

# METHOD N2 – ACCORDING TO FAJFAR

Arton Dautaj, Naser Kabashi and Hajdar Sadiku

**Abstract:** A relatively simple nonlinear method for the seismic performance evaluation of structures (the N2 method) is presented. The method combines the nonlinear static (pushover) analysis and the response spectrum approach. The method yields results of reasonable accuracy if the structure oscillates predominantly in the first mode. In the paper the method is formulated in the acceleration – displacement format. This versions combine the advantages of the visual representation of CSM developed by Freeman. By reversing the analysis process, the method can be used as a tool for the implementation of the direct displacement-based design approach.

**Keywords:** "Pushover analysis", performance evaluation, inelastic behavior, ductility etc.

## Introduction

For the rational design of buildings from seismic operations, design methods must meet the following requirements:

- a) adequately respond to requests of stiffness, strength and ductility during an expected earthquake[5], and
- b) not be complicated[5].

The methods applied in the various codes ( analysis of equivalent lateral force and modal spectral analysis) are based on the assumption of linear elastic behavior of the structure and as such, regardless of the application to them of various modifier and corrective factors, fail to satisfy the first request adequately.

On the other hand, nonlinear dynamical analysis of the system with many degrees of freedom is relatively complicated and, as such, are not very suitable for everyday design.

Method N2 (N2 designation relates to the fact that it requires the application of a nonlinear method - "Nonlinear" - and the construction of two models) meets the two requirements above

N2 method in Europe is developed in Slovenia, in the University of Ljubljana and is based on the so-called model-Q, filed by Saiidi and Sozen [23] for simple nonlinear analysis of systems with multi degrees of freedom. Later, this method has been developed through different stages and for different systems, while now has been included in the final draft of the EC 8 [3]

N2 method provides results with sufficient accuracy and can be used for systems where seismic response is dominated by the contribution of the first form of vibration.

Initially N2 method is presented for "regular" systems (Fajfar & Fischinger, 1987 ECEE [4], 1989 WCEE [5]). As basic proposals of this method are the use of two different mathematical models and the application of three main steps in the analysis. In the first step is determined the stiffness (rigidity), strength and ductility. For this applies nonlinear static analysis of the system with many degrees of freedom (MDOF) by the action of a form of distribution of loads that monotonically increasing.

In the second step is defined equivalent system with single degree of freedom. Nonlinear characteristics of equivalent system are based on the relation base shear force -displacement on the roof designated by nonlinear static analysis in the first step. While on the third step of the method N2 from nonlinear dynamic analysis of the equivalent system with single degree of freedom is determined the maximum displacement (and corresponding demand on ductility). The third step, in a simple form can be performed using inelastic spectra.

As mentioned above, the use of inelastic spectra in the third step can simplify a lot the analysis and make it highly suitable for daily practice project.

After processing of the first ideas, the method has already found numerous applications of seismic analysis and anti-seismic design (reinforced concrete buildings considering the remaining damages, according Fajfar and Gaspersic [16], bridges, asymmetric buildings; spatial buildings; and, finally assessment of performance). Actually, the N2 method is formulated in the format AD [14,17], acceleration-displacement.

The following is given recently developed version of the method N2.

---

Arton Dautaj<sup>1</sup>

Naser Kabashi<sup>1</sup>

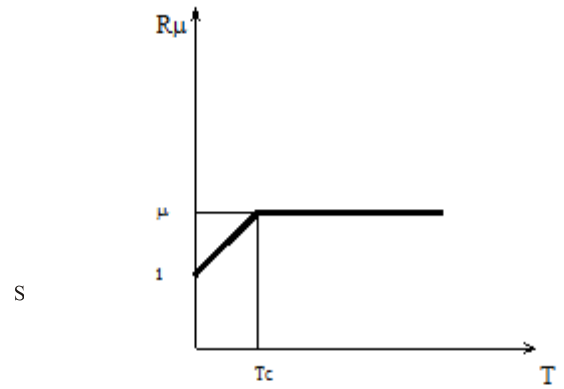
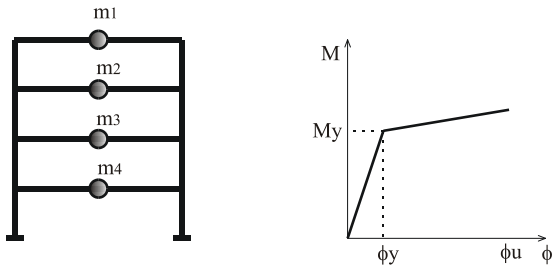
Hajdar Sadiku<sup>1</sup>

<sup>1</sup>Civil Engineering and Architectural Faculty, University "Hasan Prishtina" of Prishtina  
Republic Of Kosova,

# 1. Summary of N2 method

## I. Data

- a) Structure
- b) Moment-curved geometry relationship
- c) Elastic spectra of accelerations



$$R_{\mu} = (\mu - 1) \frac{T}{T_c} + 1, T < T_c$$

$$R_{\mu} = \mu, T \geq T_c$$

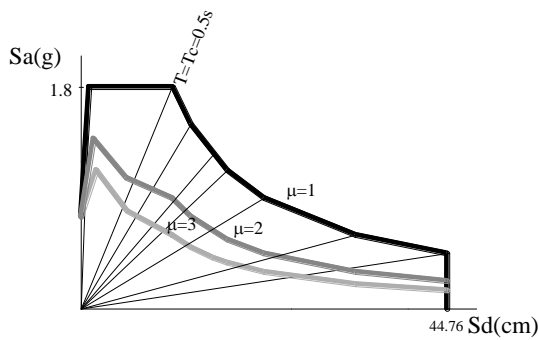
## II. Seismic demands on format AD

- a) Define elastic spectra in AD format

$$S_{de} = \frac{T^2}{4T_c^2} S_{ae}$$

- b) Define the inelastic spectra for constant ductility

$$S_a = \frac{S_{ae}}{R_{\mu}}, S_d = \frac{\mu}{R_{\mu}} S_{de}$$

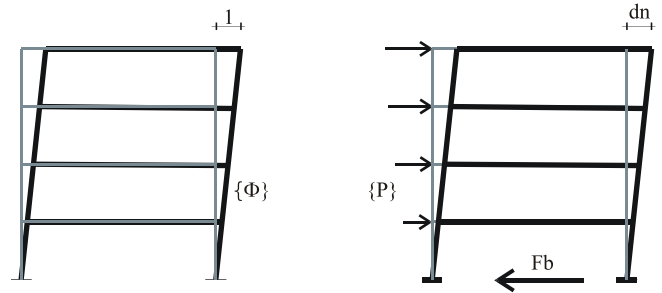


## III. "Pushover" analysis

- a) Assume the form of displacement  $\{\Phi\}$
- b) Determine the vertical distribution of lateral force

$$\{P\} = [M]\{\Phi\}, P_i = pm_i \Phi_i$$

- c) Define the relationship "based shear force - displacement on the roof".



## IV. Equivalent model with single degree of freedom.

- a) Define the mass  $m^*$

$$m^* = \sum_{i=1}^n m_i \Phi_i^2$$

b) Transform quantities (Q) of system with multi degrees of freedom in quantities ( $Q^*$ ) of the system with single degree of freedom

$$Q^* = \frac{Q}{\Gamma}, \quad \Gamma = \frac{m^*}{\sum_{i=1}^n m_i \Phi_i^2}$$

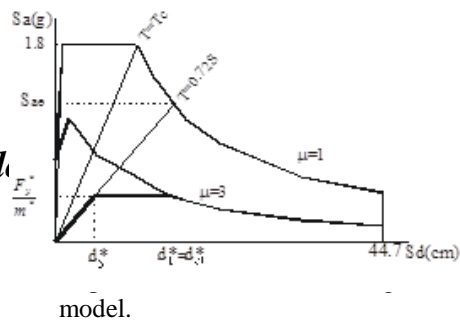
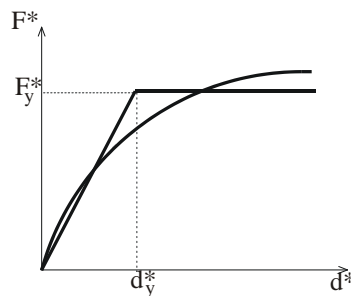
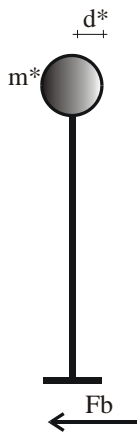
c) Determine the approximate relationship elastic-plastic force-displacement

d) Determine the strength  $F_y^*$ , yield displacement  $d_y^*$  and period  $T^*$ .

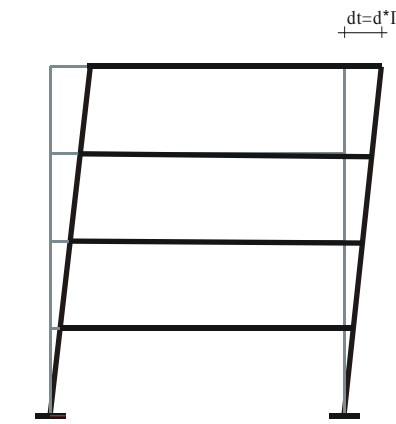
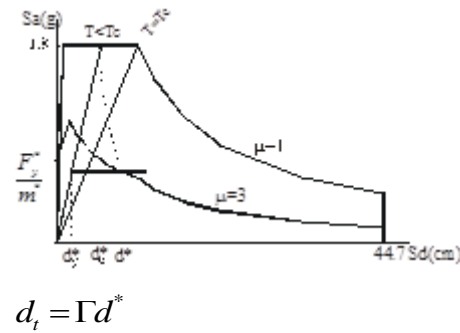
$$T^* = 2\pi \sqrt{\frac{m d_y^*}{F_y^*}}, \quad (k^* = F_y^*/d_y^*)$$

e) Determine the diagram of capacity (acceleration versus displacement)

$$S_a = \frac{F^*}{m}$$



for the  
 system  
 the top  
 freedom



### V. Seismic demand for the model with single degree of freedom.

a) Define reducing factor  $R_\mu$

$$R_\mu = \frac{S_{ae}}{S_{ay}}$$

b) Define displacement demand

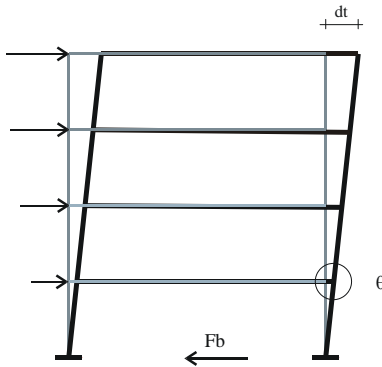
$$d^* = S_d = \frac{S_{de}}{R_\mu} \left( 1 + (R_\mu - 1) \frac{T_c}{T^*} \right), \quad T^* < T_c$$

$$d^* = S_d = S_{de}, \quad T^* \geq T_c$$

### VII. Local seismic demand

a) Apply the analysis "pushover" in the model with multi degrees of freedom ( MDOF) until reaching the displacement in  $d_t$

b) Determine local quantities (relative displacement of floors, rotation of joints etc.), corresponding to dt



Determining the displacement of the system with single degree of freedom

$$d_t^* = S_d^* = \frac{d_t}{\Gamma}$$

where  $\Gamma \cong \frac{3n}{2n+1}$  or  $\Gamma = \frac{m^*}{\sum_{i=1}^n m_i \Phi_i^2}$ , where n-

number of floors,  $m^* = \sum_{i=1}^n m_i \Phi_i$ ,

### VIII. Performance Assessment

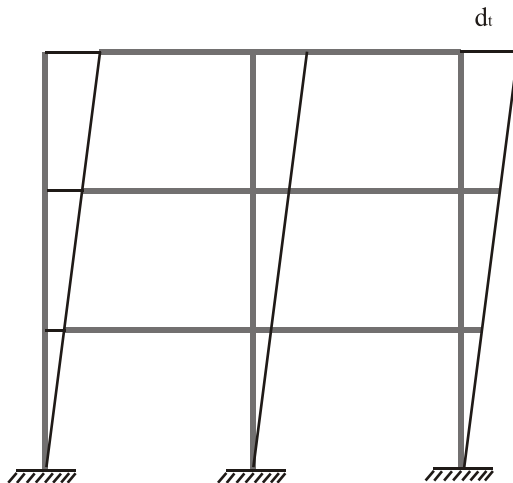
a) Compare local and global seismic demand with capacity to the required level of performance.

## 2. APPLICATION OF THE METHOD N2 IN DESIGN

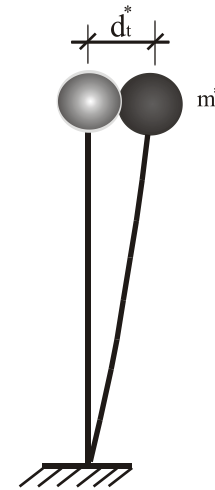
With the inversion of procedure of method N2 used for assessment of the performance, can be developed a design methodology based directly on the displacement. Practically will be done this way:

#### 1<sup>st</sup> Step.

For the required performance (defined) is given the displacement  $d_{t(n)}$ , which represents the displacement on the roof of the system with many degrees of freedom.



#### 2<sup>nd</sup> Step.



$\Phi_i$  - linear form of vibration (first form of vibration).

#### 3<sup>rd</sup> Step.

Determining the ductility or stiffness (rigidity) of the structure. To define the ductility, initially should be determined the yield displacement for the system.

$$d_{yt}^* = \frac{d_{yt}}{\Gamma}$$

Ductility will be

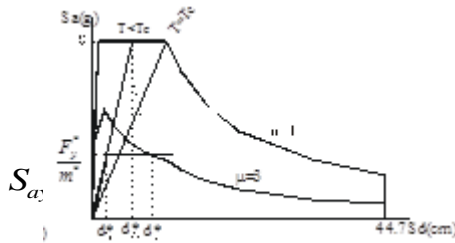
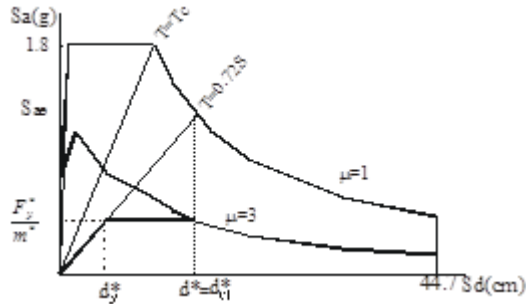
$$\mu = \frac{d_t^*}{d_{yt}^*}$$

#### 4<sup>th</sup> Step.

For the defined quantity  $d_t^*$  appreciate  $S_{ae}$  and  $T^*$ . Further apply the following expressions:

$$R_\mu = \mu \text{ per } T^* \geq T_C,$$

$$R_\mu = (\mu - 1) \frac{T^*}{T_C} + 1 \text{ per } T^* < T_C$$



Thus, base shear force of the system with single degree of freedom will be:

$$S_{ay} = \frac{F_y^*}{m^*} \Rightarrow F_y^*$$

**5<sup>th</sup> Step.**

Determine the base shear force for the system with multi degree of freedom.

$$F_b = F_y^* \cdot \Gamma$$

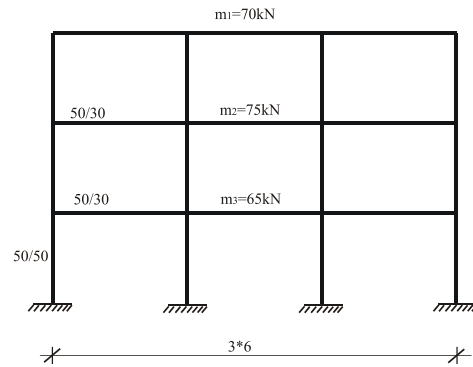
**6<sup>th</sup> Step.**

The distribution in the height of the structure of horizontal load adapting assumed displacement profile:

$$F_i = \frac{\Phi_i m_i}{\sum_{i=1}^n \Phi_i m_i} F_b$$

**Example:**

According to the Fajfar method N2 to evaluate the structure for seismic demand by EC-8,B,ag=0.3g ,04g,0.5g and 0.6g



From the "pushover" analysis is defined the relationship base shear force – displacement on the roof.

$$F_b=670 \text{ kN}, d_{yt} \cong 13 \text{ cm} \quad 4.20$$

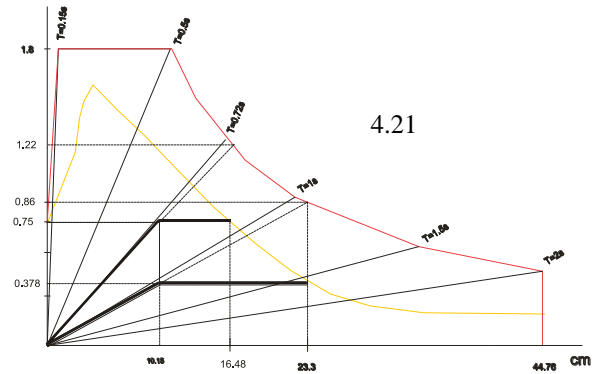


Fig.1,Displacement demand for ag=0.6g,B,EC8,v.2002

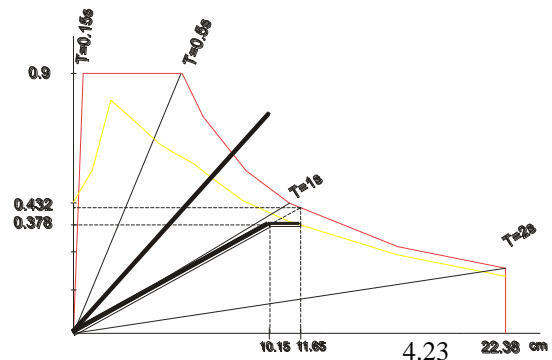


Fig.2,Displacement demand for 0.3g,B,EC8,v.2002

For four cases of seismic demand, the results are presented below in tabular form.

	0.3g	0.4g	0.5g	0.6g
--	------	------	------	------

$F_b(kN)$	670	670	670	670
$d_{yt}(cm)$	13	13	13	13
$m^*(ton)$	141	141	141	141
$F_y^*(kN)$	523	523	523	523
$d_{yt}^*(cm)$	10.15	10.15	10.15	10.15
$T^*(s)$	1.04	1.04	1.04	1.04
$k^*(kN/m)$	5153.8	5153.8	5153.8	5153.8
$S_{ay}(m/s^2)$	0.378g	0.378g	0.378g	0.378g
$S_{ae}(m/s^2)$	0.432g	0.57g	0.72	0.86
$R_\mu$	1.14	1.53	1.91	2.29
$d_t^*(cm)$	11.65	15.53	19.42	23.3
$d_t(cm)$	14.93	19.91	24.89	29.86

### According to the DDBD N2method - Fajfar,

For the defined performance  $d_{t(n)} = 15cm$  and seismic demand,  $a_g = 0.3g, 0.4g, 0.5g$  and  $0.6g$  to be determined the base shear force

$$\Gamma = 1.28$$

$$m^* = 141 \text{ ton}$$

$$d_t^* = 11.71cm$$

$$d_{yt} = 0.5 \cdot 0.002 \cdot \frac{600}{50} \cdot 1200 = 14.4cm$$

$$\pm 10\% \cong 13cm$$

$$d_{yt}^* = 10.15cm$$

$$\mu = 1.15$$

From the design spectra can be obtained (read):

$$S_{ae} = 0.43g \quad \text{dhe } T^* = 1.04$$

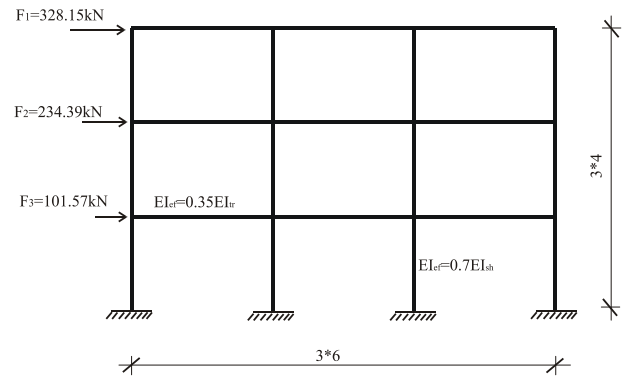
$$S_{ay} = \frac{S_{ae}}{R_\mu} = 0.37g$$

$$F_b = 664.12kN$$

$$F_1 = 328.15kN$$

$$F_2 = 234.39kN$$

$$F_3 = 101.57kN$$



For four cases of seismic demand, the results are presented below in tabular form

	0.3g	0.4g	0.5g	0.6g
$m^*(ton)$	141	141	141	141
$\Gamma$	1.28	1.28	1.28	1.28
$d_t(cm)$	15	15	15	15
$d_t^*(cm)$	11.71	11.71	11.71	11.71
$d_{yt}(cm)$	13	13	13	13
$d_{yt}^*(cm)$	10.15	10.15	10.15	10.15
$\mu$	1.15	1.15	1.15	1.15
$S_{ae}(m/s^2)$	0.43g	0.76g	1.195	1.49g
$T^*(s)$	1.04	0.78	0.627	0.52
$R_\mu$	1.15	1.15	1.15	1.15
$S_{ay}(m/s^2)$	0.37g	0.66g	1.035g	1.49g
$F_y^*(kN)$	518.28	921.37	1439.64	2073.09
$F_b(kN)$	664.12	1180.65	1844.78	2656.48

### Conclusion

N2 method can be used both for the seismic performance evaluation of newly designed or existing structures. Furthermore, by reversing the analysis process, the method can be used as a tool for the implementation of direct displacement-based design approach, in which design starts from a predetermined target displacement. The limitations of the methods based on pushover analysis should be

recognized. A detailed discussion of pushover analysis can be found in (Krawinkler and Seneviratna)

## Literature

1. A.S.Elnashai."DO WE REALLY NEED INELASTIC DYNAMICS ANALYSIS". Journal of Earthquake Engineering, Vol.6, Special issue 1(2002), pp 123-130.
2. A.D.Dautaj."VEÇORITË E APLIKIMIT TË METODIKAVE TË PROJEKTIMIT BAZUAR NË ZHVENDOSJE PËR STRUKTURAT BETONARME NË ZONAT SIZMIKE". Tema e Magjistratures, Prishtinë, 2005.
3. "Eurocode 8: Design of structures for earthquake resistance" Part 1: General rules, seismic actions and rules for buildings. DRAFT No 6, Version for translation, January 2003, CEN.
4. P.Fajfar and M.Fishinger, "Non-linear seismic analysis of RC buildings: implications of a case study", European Earthquake Engineering, 1, 31-43. (1987).
5. P.Fajfar and M.Fishinger, "N2-a method for non-linear seismic analysis of regular buildings", Proc. 9<sup>th</sup> World Conf. Earthquake Engineering, Vol. V, Tokyo, Kyoto, 1988, Maruzen, Tokyo, 1989, pp. 277-287.
6. P.Fajfar, "Elastic and inelastic design spectra". In 10<sup>th</sup> Eur. Conf. Earthquake Engineering, (ed. G. Duma), Vienna, 1994, Balkema, Rotterdam, Vol. 2, pp. 1169-1178.
7. M.Fischinger, P.Fajfar and T.Vidic, "Factors contributing to the response reduction." Proc. 5<sup>th</sup> U.S. Conference on Earthquake Engineering, Vol. 1. Chicago, IL, 1994, pp 97-106.
8. P.Fajfar and P.Gaspersic, "The N2 method for the seismic damage analysis for RC buildings", Earthquake Engineering and Structural Dynamics, 25, 23-67 (1996).
9. P.Fajfar and H.Krawinkler (1992), "Nonlinear Seismic Analysis and Design of Reinforced Concrete Buildings", Elsevier Applied Science, New York, 1992.
10. P.Fajfar, "Equivalent ductility factors taking into account low-cycle fatigue", Earthquake Engineering and Structural Dynamics, 21, 837-848 (1992).
11. P.Fajfar. (1996): "Design spectra for new generation of codes. In 11<sup>th</sup> World Conference on Earthquake Engineering, Acapulco, 28-28 June 1996, CD-ROM 2127.
12. P.Fajfar, "Trends in seismic design and performance evaluation approaches". In 11<sup>th</sup> European Conference on Earthquake Engineering, Paris, 6-11 September 1998, Balkema, Rotterdam, Invited Lectures, 237-249.
13. P.Fajfar and H.Krawinkler (eds), Seismic Design Methodology for the Next Generation of codes", Balkema, Rotterdam, 1997.
14. P.Fajfar, 1999, Capacity spectrum method based on inelastic demand spectra", Earthquake Engineering and Structural Dynamics, 28, 979-993.
15. P.Fajfar and D.Drobic, 1998, "Nonlinear seismic analysis of ELSA buildings", Proceedings of the 11<sup>th</sup> European Conference on Earthquake Engineering, Paris, CD-ROM, Balkema, Rotterdam.
16. P.Fajfar, P.Gaspersic, D.Drobic, 1997, "A simplified nonlinear method for seismic damage analysis of structures", in P.Fajfar and H.Krawinkler (eds), Seismic Design Methodology for the Next Generation of codes, Balkema, Rotterdam, 1997, pp. 183-194.
17. P.Fajfar, "A nonlinear analysis method for performance-based seismic design" Earthquake Spectra 2000, 16(3):573-592.
18. P.Fajfar, "A Practical Nonlinear Method for Seismic Performance Evaluation." Proceedings of the 2000 Structures Congress & Exposition, ed. M. Elgaaly, May 8-10, 2000, Philadelphia, Pennsylvania.
19. S.A.Freeman, "Development and use of capacity spectrum method". Proc. of 6<sup>th</sup> U.S. National Conference on Earthquake Engineering, Sttle, CD-ROM, Oakland: EERI, 1998.
20. S.A.Freeman, "The capacity Spectrum Method as a Tool for Seismic Design", Proc. 11<sup>th</sup> European Conference on Earthquake Engineering, Paris, CD-ROM, A.A. Balkema, Rotterdam, 1998.
21. H.Krawinkler and G.D.P.K.Seneviratna, "Pros and Cons of Pushover analysis for seismic performance evaluation" Engineering Structures. 20, 452-464 (1998).
22. S.R.Lawson and H.Krawinkler, "Nonlinear static pushover analysis-Why, when and how?" Proc. 5<sup>th</sup> U.S. Conference on Earthquake Engineering, Vol. 1. Chicago, IL, 1994, pp 283-292.
23. M.Saiidi and M.A.Sozen, "Simple nonlinear seismic analysis of R/C structures". Journal of the Structural Division, ASCE, 1981; 107(ST5); 937-951.