIONS

Example 1 Let us consider the Hopfield-type neural networks with time varying delays (2) of the form

$$u(k+1) = -C_iu(k) + \sum_{i=1}^{m}B_iS(u(k-h_i)),$$

where the matrices are

$$C_i = \begin{pmatrix}1.9 & 1\\ 0.3 & 2\end{pmatrix}, B_i = \begin{pmatrix}-0.9 & 0.3\\ 0.5 & -0.8\end{pmatrix}, \varepsilon = 0.5, \text{and}$$

$h_i = 59$.

Using the LMI Toolbox in MATLAB, we found that the LMIs in **Theorem 1** are feasible and

$$P_i = \begin{pmatrix}0.8973 & 0.2315\\ 0.2379 & 1.2376\end{pmatrix}, G_i = \begin{pmatrix}1.3873 & 0.5697\\ 0.3279 & 0.6971\end{pmatrix},$$

$$W_i = \begin{pmatrix}0.7325 & 0.1237\\ 0.3214 & 0.9668\end{pmatrix}$$

are the set of solutions to the LMIs(5).

Therefore, the system is asymptotically stable.

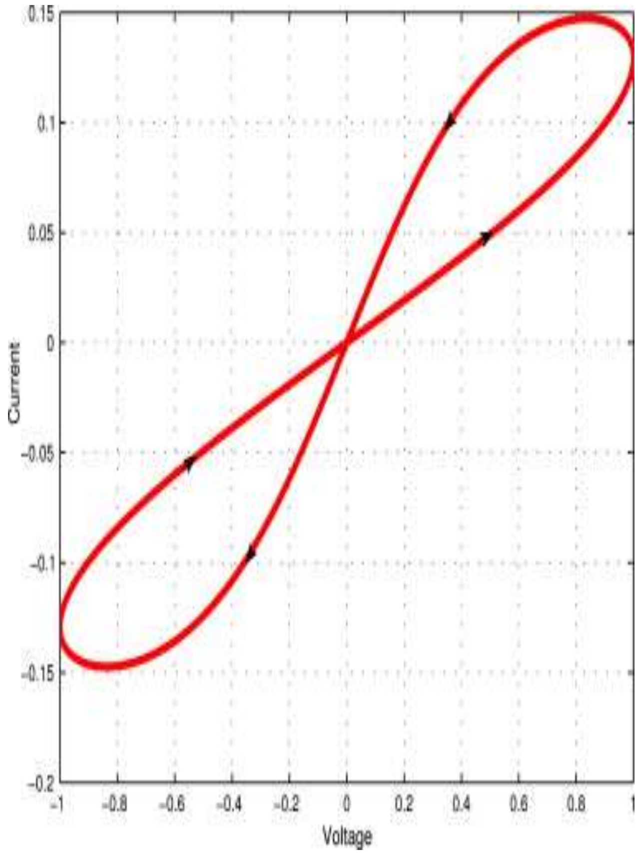

Fig. 1 Numerical simulation of a solution for the **example 1.**

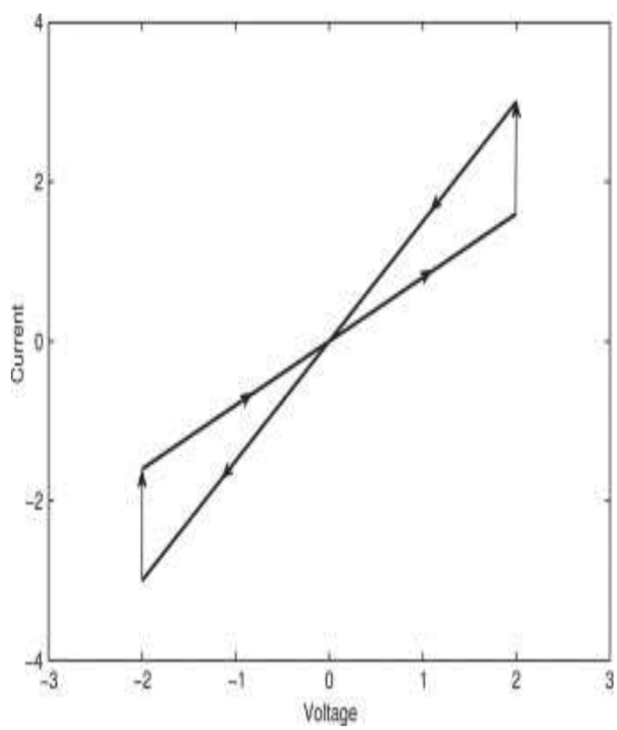

Fig. 2 Numerical simulation of a solution for the **example 1.**

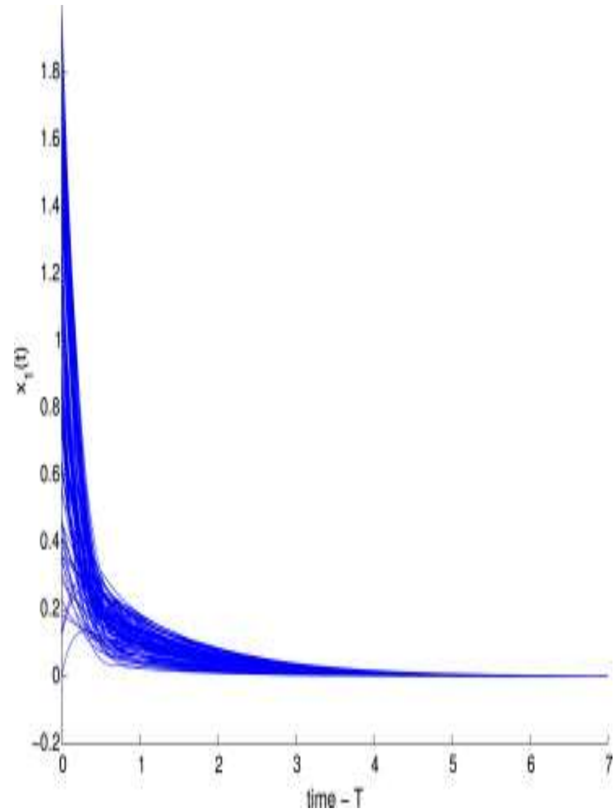

Fig. 3 Numerical simulation of a solution for the **example 1.**

For a given initial condition $x(\theta)=[1,1]^T$, convergence behavior of is shown in Fig. 1. As we can see from this figure, the steady state of nonlinear time varying delay-difference system is indeed asymptotically stable.

V. Conclusion

This paper was dedicated to the delay-dependent stability of delay-difference system with multiple delays of Hopfield neural networks. A less conservative LMI-based globally stability criterion is obtained with quadratic Lyapunov functional approach and free-weighting matrix approach for periodic delay-difference system with multiple delays of Hopfield neural networks.

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