

Financial Market Fluctuations in Econophysics: FTSE, DJIA & BIST-100

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ABSTRACT

This article aims at reviewing recent empirical and theoretical structures of entropy, temperature and energy of stock markets in the manner of statistical physics are obtained. What are the main characteristics of Econophysics? In what follows, we will try to summarise some basic principles. Each of them will be illustrated by one or several researches performed by econophysicists.

On the basis of some hypothesis of quantum mechanics, this paper considers stock markets as quantum systems and investors as particles. A quantum model of stock price fluctuations is defined in a theoretical framework. Essentially, the models are based upon models of statistical physics and quantum mechanics in which energy is conserved in exchange processes. The relative entropy is used as a measure of stability and maturity of financial markets from financial information about some considered emerging markets (Turkey) and some considered mature markets (England, United States). The model analytically calculates and simulate the system in FTSE-100, DJIA and BIST-100 indexes basic predictive model in Econophysics is discussed.

Keywords: *Finance, Physics, Econophysics, Entropy, Temperature and energy of stock markets*

I. Introduction

The science explaining the reasons and the results of economy with physical reasons and results more to define these more concretely is called “*Econophysics*” [1]. Econophysics is a new explanation of financial complex systems, Complex systems have many different properties which can be adapted to many different studies such as economics and business. An important property of a complex system is the ability to change volatility, entropy and volume, which is known as the phase diagram. A complex market mostly displays the motions of a fluid or gas. Bachelier’s, conducting studies on price fluctuations in Paris stock Exchange during 1900s, laid the foundations of the financial analysis of nowadays with his study on

the association of physics and mathematics [2] then, Einstein had developed formulations enlightening the behaviours of today’s investors with his study on the behaviours of particles in gasses [3].

In the 60s and 70s, a team consisted of three Americans: physicist Fischer Black, economist Myron Scholes and engineer Robert Merton have developed the Black-Scholes equation that can anticipate the price changes including financial variables such as time, price, interest rate and volatility.

The research they have carried comprised the basis for a huge as well as very new financial industry for the composition of derivative instruments and even more they have succeeded to receive the Nobel Prize in economics field thanks to the equations derived from physics [4].

Although it is a proved fact that physics and money have many common points¹, the point of departure can be considered in this way: When management of money is considered as a physical test, both requires involving with equations and numbers. Physics is based on developing some physical events in the world utilizing mathematical equations. Scientists have worked on how to implement the models developed in physics to “*financial world*” during recent years and they are focused on how to adapt some mathematical models obtained in “*molecular world*” to financial literature. Models are created utilizing processes where random results are formed and some research is carried on the pricing of financial instruments, the future movements of money and capital markets, trend of market and starting from their research efforts are made to make it possible to utilize these results by investors to reach profitable investment strategies.

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¹ Dominant method in neo-Classical economy is adapting thermodynamics to economics. At the same time, Neo-Classical economics is the start of engineering-economy tradition. Especially for L. Walras and I.Fisher interaction with thermodynamics is almost exact. Walras and Fisher have applied 1st Law of thermodynamics to economics

II. Thermo-Economy: Carnot Cycle and Finance Relationship

It is known that in the second law of thermodynamics it is said that all systems left by itself in the universe, to natural conditions will go through irregularity, disorder and degradation directly proportional with time. It is known that the transition of a system from a regular organized and planned structure to an irregular, dispersed and unplanned condition increases the entropy of the system and the high level of irregularity of a system leads the increase of entropy of the system at the same level. If we investigate Carnot Cycle in finance, it is possible to give following information [5].

With closed integration where W is the made investment (suffered obligation by investor) Q is the *return/profit* in consequence of the investment:

$$-\oint \delta W = \oint \delta Q$$

is reached.

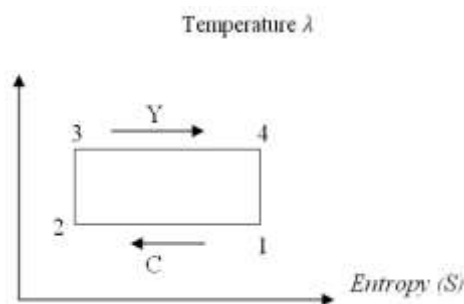


Figure 1: Carnot Cycle

$$ds = \frac{1}{\lambda} \delta Q$$

When the opposite value of the temperature is placed:

$$-\oint \delta W = \oint \lambda ds \quad (1)$$

equation is reached.

When the difference of the inputs to and outputs from the portfolio of investment Y is the return of the investment, C all expenses suffered for investment then delta Q (ΔQ) indicates net profit:

$$\begin{aligned} -\oint \delta W &= \oint \delta Q = \oint \lambda ds \\ &= \int_1^2 \lambda_1 ds + \int_3^4 \lambda_2 ds = Y - C = \Delta Q \end{aligned} \quad (2)$$

the equation gives the net remaining investment amount at the end of the investment. Let's try to see how adiabatic processes and isothermal processes available in thermal physics at the closed area integration shown in "Figure 1" are explained in the financial investment field [6]. During 1-2 process, the investor is buying securities for his portfolio. Since the work carried in the financial vessel yet collecting the securities together the change with entropy is $\Delta S < 0$. For instance, the cost (carried work) for n_1 units of shares at λ_1 the price level should be:

$$C = \lambda_1 \Delta S \quad (3)$$

This section corresponds to "reversible isothermal compression of financial instruments" in financial thermodynamics. At this stage heat profit of investor) is given out. The entropy change here:

$$\Delta S = \frac{q_1}{\lambda_1} \approx q_1 < 0 \quad (4)$$

At 2-3 process, the investor has formed his portfolio with those allocated from revenue. In order to arrive selling stage the occurrence of temperature at $\exists \lambda_2 > \lambda_1$ is waiting for. The thing indicated with λ_1 here is the price/temperature of securities after t time. The investor is not going forward to un-negligible changes. Since there is an entropy fixation in the process, entropy change is zero. This step corresponds to "reversible adiabatic compression of financial instruments" in financial thermodynamics.

Heat is not input to the system at this step. Here it's assumed that the temperature of security, increased from λ_1 to λ_2 due to external factors. During 3-4 process the investor wants to sell security to financial market with λ_2 price, assuming that the market has reached expected temperature where investor expectations are met. The revenue after sales will be:

$$Y = \lambda_2 \Delta S \quad (5)$$

This step corresponds to "reversible isothermal financial expansion". The change with entropy is now positive. It is equal to:

$$\Delta S = \frac{q_2}{\lambda_2} \approx q_2 < 0 \quad (6)$$

Here there is now financial heat flowing from hot portfolio to the system.

At 4-1 step, since the sale is realized there is portfolio stability. This section shows reversible adiabatic expansion. There is no heat output from the system. Therefore entropy change is zero. After this expansion fever is cooled down, in other words, formation of required base price is waited to the repurchase of the security. No input will occur to the portfolio of an investor until $\exists \lambda_1 > \lambda_2$ price is seen.

As the concepts of heat, entropy is at the forefront in *Carnot cycle*, it is seen possible to make these determinations:

1- The efficiency of investment can be possible with $\Delta\lambda$ temperature change.

2- If ΔS entropy change is linked with the probabilities of the system at different price steps, it may be possible to obtain sound assumptions for stock exchange market.

3- If sales-purchases in the portfolio of the investor are carried under the assumption of obtaining low temperatures under high entropy changes, this indicates a result that investment is made varied price ranges with relatively low risk. Purchase-sell processes are leading high probability continuity result. In such a case, it is possible to say that an investor can be readily entering exchange transactions in financial markets.

4- Let us consider the opposite condition. If the purchases- sells in the portfolio of investor are carried under the assumption of achieving high temperatures under low entropy changes, this gives a result indicating that investment is made with not fully diversified price ranges with higher risk. Purchase- Sell processes indicate that the investor is tending to investment under low probability continuity assumption. In such a case it is not possible to say that investor can easily make exchange transactions in the financial market. Because, the risk was taken over a certain level.

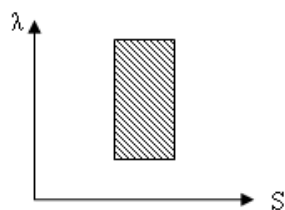


Figure 2: Risky Investment Cycle

Grey area states $\Delta\lambda\Delta S$ multiplication lower entropy change under high temperature change. It is possible for investors to take high risk, high return expectation. Therefore, it should be deemed normal to have a trend through such investment. But the portfolio of the investor is not secure under this situation. When there are risky investments, high losses or high returns are very probable. In this section investor did not invest in financial instruments with complex returns and may be hedged, but instead invested in more speculative assets.

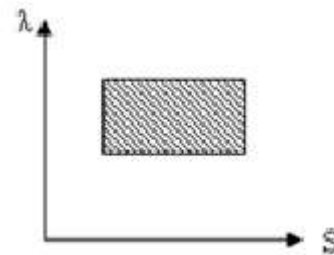


Figure 3: Non-Risky Investment Cycle

Grey area states $\Delta\lambda\Delta S$ multiplication for higher entropy change under low temperature change. The investor did not take risks here, preferring more volatile (easily sellable) securities with an acceptable level of return. Therefore, such an investment cannot be expected to give high returns. The portfolio of investor is safe in this condition. Since the subject matter is risk free investment, the probability of high losses and high returns are low (the probability is close to zero but not zero). In this part, the investor was preferred financial instruments, giving complex returns and can be hedged.

III. Microscopic Modelling of Volatility and Effects of Temperature [7]

Any of calendar time stock's returns of volatility denoted by σ_i is multiplied by square-root of stock's trading frequency. This brings us the notion temperature of stock denoted by X_i

$$X_i = \sigma_i \sqrt{V_i} \quad (7)$$

The right side of this equality denotes multiplication of calendar time volatility with trading frequency. This multiplication shows a stocks temperature. Furthermore, in this research, an assumption is thrown out for consideration

under this formula. If we can take a calendar time hypothesis with calculation of formula as a temperature of the stock market using volatilities, will the results significant? Providing that trading times of stock market are taken out to be 1 within a basis of daytime and stock market trading frequency is discussed as a normalized volume of the stock market, will it be possible for the research to give out significant results?

First evaluation was carried out between January 2003 and May 2012 in 2337 trading days and 4674 open-to-trade sessions regarding these days. The graphics about these have been discussed in the next section of the study. Certain data subject to the evaluation has been kept. With this data, temperature and entropy values have been calculated over the volume obtained during the day and FTSE-100, DJIA and BIST-100 index closing session values.

Volume and index closing values have been taken into account at the end of each session and they were accompanied with graphics. By doing so, there has been an attempt to determine the current direction of index trend. Intermediate values have been flattened for the date mentioned with the aim of avoiding any possible confusion.

Under separation of temperature formula, we have 2 parts. The one is calculated of volatility of calendar time returns of the index, the other one is production of trading frequency of the stock market.

Assumption under number of investors are constant, during the specific time t , every investor of financial system denoted by i has a $w(t)$ variable, which is accredited or alleviated the investor wealth. Then cumulative wealth or in other words direction factor in market players' investments is built up denoted by variable

$$W(t) = \sum_{i=1}^n w_i(t) \quad (8)$$

During changeovers from t_0 to t_1 , each investors wealth change of $W_i(t) \rightarrow W_i(t+1)$. From this point every investor has $W(t)$ variable. In fact, even if an economic coefficient such as the growth rate, taxes, social yields and interest rates, which is the same for each $W_i(t)$, is at work during these transitions, they have been excluded from the

study. If the value $W(t)$ which represents the increases and decreases statistically is considered as index value in the study, the market yield over t time will be

$$r(t) = \ln \left(\frac{W(t+1)}{W(t)} \right) \quad (9)$$

Variation of W within each trading time range is very small. Then, the volatility will be

$$\text{Volatility} = \left[\ln \frac{W(t+1)}{W(t)} \right]^2 / N \quad (10)$$

when one takes the average of the squares of yields over a certain time range. If it is considered as N step and transition step is taken as $N = 1$, new calculation formula of volatility will be

$$\text{Volatility} = \left[\ln \frac{W(t+1)}{W(t)} \right]^2 \quad (11)$$

Constitute the basics of calculating, temperature formula have been formed as 2 parts. If one rewrites the volatility of calendar time returns, first one of the two parts of temperature formula which is subject to the calculation, the outcome is

$$\text{Volatility} = \left[\ln \frac{\text{Index_Values}_{(t+1)}}{\text{Index_Values}_{(t)}} \right]^2 \quad (12)$$

We need for a variable called trading frequency of market in the section which constitutes for the secondary part in calculation of temperature.

According to the intrinsic time hypothesis, trading frequency means the number of trades through which related stocks pass over a certain time period. According to the calendar time hypothesis, on the other hand, it means trades experienced within a certain session. Since calendar time hypothesis constitutes for the basis of the assessment here, the number of contracts or the number of orders has been taken into account for trading frequency and they have been normalized by using a coefficient, 10^{12} , in order to make the calculation easier.

$$\text{Trading Frequency of Market}_{t+1} = \sqrt{\text{Market_Volume}_{t+1} \cdot 10^{12}} \quad (13)$$

To sum up, the temperature of the market during $t+1$ time, λ_{t+1} , will be

$$\lambda_{t+1} = \left[\ln \frac{\text{Index_Values}_{(t+1)}}{\text{Index_Values}_{(t)}} \right]^2 \cdot \sqrt{\text{Market_Volume}_{t+1} \cdot 10^{12}} \quad (14)$$

The notion “entropy” has been discussed in previous chapters. So, as an element of temperature, entropy has been considered as the variation of trend direction of the market. According to Bose-Einstein statistic, entropy of a Bose system, (Φ), is calculated as **Entropy of the market**

$$\Phi = \frac{\varepsilon}{\lambda} \cdot \frac{1}{e^{\lambda-1}} + \log \left(1 + \frac{1}{e^{\lambda-1}} \right) \quad (15)$$

If one takes the index closing value in $\varepsilon = t + 1$ time as a temperature value in $\lambda = t$ time, the entropy value in $\Phi = t + 1$ time is calculated.

Entropy minimum and entropy maximum points provide further information for the trade in cyclic analyses. Entropy in this study appears as an application of the Second Law of Thermodynamics. This law, as mentioned previously, suggests that each event will ultimately present itself on a stationary level.

Like human beings and other substances, assets traded in financial markets have also a life-span. Forward movement of assets, i.e. towards a positive direction, or their backward movement, that is in a negative direction, can account for these life-spans. When entropy is applied to the equity market, it proves an indicator of the fact that movement of a financial instrument in one direction has ended and goes towards another direction.

In the simulation designer, entropy ceiling and entropy base points have been applied to equity markets. Laws have been displayed together with temperature and Bose Einstein condensation. Also, temperature and entropy values have been composed in different sessions, not as weekly, monthly or yearly.

In daily overview, entropy calculations have been carried out by using trading frequency in calculation, but in order to make the presentation robust, session times, which can be accepted as not trading times but calendar times have been taken as the basis. Furthermore, graphics regarding volatility-index and risk-temperature have been calculated daily on a monthly basis and represented graphically. And this allowed the study to make

comparisons with other graphics. Also, by means of a polynomial on related graphics, interpolation which gives flattening of intermediate value has been undertaken for all values appearing so as to make monitoring easier and to enable rather scattered values to be studied all together. It has been of great importance to calculate intermediate values for enabling a comparison of two values on the same graphic.

IV. Conclusion

This paper investigates the relation between Entropy of the market, temperature and energy of stock markets, structural change and the size of the stock market investigations.

Entropy is not restricted to natural science, but is a function of mathematical and physical statistics. Econophysicists often use the concept of entropy to characterise the idea of uncertainty in Physical, Mathematical approach, economics and finance. As Dionisio [8] explain, “*entropy is a measure of dispersion, uncertainty, disorder and diversification used in a dynamic process, in statistics and information theory, and has been increasingly adopted in financial theory*”. This investigation, the diversity approach is more useful in the cases of new, unrecognised physical phenomena.

It is a fact that the investors, in contrast to gas particles, have enough recall ability to modify their behaviours constantly. In this context, it is already unlikely to totally apply gas particle behaviour to financial markets. However, it is more sensible to carry out trading possibilities and risk estimation through a prediction of the direction of the existing trend via modern physical theories called the *statistical structure of quantum*.

Therefore, this paper aims to achieve a different perspective, not found in other traditional studies on equity pricing and trend analyses about markets. In fact, as argued in most of the studies on the topic, it is obvious that the prices cannot be predicted thoroughly. Yet, as can be inferred from this work, one should not forget that rise and falls on pricing follow a certain succession. From that perspective, for a proper handling of the happenings in the past and easy understanding of potential conditions in the future, it is significant to consider the market as a scientific system. Considering that the markets have a personality

around themselves, it is possible to conclude the following: Despite seeming to be formed through effects outside the market, the prices are actually determined by the internal dynamics of the market. Intra-market happenings determine the prices and external dynamics are held as reasons after the happenings have taken shape.

The fact that the prices seem to be reacting to the external dynamics affects all the investors at the same time. However, what is important in a neutral situation is how the investors affect each other and their demands for being/not being in the same energy condition. This is, on the other hand, a condition to be evaluated within the statistical background of Econophysics.

Figures

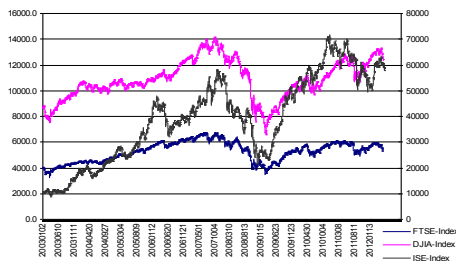


Figure 1: Ten Year Index Comparison for DJIA-FTSE100

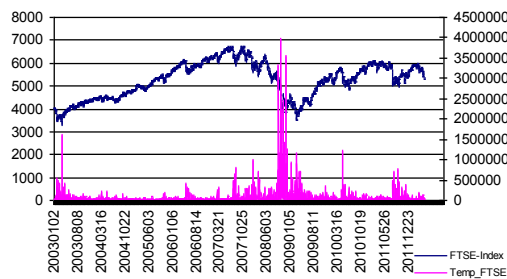


Figure 2: Ten Year Comparisons of FTSE 100 Index and FTSE Index Temperatures

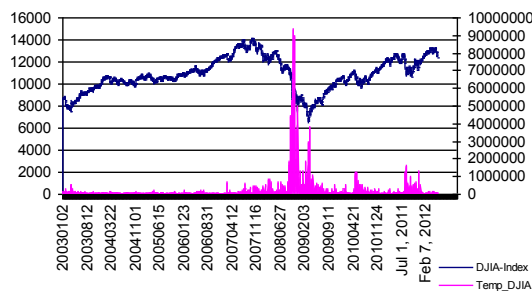


Figure 3: Ten Year Comparisons of DJIA Index and DJIA Index Temperatures

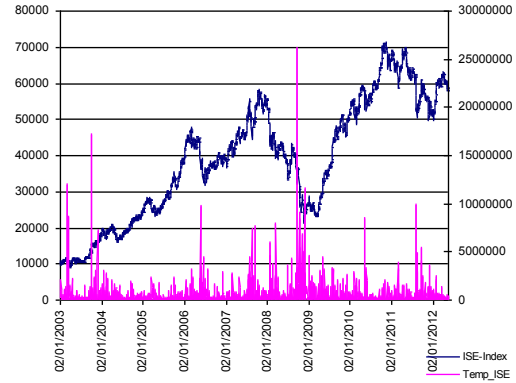


Figure 4: Ten Year Comparisons of ISE100 Index and ISE 100 Index Temperatures

Annual Analogy Between FTSE Temperature and FTSE Risk for 2003

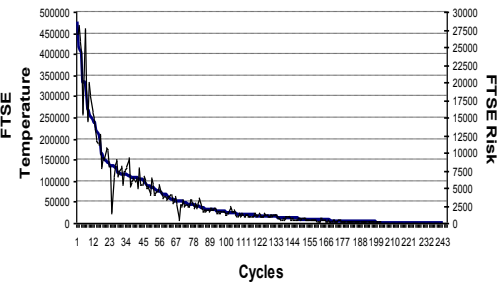


Figure 5: Annual Analogy Between FTSE Temperature and FTSE Risk for 2003

Biennial Analogy Between FTSE Temperature and FTSE Risk for 2004-2005

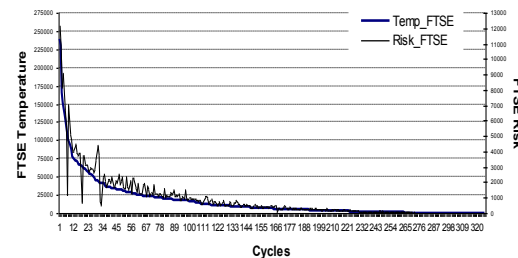


Figure 6: Biennial Analogy Between FTSE Temperature and FTSE Risk for 2004-2005

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