

A Frequency Allocation Scheme with Controllable Fairness in FFR Aided OFDMA Networks

Lang Zhong, Guangjun Li, Xuemin Yang, Zhi Zheng, Jie Zhang

Abstract—a novel distributed frequency allocation scheme named as exponential distribution based frequency allocation (EDFA) is proposed in Fractional Frequency Reuse (FFR) aided OFDMA networks. EDFA has a flexible adjustment between fairness and throughput. Simulation results demonstrate that compared to the proportional fairness (PF) allocation and equal rate allocation (ERA) schemes, EDFA could achieve notable performance improvement and also has continuous controllability between “absolutely fair” and “absolutely unfair”.

Keywords—controllable fairness, fractional frequency reuse, frequency allocation, OFDMA

I. Introduction

In commercial cellular networks, the shortage of frequency is becoming a serious problem. Therefore, people have to look for high spectral efficiency (SE) solutions. One of these schemes is OFDMA, in which all available sub-carriers are divided into many subsets and allocated to mobile stations (MSs) by some specific schemes.

A number of researches have been achieved within the resource allocation scheme of OFDMA. The power and frequency allocation in single cell are discussed in literatures [1][2][3]. Optimization schemes of allocation in heterogeneous networks are studied by [4]. To enhance cell-edge MSs' Quality of service (QoS) and increase system throughput, [5] and [6] give their solution, respectively. Cooperation among base stations (BSs) is discussed in [7].

to further improve the spectral efficiency, unity frequency reuse (UFR) is also adopted. But this method makes the Signal to Interference and Noise Ratio (SINR) of cell-edge MSs gets worse. FFR is a good tradeoff to this problem [8][9]. In this scheme, each cell is divided into center region and edge region. The frequency band F is also divided into two parts, noted as F_c and F_e . In each cell, F_c is reserved for the MSs within the center region, noted as cell-center group, while F_e is partitioned into three equal part, namely F_{e1} , F_{e2} and F_{e3} , and respectively used by three adjacent cells for MSs of edge

region, noted as cell-edge group. Based on FFR, there are also literatures devote to enhance the performance. In [10], a dynamic FFR architecture wherein the cell surface is divided into two overlapping geographical regions is proposed to improve the performance of conventional FFR. A distributed allocation scheme combines Relay based cellular networks are considered in [11] to improve the SE. Authors of [12] introduced adaptive FFR to achieve reasonably high ergodic system spectral efficiency, while assuring a desired performance near the cell boundary.

Actually, all the above-mentioned schemes are subject to constraint of fairness. Without considering the fairness, the throughput is maximized when all available bandwidth is assigned to the MS with the best average SINR [2], called maximum rate allocation (MRA), but the fairness is destroyed completely. The most common constraint of fairness is PF [13], it is applied into researches of resource allocation as a basic criterion [2], [14]. Some adjustment factors are also introduced into allocation policies [7][15]. All the schemes above can either achieve fixed fairness index or limited flexibility, but none could reach a wide range controllability. This paper proposes a novel frequency allocation scheme to continuously adjust the fairness from “absolutely fair” to “absolutely unfair”.

II. System Description

At first, we highlight our cellular architecture, channel model, assumptions and some specifications.

A. Cellular System Topology

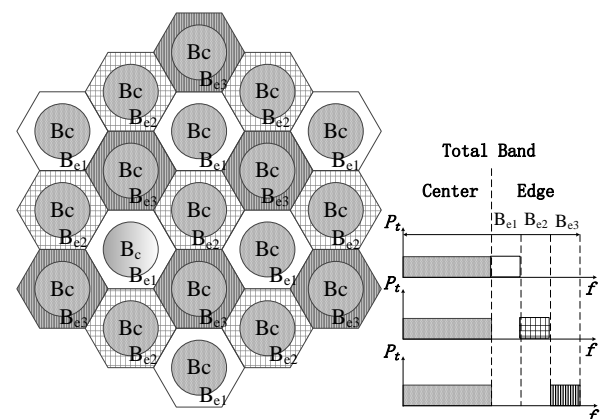


Figure 1. Cellular System with FFR Manners

We consider the FFR cellular network in Figure 1, where 19 hexagonal cells are employed. Each cell has only one BS which locates in the center with single omnidirectional

Lang Zhong, Guangjun Li, Xuemin Yang, Zhi Zheng
University of Electronic Science Technology of China
China

Jie Zhang
China Electric power research institute
China

antenna. Bandwidth F is equally divided into N orthogonal sub-carriers noted as set \mathbb{N} . K MSs are uniformly distributed in each cell. C_1 and C_2 denote the sets containing 6 tier-one cells and 12 tier-two cells, respectively. Interference from further cells is ignored [11].

B. Channel Model

The channel model is made up of path loss, shadow fading and multi-path fading as [16]

$$h = \sqrt{\Omega\psi}h^f, \quad (1)$$

where h^f is the Rayleigh fast fading envelope. The path loss Ω is given by $\Omega = \Omega_\beta d^{-\Omega_\alpha}$, where $(\Omega_\beta, \Omega_\alpha) = (1.35 \times 10^7, 3)$ and $d=1\text{m}$. The large-scale shadowing ψ is typically modeled as lognormal random variable having a probability density function (PDF) of

$$p(\psi) = \frac{\xi}{\sqrt{2\pi}\sigma_\psi\psi} \exp[-(10\log_{10}\psi - \mu_\psi)^2 / 2\sigma_\psi^2], \quad (2)$$

where $\xi=10/\ln 10$, $\mu_\psi=0$ and $\sigma_\psi=6\text{dB}$.

The subsequent discussions in this paper are based on the following assumptions: 1) BSs could obtain the channel state information (CSI) completely. 2) There is no inter-carrier interference. 3) All sub-carriers have equal average transmit power. 4) Noise is ignored, only the Signal to Interference Ratio (SIR) is considered.

C. Downlink Transmission

In FFR cellular networks, \mathbb{N} is partitioned into two parts as $\mathbb{N} = \mathbb{N}_c \cup \mathbb{N}_e$, which represent F_c and F_e , where \mathbb{N}_e is separated into three sub-sets equally as $\mathbb{N}_e = \mathbb{N}_{e1} \cup \mathbb{N}_{e2} \cup \mathbb{N}_{e3}$, corresponding to band F_{e1} , F_{e2} and F_{e3} , respectively. MSs of each cell are classified as cell-center group and cell-edge group, noted as K_c and K_e .

When MS $k \in K_c$, the received signal of MS k in center cell can be written as

$$\mathbf{y}_{0,k} = h_{0,k} \mathbf{x}_{0,k} + \sum_{c=1}^{M-1} h_{c,k} \mathbf{x}_{c,k_c}, \quad (3)$$

where $h_{c,k}$ describes the downlink channel between BS c and MS k . $\mathbf{x}_{c,k}$ is the signal vector transmitted from cell c 's BS which loaded on the set of sub-carriers allocated to MS k , obeying $E[x_{i,c,k} x_{i,c,k}^*] = 1$.

In line with (3), the average SIR of MS k is formulated as

$$S_k = E[|h_{0,k}|^2 / \sum_{c=1}^{M-1} |h_{c,k}|^2]. \quad (4)$$

The average capacity of MS k may be written as

$$C_k = \log_2(1 + S_k). \quad (5)$$

Assuming there are n ($n \leq N_c$) sub-carriers belong to MS k , where $N_c = |\mathbb{N}_c|$. The average rate of this MS could be written

as $R_k = nC_k/N_c$. Let $\rho_k = n/N_c$ be the frequency allocation factor of MS k , which obeying $\sum \rho_k = 1$. Then we have

$$R_k = \rho_k C_k. \quad (6)$$

From(6), we can see that the normalized average throughput of cell-center group is

$$R_c = \sum_{k \in K_c} R_k = \sum_{k \in K_c} \rho_k C_k. \quad (7)$$

If MS $k \in K_e$, the received signal and average SIR are

$$\mathbf{y}_{0,k} = h_{0,k} \mathbf{x}_{0,k} + \sum_{c \in \mathbb{I}_{2,1}} h_{c,k} \mathbf{x}_{c,k_c}, \quad (8)$$

and

$$S_k = E[|h_{0,k}|^2 / \sum_{c \in \mathbb{I}_{2,1}} |h_{c,k}|^2]. \quad (9)$$

Similarly, by defining $N_e = |\mathbb{N}_e|$ and $\rho_k = n/N_e$, we could get the normalized average throughput of cell-edge group:

$$R_e = \sum_{k \in K_e} R_k = \sum_{k \in K_e} \rho_k C_k \quad (10)$$

Let $\eta = N_c/N$, based on (7) and (10), we could get the normalized average throughput of center cell as

$$R_{\text{cell}} = \eta R_c + \frac{1}{3}(1-\eta)R_e. \quad (11)$$

D. Fairness

Jain's index is often used to measure the fairness among users [2], [7], [15]. However, its domain is $[1/K, 1]$, which affected by the number of MSs. In order to unify the range of judge standard, we introduces Gini coefficient [17] as the fairness indicator. Without loss of generality, we assume that all the R_k are arranged in ascending order. The calculation of Gini coefficient is

$$G = \sum_{k=1}^K \left(\frac{k}{K} - \frac{\sum_{l=1}^k R_l}{\sum_{k=1}^K R_k} \right) / \sum_{k=1}^K \frac{k}{K}. \quad (12)$$

where $G \in [0, 1]$ is negative correlated with the fairness.

E. Coverage

For mobile communication networks, operators always want to minimize the blind spots of covering. To measure the system coverage, we divide center cell into Z small grids and any sampling point in grid z ($z \in \mathbb{Z}$) could be considered have the same SIR value. Coverage is calculated as follows.

$$\phi = \frac{1}{Z} \sum_{z=1}^Z 1(\gamma_z - \gamma_{th}), \quad (13)$$

where

$$1(x) = \begin{cases} 1, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0 \end{cases}. \quad (14)$$

The notations γ_z and γ_{th} describe the SIR of grid z and threshold of SIR, respectively.

III. Problem Formulation

Now, let us analyze the factors which affect the capacity, the constraints of allocation scheme, and the relationship among them.

A. Crucial Parameters

In the presented model, there are three parameters as follows, which could affect the system indicators.

The ratio of radii of center region and the whole cell, noted as $\mu=r_c/R$, where r_c is the radius of center region. μ determines the elements of sets K_c and K_e .

MS k 's resource allocation factor ρ_k . According to (11) and (12), the distribution of ρ_k determines the intra-group throughput and fairness.

The ratio $\eta=N_c/N$. Its value determines the proportion of bandwidth allocated to two groups.

When μ and ρ_k are determined, the throughput of two groups could be calculated separately. After getting η , the total throughput can be calculated by (11).

B. Constraints

1) Fairness Constraint

First, we impose some restrictions on the intra-group fairness. According to (12), we can get the Gini coefficients of two groups as

$$G_c = \sum_{k \in K_c} \left(\frac{k}{K_c} - \frac{\sum_{l=1}^k R_l}{\sum_{k \in K_c} R_k} \right) / \sum_{k \in K_c} \frac{k}{K_c} \quad (15)$$

and

$$G_e = \sum_{k \in K_e} \left(\frac{k}{K_e} - \frac{\sum_{l=1}^k R_l}{\sum_{k \in K_e} R_k} \right) / \sum_{k \in K_e} \frac{k}{K_e}, \quad (16)$$

where $R_k=\rho_k C_k$. It can be seen that the intra-group fairness is only affected by ρ_k . We set an upper bound and a lower bound for G_c and G_e , namely

$$G_{low} \leq G_c, G_e \leq G_{up}. \quad (17)$$

to ensure necessary fairness.

Second, the inter-group fairness need to be considered as well. When μ is determined, the inter-group fairness mainly influenced by η . From (7) and (10) we learn that the average data rate per MS of the two groups are $R_c=F_c R_c/K_c$ and $R_e=F_e R_e/K_e$, respectively. So we can get

$$\alpha = \frac{R_{c,av}}{R_{e,av}} = \frac{N_c R_c / K_c}{N_e R_e / K_e / 3} = \frac{3\eta R_c / K_c}{(1-\eta) R_e / K_e}. \quad (18)$$

To avoid too large difference between $R_{c,av}$ and $R_{e,av}$, we set upper bound and lower bound for α as $\alpha_{low} \leq \alpha \leq \alpha_{up}$, so we obtain the following expression.

$$\frac{\alpha_{low} R_c / K_c}{R_c / K_c + \alpha_{low} R_e / K_e} \leq \eta \leq \frac{\alpha_{up} R_c / K_c}{R_c / K_c + \alpha_{up} R_e / K_e} \quad (19)$$

Let $\beta_{low}=\alpha_{low} R_c / (R_c / K_c + \alpha_{low} R_e / K_e) / K_e$ and $\beta_{up}=\alpha_{up} R_c / (R_c / K_c + \alpha_{up} R_e / K_e) / K_e$, finally we have

$$\beta_{low} \leq \eta \leq \beta_{up}. \quad (20)$$

2) Coverage Constraint

When μ is determined, (13) can be rewrite as

$$\varphi = \frac{1}{Z} \left(\sum_{\{z|r_z \leq \mu R\}} 1(\gamma_z - \gamma_{th}) + \sum_{\{z|r_z > \mu R\}} 1(\gamma_z - \gamma_{th}) \right), \quad (21)$$

where r_z is the distance between the BS of center cell and grid z . So the coverage is mainly related to μ . To meet the requirement of coverage, a threshold could be set as

$$\varphi \geq \varphi_{th}, \quad (22)$$

where $\varphi_{th} \in (0, 1)$.

C. Optimization Problem

With considering of (11), (17), (20) and (22), the optimization problem can be described as

$$\max C = \eta F \sum_{k \in K_c} \rho_k R_{k,c} + \frac{1}{3} (1-\eta) F \sum_{k \in K_e} \rho_k R_{k,e} \quad (23)$$

$$\text{s.t. } \varphi(\mu) \geq \varphi_{th} \quad (24)$$

$$G_c(\rho_{k,c}), G_e(\rho_{k,e}) \leq G_{th} \quad (25)$$

$$\beta_{low} \leq \eta \leq \beta_{up}. \quad (26)$$

IV. Proposed Solution

Based on the system shown in Figure 1, the frequency allocation process could be split into steps as grouping, intra-cell allocation and inter-cell allocation.

A. Grouping

Figure 2a shows the relationship of φ and μ at $R_{cell}=800m$ and the threshold of SIR $\gamma_{th}=5dB$. To satisfy the requirement of coverage, there must be an upper bound of μ . Based on ERA, Figure 2b compares the normalized average throughput of two groups and the whole cell. It could be easily found that the throughput of two groups and the whole cell decreases with the increase of μ . To maintain performance as well as possible, μ should not be too large. According to (24), φ_{th} gives this upper bound. The search process of μ could be expressed as pseudo code as follows.

Algorithm 1: searching scheme of μ

initialize: $\mu_a = 0, \mu_c = 1, \mu_b = (\mu_a + \mu_c) / 2$;

calculate $\varphi(\mu_b)$;

while $|(\varphi(\mu_b) - \varphi_{th})| / \varphi_{th} > \varepsilon_\varphi$

if $\varphi(\mu_a) < \varphi_{th}$

$$\mu_c = \mu_b;$$

else

$$\mu_a = \mu_b;$$

end

$$\mu_b = (\mu_a + \mu_c) / 2$$

calculate $\varphi(\mu_b)$;

end

where ε_φ is The maximum allowed relative error of φ_{th} . The final value of μ_b is our needed μ .

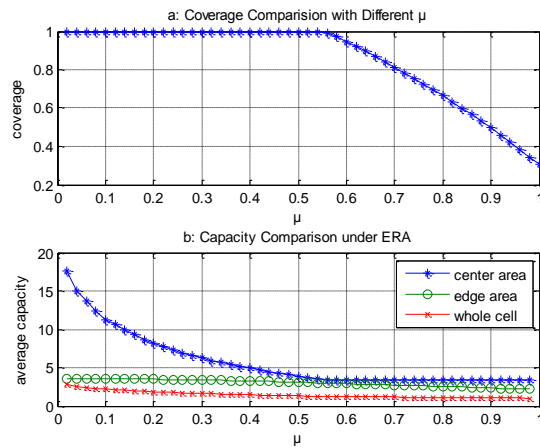


Figure 2. Coverage and Capacity Comparison with different μ

B. Intra-Group Allocation

Considering the SE and fairness requirements, the intra-group allocation should follow principles as: a) If $SIR_k \geq SIR_l$, then $R_k \geq R_l$. b) The intra-group fairness should be adjustable.

An extreme case which satisfies principle a) is ERA, called “absolutely fair”, but its SE is too low. Another extreme is MRA, called “absolutely unfair”. What we should do is to find schemes between the two extremes and also meets principle b).

Let R_{ERA} be the average bit rate per MS in ERA mode, means $\rho_k C_k = R_{ERA}$, $k=1, 2, \dots, K$, we have

$$\rho_k = R_{ERA} / C_k. \quad (27)$$

So R_{ERA} and ρ_k could be calculated as

$$R_{ERA} = \left(\sum_{k=1}^K C_k^{-1} \right)^{-1} \quad (28)$$

and

$$\rho_k = \left(C_k \sum_{i=1}^K C_i^{-1} \right)^{-1}. \quad (29)$$

The total normalized average throughput is

$$R_{ERA}^K = K \left(\sum_{k=1}^K C_k^{-1} \right)^{-1}. \quad (30)$$

Without loss of generality, we assume that MSs $\{1, 2, \dots, K\}$ is in descending order of SIR, so the vector

$$\mathbf{p} = [\rho_1, \rho_2, \dots, \rho_k, \dots, \rho_K] \quad (31)$$

is in ascending order.

Now let's observe the PDF of Exponential distributed random variable with the rate parameter λ :

$$f(x; \lambda) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases}. \quad (32)$$

As we know, when $\lambda \rightarrow +\infty$, $f(x; \lambda)$ becomes the impulse function $\delta(x)$; and when $\lambda \rightarrow 0$, $f(x; \lambda)$ trends to uniformly distribution. If we deem $f(x; \lambda)$ as a frequency allocation curve of MSs, its feature exactly meets principle 1) and 2).

For practical purpose, $f(x; \lambda)$ need to be discretized. By taking K samples equally spaced in $[0, X]$, we get

$$\mathbf{x} = [x_1, x_2, \dots, x_k, \dots, x_K], \quad (33)$$

where $x_k = (k-1)X/K$, there should be a vector as

$$\mathbf{f}(\mathbf{x}; \lambda) = [f(x_1; \lambda), f(x_2; \lambda), \dots, f(x_k; \lambda), \dots, f(x_K; \lambda)]. \quad (34)$$

Obviously, the elements of $\mathbf{f}(\mathbf{x}; \lambda)$ are in descending order. By multiplying the corresponding elements of (31) and (34), then normalized the results, we could obtain

$$\mathbf{p}_{exp} = [\rho_{exp,1}, \rho_{exp,2}, \dots, \rho_{exp,k}, \dots, \rho_{exp,K}]^T, \quad (35)$$

where $\rho_{exp,k} = f(x_k; \lambda) \rho_k / \sum_{i=1}^K f(x_i; \lambda) \rho_i$. The rate of MS k can be rewrite as

$$R_k = \rho_{exp,k} C_k = C_k f(x_k; \lambda) \rho_k / \sum_{i=1}^K f(x_i; \lambda) \rho_i. \quad (36)$$

The throughput of group is

$$R_K = \sum_{k=1}^K R_k = \sum_{k=1}^K C_k f(x_k; \lambda) \rho_k / \sum_{i=1}^K f(x_i; \lambda) \rho_i. \quad (37)$$

When $\lambda \rightarrow +\infty$, there is

$$\lim_{\lambda \rightarrow +\infty} f(x, \lambda) = \delta(x). \quad (38)$$

So $\mathbf{p}_{exp} = [1, 0, \dots, 0]^T$. It is MRA. When $\lambda \rightarrow 0$, we have

$$\rho_{exp,k} = f(x_k; 0) \rho_k / \sum_{i=1}^K f(x_i; 0) \rho_i = \rho_k / \sum_{i=1}^K \rho_i = \rho_k. \quad (39)$$

Here $\mathbf{p}_{exp} = \mathbf{p} = [\rho_1, \rho_2, \dots, \rho_K]^T$. This is ERA. When $\lambda \in (0, +\infty)$, we call this approach as EDFA.

Through (12), the Gini coefficient of two groups could be written as

$$G_c = \sum_{k=1}^{K_c} \left(\frac{k}{K_c} - \frac{\sum_{l=1}^k \rho_{\text{exp},l} C_l^c}{\sum_{k=1}^{K_c} \rho_{\text{exp},k} C_k^c} \right) / \sum_{k=1}^{K_c} \frac{k}{K_c} \quad (40)$$

and

$$G_e = \sum_{k=1}^{K_c} \left(\frac{k}{K_c} - \frac{\sum_{l=1}^k \rho_{\text{exp},l} C_l^e}{\sum_{k=1}^{K_c} \rho_{\text{exp},k} C_k^e} \right) / \sum_{k=1}^{K_c} \frac{k}{K_c} \quad (41)$$

It is difficult to write the analytical expression of λ about G . To solve the range of λ of both groups, we use the searching method shown as:

Algorithm 2: searching scheme of λ

1. initialize: let $\lambda_a = 0$, arbitrary $\lambda_b, \lambda_c \in (0, +\infty)$ which satisfy $G(\lambda_c) > G_{\text{th}}$;

2. calculate $G(\lambda_b)$;

3. while $|G(\lambda_b) - G_{\text{th}}| / G_{\text{th}} > \varepsilon_G$

if $G(\lambda_b) > G_{\text{th}}$

$\lambda_c = \lambda_b$;

else

$\lambda_a = \lambda_b$;

end

$\lambda_b = (\lambda_a + \lambda_c) / 2$;

calculate $G(\lambda_b)$;

end

where ε_G is the maximum relative error allowed for G_{th} . The final value of λ_b is the solution of λ .

C. Inter-Group Allocation

The inter-group allocation is only determined by η . The judge process of η is much simpler and could be described as following pseudo-code:

calculate $[\beta_{\text{low}}, \beta_{\text{up}}]$ and $[R_c, R_e]$

if $R_c > R_e / 3$

$\eta = \beta_{\text{up}}$

else

$\eta = \beta_{\text{low}}$

end

V. Performance Evaluation

According to the system of Figure 1, we consider $R=800\text{m}$, $\varphi_{\text{th}}=95\%$, $\gamma_{\text{th}}=5\text{dB}$, $\alpha_{\text{low}}=1/3$, $\alpha_{\text{up}}=3$, and $\varepsilon_\varphi=\varepsilon_G=0.02$.

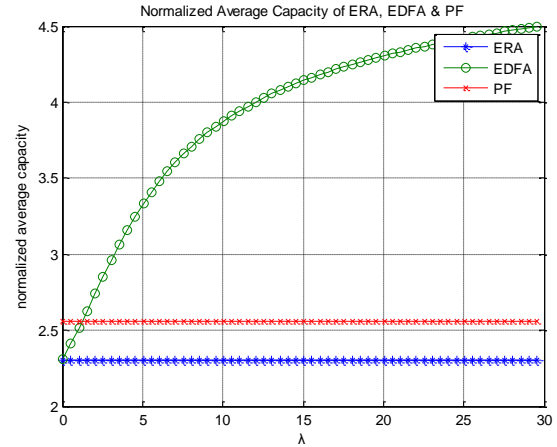


Figure 3. Normalized Average Throughput of EDFA, PF and ERA

Figure 3 illustrates the throughput of EDFA, PF and ERA. It is easy to see that compared to ERA and PF, EDFA has significant performance advantage when $\lambda > 1.5$. In addition, The larger λ is, the greater throughput obtained.

From Figure 4 we could see a fairness comparison of three allocation schemes. It is shown that with the change of λ , the Gini Coefficient of EDFA could vary between 0 and 1, which shows tremendous flexibility of fairness. In addition, the two groups could adjust their intra-fairness by changing their own λ , respectively. Compared to Figure 3 we find that this performance improvement is based on the sacrificing of fairness among MSs.

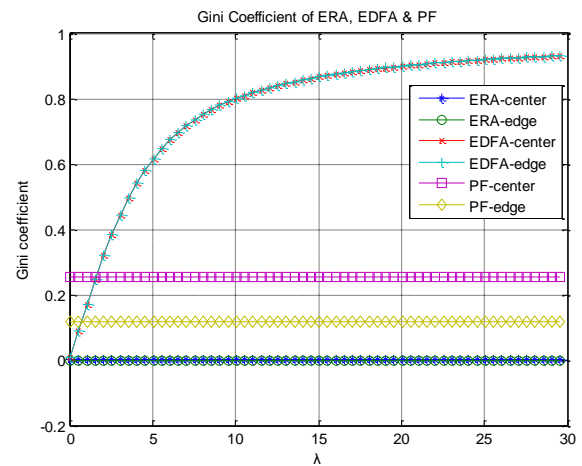


Figure 4. Fairness Comparison of Different Allocation Schemes

A fairness comparison with different η in EDFA is given in Figure 5. Since both groups have the same λ , the curves of two groups overlap completely, called as the reference curve. It is easy to note that whether η is too small (0.01, 0.05) or too large (0.5, 0.8), the fairness curves of center cell are upward from the reference curve, which means the system fairness

declines. Only when η takes a particular value, 0.1 in this example, the curve could overlap with the reference one, and the overall fairness achieves the best.

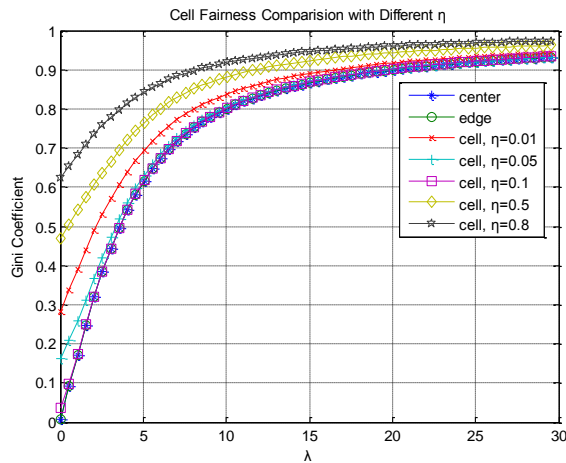


Figure 5. Total Fairness Comparison with Different η in EDFA

VI. Conclusion

This paper discussed the frequency resource allocation problem of the downlink in FFR-OFDMA cellular network. A flexible distributed frequency resource allocation scheme, which called EDFA, is proposed. This scheme divides the allocation process into three steps as grouping, intra-group allocation and inter-group allocation. By adjusting the values of parameters such as μ , λ and η , the proposed scheme could adjust the coverage, throughput, intra-group fairness and inter-group fairness of system, and finally balance the relationship between performance and fairness. Theoretically, this scheme has continuous controllability between “absolutely fair” and “absolutely unfair”. Simulation results illustrate that under the constraints of coverage, intra-fairness and inter-fairness, the presented frequency resource allocation scheme is capable of obtaining much higher throughput than ERA and PF.

Acknowledgement

This research was supported by the National Natural Science Foundation of China under Grant No. 61301155 and 61176025.

Reference

- [1] C. Y. Wong, R. S. Cheng, K. B. Lataief and R. D. Murch, “Multiuser OFDM with Adaptive Subcarrier, Bit, and Power Allocation,” *IEEE J. Sel. Area. Comm. Piscataway*, vol. 17, pp. 1747-1758, October 1999.
- [2] Z. Shen, J. Andrews and B. Evans, “Adaptive Resource Allocation in Multiuser OFDM System with Proportional Rate Constraints,” *IEEE Trans. Wirel. Commun. New York*, vol. 4, pp. 2726-2737, November 2005.
- [3] W. Rhee and J. M. Cioffi, “Increase in Capacity of Multiuser OFDM System Using Dynamic Subchannel Allocation,” *IEEE Veh. Technol. Conf. Tokyo*, vol. 2, pp. 1085-1089, May 2000.
- [4] Q. Li, R. Q. Hu, Y. Qian and G. Wu, “Intracell Cooperation and Resource Allocation in a Heterogeneous Network with Relays,” *IEEE T. Veh. Technol. Piscataway*, vol. 62, pp. 1770-1784, November 2012.
- [5] G. P. Koudouridis, C. Qvarfordt, T. Cai and J. Johansson, “Partial Frequency Allocation in Downlink OFDMA based on Evolutionary Algorithms,” *IEEE Veh. Technol. Conf. Ottawa*, pp. 1-5, September, 2010.
- [6] C. Singhal, S. Kumar, S. De, N. Panwar, R. Tonde and P. De, “Class-Based Shared Resource Allocation for Cell-Edge Users in OFDMA Networks,” *IEEE Trans. Mobile Comput. Los Alamitos*, vol. 13, pp. 48-60, October 2012.
- [7] S. Han, B. H. Soong and Q. D. La, “Subcarrier Allocation in Multi-cell OFDMA Wireless Networks with Non-Coherent Base Station Cooperation and Controllable Fairness,” *PIMRC, Sydney*, pp. 524-529, September 2012.
- [8] T. D. Novlan, J. G. Andrews, I. Sohn, R. K. Ganti and A. Ghosh, “Comparison of Fractional Frequency Reuse Approaches in the OFDMA Cellular Downlink,” *IEEE GLOBECOM. Miami*, pp. 1-5, December 2010.
- [9] T. D. Novlan, R. K. Ganti, A. Ghosh and J. G. Andrews, “Analytical Evaluation of Fractional Frequency Reuse for OFDMA Cellular Networks,” *IEEE Trans. Wirel. Commun. New York*, vol. 10, pp. 4294-4305, October 2011.
- [10] S. H. Ali and V. C. M. Leung, “Dynamic Frequency Allocation in Fractional Frequency Reused OFDMA Networks,” *IEEE Trans. Wirel. Commun. New York*, vol. 8, pp. 4286-4295, August 2009.
- [11] H. Mei, J. Bigham, P. Jiang and E. Bodanese, “Distributed Dynamic Frequency Allocation in Fractional Frequency Reuse Relay Based Cellular Networks,” *IEEE Trans. Commun. Piscataway*, vol. 61, pp. 1327-1336, January 2013.
- [12] H. C. Lee, D. C. Oh and Y. H. Lee, “Mitigation of Inter-Femtocell Interference with Adaptive Fractional Frequency Reuse,” *IEEE ICC. Cape Town*, pp. 1-5, May 2010.
- [13] F. P. Kelly, A. K. Maulloo and D. K. H. Tan, “Rate Control for Communication Networks: Shadow Prices, Proportional Fairness and Stability,” *J. Oper. Res. Soc. Basingstoke*, vol. 49, pp. 237-252, November 1998.
- [14] Y. Ma, “Rate Maximum for Downlink OFDMA with Proportional Fairness,” *IEEE Trans. Veh. Technol. Piscataway*, vol. 57, pp. 3267-3274, September 2008.
- [15] M. George and R. D. Koilpillai, “Fairness-Based Resource Allocation in OFDMA Downlink with Imperfect CSIT,” *WCSP. Hangzhou*, pp. 1-6, October 2013.
- [16] R. Steele and L. Hanzo, “Mobile Radio Communications,” *IEEE Press* 1999.
- [17] C. Kleiber and S. Kotz, “A Characterization of Income Distributions in Terms of Generalized Gini Coefficients,” *J. Social Choice and Welfare. Berlin*, vol. 19, pp. 789-794, October 2002.

About Author (s):



Lang Zhong was born in Sichuan, China, on November, 1981. He received the M.S. degree in Military Equipment from Xi'an Communication institute, Xi'an, China in 2008 and is currently studying towards the Ph.D. degree in Communication and Information systems at University of Electronic Science and Technology of China. His research interests are focused on resource allocation in wireless communication.



Jie Zhang was born in Sichuan, china, on June, 1984. He received the B. Eng. degree in Electronics Engineering, the M. Eng. degree in Signal Processing and the Ph. D. degree of Communication and Information System from the University of Electronic Science and Technology of China, Chengdu, China, in 2005, 2008 and 2013, respectively.



Xuemin Yang was born in Sichuan, China, on October, 1986. He received the M.S. degree in Signal and Information Processing from Chongqing University, Chongqing, China, in 2012 and is currently studying towards the Ph.D. degree in Communication and Information systems at University of Electronic Science and Technology of China. His research interests are focused on array signal processing and wireless communication.



Guangjun Li was born in 1950. He is currently a full professor in School of Communication and Information Engineering, University of Electronic Science and Technology of China. His research interests mainly include communication signal processing and wireless personal communication.



Zhi Zheng was born in Sichuan, China, on April, 1980. He received the M.S. degree in Circuit and Systems from University of Electronic Science and Technology of China, Chengdu, China, in 2007 and received the Ph.D. degree in Communication and Information systems at the same university in 2011. His research interests are focused on array signal processing, smart antenna and wireless communication.