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Comparison between Fuzzy Risk Assessment Methods

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***Abstract*— Contracting for construction services is an inherently risky venture for the owner, design agent and contractor. All of these parties are exposed to unanticipated risks, exposure to economic loss and unforeseen contract liability while performing under the contract. Project risk management, therefore, has been recognised critical for the construction industry to improve their performance and secure the success of projects. Risk assessment is the most important step in risk management. Classical methods for risk assessment are no longer accurate and effective, therefore, many papers introduced fuzzy logic as a more accurate and effective technique in risk assessment. In this paper, a comparison between two fuzzy risk assessment methods; Nieto-Morote and Ruz-Vila [1] and Kuo and Lu [2] is done using the same input parameters which are risk probability (RP), risk impact (RI) and risk discrimination (RD) to determine if these methods give the same risk ranking or not. Actually, the comparison results in different risk ranking, because the Nieto-Morote and Ruz-Vila [1] method depends on minimization error tool to minimize inconsistency in results, and this tool always doesn't give optimum results, while, we can consider Kuo and Lu [2] method more accurate because it depends on eliminating the inconsistency in results using a transformation process step to remain the decision matrix with reciprocity and additive consistency.**

***Keywords*— risk assessment, linguistics variables, fuzzy logic, fuzzy numbers, risk factor.**

I. Introduction

The construction industry is plagued by uncertainty and risks and this returns to the fact that construction projects are complex, dynamic in their nature, governed by complicated contracts and involving complex relationships. This makes it very important to plan how to deal with these risks during the project, because if risks are not dealt with adequately, they will result in poor performance with increased costs and time delays [3].

First of all let us define the meaning of risk from different perspectives; engineering, technological, financial and economical points of view [4]. Project risk is defined by Project Management Body of Knowledge (PMBOK) [5] as an uncertain event or condition that, if it occurs, it has a positive or negative impact on, at least, one of the project objectives such as cost, time and quality.

Mark, et al. [6] defined "risk" as the potential for complications and problems with respect to the completion of a project and the achievement of a project goal. Chapman and Ward [7] defined risk as "the exposure to the possibility of economic or financial loss or gain, physical damage or injury, or delay, as a consequence of the uncertainty associated with pursuing a particular course of action".

Bufaied [8] has described risk in relation to construction as "a variable in the process of a construction project whose variation results in uncertainty as to the final cost, duration and quality of the project". According to Dey [9], such variation is due to the absence of risk management techniques in project management. Hence, risk management, as defined by Toakley [8] is a procedure which controls the level of risk and mitigates its effects.

Risk management process consists of three main steps; risk identification, risk assessment and risk response. Risk identification involves identifying the source and type of risks and classifying risk according to risk source. Risk assessment is the process of prioritizing risks for further analysis by assessing and combining, generally, their probability of occurrence and impact. Risk response is the choice of a proper strategy to reduce the negative impact of the risk [8].

II. Fuzzy Risk Assessment

The concept of fuzzy sets and theory was first introduced by Zadeh[10] where data are defined in terms of mathematical logic rather than vague and linguistic terms such as low probability or high impact.

Many papers introduced fuzzy logic as a risk assessment tool in construction process but with different methodologies. Kangari and Riggs [11] use fuzzy sets in natural language computation, risk evaluation and linguistic approximation. Also Wirba, et al. [12] assesses the risk`s likelihood of occurrence by using linguistic variables. Choi, et al. [13] fuzzy-based uncertainty assessment system considers

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uncertainty as objective probabilities and subjective judgment by incorporating probabilistic or linguistic variables.

While Carr and Tah [3] implements a hierarchical risk breakdown structure in a fuzzy risk assessment model using linguistic variables in definition of risk factors which are the likelihood of occurrence (L), the severity (V), and the effect of the risk factor (E) and formulation of the rules describing the relationship between these factors to determine the value of risk. Also Zeng, An and Smith [14] applies the modified analytical hierarchy process with fuzzy reasoning techniques to provide an effective tool to handle the uncertainties and subjectivities arising in the construction projects.

From the most popular papers, Wang and Elhag [15] who uses fuzzy group in designing a decision making approach (FGDM) to evaluate risk factors in bridge construction. The FGDM approach introduces the values of risk factors which are risk likelihood and risk consequences in terms of triangular fuzzy sets. All the consequences are assessed against four criteria; safety, functionality, sustainability and environment. The risk rating is a fuzzy multiplication of risk likelihood and consequences. The model provides two alternative algorithms to aggregate the assessments of multiple bridge risk factors, one of which offers a rapid assessment and the other one leads to an exact assessment.

Both Nieto-Morote and Ruz-Vila [1] and Kuo and Lu [2] introduce fuzzy risk assessment models with Analytic Hierarchy Process (AHP) using sequential pair-wise comparison matrix to evaluate the relative impact of identified risk factors on project performance. But Nieto-Morote and Ruz-Vila [1] formulate risk rating in terms of risk impact, risk probability and risk discrimination. All these factors are expressed in terms of trapezoidal fuzzy numbers to capture the vagueness in the linguistic variables. While Kuo and Lu [2] evaluate risk in terms of relative impacts and probability of occurrence using consistent fuzzy preference relations (CFPR) to investigate the relative impact on project performance of identified risk factors and dimensions and the fuzzy multiple attributes direct rating (FMADR) approach to analyze the multiple risk factors' probability of occurrence which is defined in terms of triangular fuzzy numbers.

III. Basic concepts on fuzzy sets

A fuzzy set is a collection of elements in a universe of information where the boundary of the set contained in the universe is ambiguous, vague and otherwise fuzzy. Each fuzzy set is specified by a membership function, which assigns to each element in the universe of discourse a value within the unit interval [0, 1].

Let X be the universe of discourse. A fuzzy set $\tilde{A}$ of the universe of discourse X is said to be convex if and only if for all x_1 and x_2 in X there always exists:

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \mathrm{Min}(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)), \quad (1)$$

where $\mu_{\tilde{A}(x)}$ is the membership function of the fuzzy set $\tilde{A}$ and $\lambda \in [0, 1]$.

Fuzzy numbers are special cases of fuzzy sets that are both convex and normal. A fuzzy number is a convex fuzzy set, characterized by a given interval of real numbers, each with a grade of membership between 0 and 1. Its membership function is piecewise continuous and satisfies the following conditions:

- $\mu_{\tilde{A}}(x) = 0$ for each $x \notin [a,d]$;
- $\mu_{\tilde{A}}(x)$ is non-decreasing (monotonic increasing) on [a,b] and non-increasing (monotonic decreasing) on[c,d]; (2)
- $\mu_{\tilde{A}}(x) = 1$ for each $x \in [b,c]$;

where $a \leq b \leq c \leq d$ are real numbers in the real line R = $(-\infty, +\infty)$ [14].

The most commonly used fuzzy numbers are triangular and trapezoidal fuzzy numbers, in this study, the trapezoidal fuzzy numbers are used whose membership functions are respectively defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} (x-a)/(b-a), & a \leq x \leq b, \\ 1, & b \leq x \leq c, \\ (d-x)/(d-c), & c \leq x \leq d, \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$

By the extension principle [10], the fuzzy arithmetic operations of any two trapezoidal fuzzy numbers follow these operational laws:

Fuzzy addition:

$$A_1 \oplus A_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2) \quad (4)$$

Fuzzy subtraction:

$$A_1 \Theta A_2 = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2) \quad (5)$$

Fuzzy multiplication:

$$A_1 \otimes A_2 \approx (a_1 \text{ x } a_2, b_1 \text{ x } b_2, c_1 \text{ x } c_2, d_1 \text{ x } d_2) \quad (6)$$

Fuzzy division:

$$A_1 \varnothing A_2 = (a_1 / d_2, b_1 / c_2, c_1 / b_2, d_1 / a_2) \quad (7)$$

The scalar multiplication of a trapezoidal fuzzy number is also a trapezoidal fuzzy number defined as:

$$\begin{aligned} k \text{ x } A &= (k \text{ x } a, k \text{ x } b, k \text{ x } c, k \text{ x } d) && \text{if } k > 0 \\ k \text{ x } A &= (k \text{ x } d, k \text{ x } b, k \text{ x } c, k \text{ x } d) && \text{if } k < 0 \end{aligned} \quad (8)$$

Fuzzy sets can also be represented by intervals, which are called α-level sets or α-cuts. The α-level sets A_α of a fuzzy set $\tilde{A}$ are defined as [15]

$$A_\alpha = \{x \in X | \mu_{\tilde{A}}(x) \geq \alpha\} = [\min\{x \in X | \mu_{\tilde{A}}(x) \geq \alpha\}, \max\{x \in X | \mu_{\tilde{A}}(x) \geq \alpha\}] \quad (9)$$

According to Zadeh's extension principle [10], the fuzzy set Ã can be expressed as

$$\tilde{A} = \mathrm{U}_\alpha \alpha A_\alpha, \quad 0 < \alpha \leq 1 \quad (10)$$

where U denotes the standard fuzzy union and α^{α}_{A} denotes the special fuzzy set which membership function is defined as:

$$\mu_{\alpha\_}A_{\alpha} = \begin{cases} \alpha & \text{for } x \in A^{\alpha} \\ 0 & \text{for } x \notin A^{\alpha} \end{cases} \quad (11)$$

Therefore, the multiplication and division operations of any two positive fuzzy numbers A and B, which α-cuts are denoted as $A^{\alpha} = |A^{\alpha}_{1},\ A^{\alpha}_{r}|$ and $B^{\alpha} = |\ B^{\alpha}_{1},\ B^{\alpha}_{r}|$ respectively, can be expressed as:

Fuzzy multiplication

$$A \otimes B = \bigcup_{\alpha \in [0,1]} \alpha\,(A \times B)^{\alpha}(x) \quad (12)$$

Fuzzy division

$$A \oslash B = \bigcup_{\alpha \in [0,1]} \alpha\_(A / B)^{\alpha}(x) \quad (13)$$

IV. Methodology

The aim of this paper is to make a comparison between two Fuzzy risk assessment methods introduced by Nieto-Morote and Ruz-Vila [1] and Kuo and Lu [2] using the same values for input parameters which are risk probability (RP), risk impact (RI) and risk discrimination (RD) to determine if these methods give the same risk ranking or not.

The risk impact parameter investigates the potential effect of the risk on a project objective such as schedule, cost, quality or performance. The risk probability parameter investigates the likelihood that each specific risk will occur. While the risk discrimination investigates the impact of the risk to the overall framework of the project.

A. *Steps applied in both Nieto-Morote and Ruz-Vila* [1] *and Kuo and Lu* [2] *methods*

1) Identify the risk sources and construct hierarchical structure of risks: *In this paper, risk sources are divided into five groups (dimensions), each group consists of four risks (factors) as in Fig. 1.*

2) Measure of RI and RP parameters and aggregate individual fuzzy numbers into group fuzzy number: *The measurement of RI and RP for each risk in both methods, involves the following steps.*

a) linguistic measure of RI and RP by using the linguistic terms shown in Table 1.

b) Convert these measures into fuzzy numbers using the score system shown in Table I.

c) Aggregate the fuzzy numbers into a group fuzzy number by applying the fuzzy arithmetic average as shown Equations (14) and (15),respectively. Table II shows aggregated fuzzy values of RI and RP parameters for group (D1).

$$RI_{i} = \frac{1}{m} \times \sum_{n=1}^{m} RI_{i}^{n} = \frac{1}{m} \times (RI_{i}^{1} \oplus RI_{i}^{2} \oplus \cdots \oplus RI_{i}^{m}) \quad (14)$$

$$RP_{i} = \frac{1}{m} \times \sum_{n=1}^{m} RP_{i}^{n} = \frac{1}{m} \times (RP_{i}^{1} \oplus RP_{i}^{2} \oplus \cdots \oplus RP_{i}^{m}) \quad (15)$$

where i is each one of the risks at the bottom level of the hierarchy, m is the number of experts providing RI and RP values, x is the scalar multiplication defined in Equation (8) and $\oplus$ is the fuzzy addition defined in Equation (4).

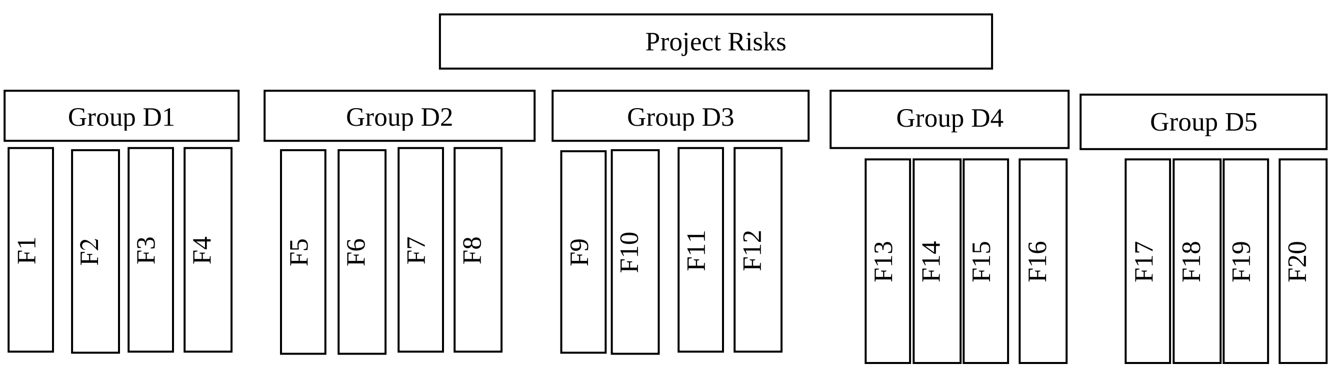

Figure 1. Generic hierarchical structure of risks.

TABLE I. DESCRIPTIONS OF RI, RP AND RD COMPARISON

Description of RI	General Interpretation	Fuzzy Number
Critical (C)	Involved very highly impact	(0.8, 0.9, 1, 1)
Serious (S)	Involved highly impact	(0.6, 0.75, 0.75, 0.9)
Moderate (Mo)	Involved moderate impact	(0.3, 0.5, 0.5, 0.7)
Minor (Mi)	Involved only small impact	(0.1, 0.25, 0.25, 0.4)
Negligible (N)	Involved no substantive impact	(0, 0, 0.1, 0.2)
Description of RP	**General interpretation**	**Fuzzy number**
High (H)	Very likely to occur	(0.7, 0.9, 1, 1)
Medium (M)	Likely to occur	(0.2, 0.5, 0.5, 0.8)
Low (L)	Occurrence is unlikely	(0, 0, 0.1, 0.2)
Description of RDC	**General interpretation**	**Fuzzy number**
Much more	Much more impact on overall framework of project than	(0, 0, 0, 0.3)
More	More impact on overall framework of project than	(0, 0.25, 0.25, 0.5)
Same	Same impact on overall framework of project than	(0.3, 0.5, 0.5, 0.7)
Less	Less impact on overall framework of project than	(0.5, 0.75, 0.75, 1)
Much less	Much less impact on overall framework of project than	(0.7, 1, 1, 1)

3) Measure of RDC parameter: Compare risk factors pair-wise and aggregate individual fuzzy numbers into group fuzzy number.

RDC which is the comparative judgment on the impact on overall framework of the project is provided for every risk pair-wise in each group defined in the hierarchy using the linguistic terms shown in Table I. Each one of the comparative judgments provided, is converted into its corresponding fuzzy number using the score system shown in Table I. And then aggregate the fuzzy numbers into a group fuzzy number by applying the fuzzy arithmetic average as shown in Equation (16). Table III shows aggregated fuzzy numbers of RDC pair wise comparison for group (D1).

$$RDC_{ij} = \frac{1}{m} \text{ x } \sum_{n=1}^{m} RDC^{n}_{ij}$$

$$= \frac{1}{m} \text{ x } (RDC^{1}_{ij} \oplus RDC^{2}_{ij} \oplus \ldots \oplus RDC^{m}_{ij}) \quad (16)$$

where i and j are the risks of the group g and the level l in the hierarchy and m is the number of experts providing RDC, x is the scalar multiplication defined in Equation (8) and $\oplus$ is the fuzzy addition defined in Equation (4).

TABLE II. AGGREGATED FUZZY VALUES OF RI AND RP PARAMETERS FOR GROUP (D1)

Risks	Measure of RI	Measure of RP
F1	(0.25, 0.375, 0.4, 0.5)	(0.1, 0.25, 0.3, 0.5)
F2	(0.175, 0.3125, 0.3375, 0.5)	(0.275, 0.475, 0.525, 0.7)
F3	(0.45, 0.625, 0.625, 0.8)	(0.15, 0.375, 0.4, 0.65)
F4	(0.325, 0.5, 0.5, 0.675)	(0.575, 0.8, 0.875, 0.95)

B. *Steps Applied in Nieto-Morote and Ruz-Vila [2] method*

1) Calculate RD* Value

$RD^{*}i$ is the value of aggregated risk discrimination for each risk which is corresponding to the minimum error between the aggregated input RDCij calculated from Eq.(16) and the consistent RDC'_{ij} calculated from Equation(18). The minimum error between the input RDC and consistent RDC is calculated using Equation(17). The values of RD^{*}_{i} corresponding to the risks of all the groups and levels in the hierarchy are shown in Table IV.

$$\min \left[\sum_{i=1}^{m} \sum_{\substack{j=1 \\ i=1}}^{m} (RDC'_{ij} \ominus RDC_{ij})^2 \right] \quad (17)$$

$$RDC'_{ij} = \frac{RD^{*}_{i} \oplus (1 \ominus RD^{*}_{j})}{2} \quad (18)$$

where i and j are risks of the group g and the level l in the hierarchy and $\oplus$ and $\ominus$ represents the fuzzy addition and subtraction using Equations (4) and (5).

2) Aggregate RD* in Hierarchy

Assume the risk r_i has t upper groups at different level in the risk structure hierarchy and $RD^{*(j)}_{group}$ is the value RD* of the j[th] upper group which contain the risk r_i in the hierarchy. The final value of RD for each risk r_i can be calculated using Equation (19). The obtained values are shown in Table IV.

$$RD_i = RD^{*}_{i} \otimes \prod_{j=1}^{i} (RD^{*})^{(j)}_{group} \quad (19)$$

where i is each one of the risks at the bottom level of the hierarchy and $\otimes$ represent the fuzzy multiplication using arithmetic operations on their a-cuts in Equation (12).

TABLE III. RDC COMPARISON AGGREGATED FUZZY NUMBERS OF GROUP (D1)

D1	F1	F2	F3	F4
F1		(0, 0.125, 0.125, 0.4)	(0.15, 0.375, 0.375, 0.6)	(0, 0.25, 0.25, 0.5)
F2	(0.6, 0.875, 0.875, 1)		(0.5, 0.75, 0.75,1)	(0.4, 0.625, 0.625, 0.85)
F3	(0.4, 0.625, 0.625, 0.85)	(0, 0.25, 0.25, 0.5)		(0.15, 0.375, 0.375, 0.6)
F4	(0.5, 0.75, 0.75,1)	(0.15, 0.375, 0.375, 0.6)	(0.4, 0.625, 0.625, 0.85)	

3) **Fuzzy inference step**

a) Calculate ORF

Once all parameters RI, RP and RD are valued in form of fuzzy numbers, the overall risk factor of each risk at the bottom level of the hierarchy is calculated using Equation (20). The obtained values are shown in Table IV.

$$ORF_i = (RI_i \otimes RP_i) \oslash RD_i \qquad (20)$$

where i is each one of the risks at the bottom level of the hierarchy and $\otimes$ and Ø represent the fuzzy multiplication and the fuzzy division using arithmetic operations on their α-cuts in Equations (12) and (13), respectively.

b) Defuzzification

The last step is to convert the fuzzy output ORF of each risk at the bottom level of the hierarchy into a numerical value $ORF_ɤ$ by using the defuzzification centroid method as defined in Equation (21). The obtained values of $ORF_ɤ$ and risk ranking are shown in Table IV.

TABLE IV. VALUES OF RD*, RD, ORF, ORFɤ AND RISK RANK

Risk	RD*	RD	ORF	ORFɤ	Rank
D1	(0, 0.175, 0.175, 0.375)				
F1	(0.015, 0.015, 0.015, 0.015)	(0, 0.0027, 0.0027, 0.0058)	(4.3275, 34.7746, 44.5115, 16228296)	5409444.98	3
F2	(0.36, 0.36, 0.36, 0.36)	(0, 0.063, 0.063, 0.135)	(0.3565, 2.3565, 2.8129, 972359)	324120.5955	11
F3	(0.298, 0.298, 0.298, 0.298)	(0, 0.0521, 0.0521, 0.1117)	(0.6044, 4.4973, 4.7971, 1746174)	582059.8186	9
F4	(0.33, 0.33, 0.33, 0.33	(0, 0.0578, 0.0578, 0.1238)	(1.5097, 6.9246, 7.5738, 1942697)	647568.3544	8
D2	(0, 0.175, 0.175, 0.375)				
F5	(0, 0.175, 0.175, 0.375)	(0, 0.0306, 0.0306, 0.1406)	(0.1778, 3.0612,3.9183,25000161463)	83333871546.1	2
F6	(0, 0.175, 0.175, 0.375)	(0, 0.0306, 0.0306, 0.1406)	(0.3422, 4.8469,5.7856,35000226048)	116667420164.7	1
F7	(0.175, 0.375, 0.375, 0.675)	(0, 0.0656, 0.0656, 0.2531)	(0.2667, 3.5714, 3.8095, 2971421)	990475.0085	4
F8	(0.3, 0.3, 0.3, 0.3)	(0, 0.0525, 0.0525, 0.1125	(1.6611, 7.619, 8.3333, 2137500)	712503.0192	6
D3	(0, 0.175, 0.175, 0.375)				
F9	(0, 0.122, 0.122, 0.141)	(0, 0.0214, 0.0214, 0.0527)	(0.4743, 4.3763, 5.6016, 2042279)	680761.1566	7
F10	(0.554, 0.554, 0.554, 0.554)	(0, 0.0969, 0.0969, 0.2076)	(0.2318, 1.5322, 1.829, 632254)	210752.1118	12
F11	(0.299, 0.299, 0.299, 0.311)	(0, 0.0523, 0.0523, 0.1166)	(0.5791, 4.4797, 4.7784, 1739351)	579785.3646	10
F12	(0.299, 0.299, 0.299, 0.311)	(0, 0.0523, 0.0523, 0.1166)	(1.6032, 7.6454, 8.3622, 2144920)	714976.7054	5
D4	(0.175, 0.425, 0.425, 0.675)				
F13	(0.405, 0.405, 0.405, 0.405)	(0.0709, 0.1722, 0.1722, 0.2734)	(0.0914, 0.5445, 0.697, 3.5)	1.396	19
F14	(0.405, 0.405, 0.405, 0.405)	(0.0709, 0.1722, 0.1722, 0.2734)	(0.176, 0.8621, 1.0291, 4.937)	2.0013	18
F15	(0.273, 0.358, 0.358, 0.358)	(0.0477, 0.1523, 0.1523, 0.2419)	(0.2791, 1.5389, 1.6415, 10.9019)	4.244	17
F16	(0.273, 0.358, 0.358, 0.358)	(0.0477, 0.1523, 0.1523, 0.2419)	(0.7726, 2.6264, 2.8727, 13.4439)	5.6276	15
D5	(0.3, 0.3, 0.3, 0.3)				
F17	(0.023, 0.023, 0.023, 0.023)	(0.0069, 0.0069, 0.0069, 0.0069)	(3.6013, 13.5047, 17.2861, 36)	18.1828	14
F18	(0.491, 0.597, 0.597, 0.597)	(0.1473, 0.1791, 0.1791, 0.1791)	(0.2687, 0.8287, 0.9892, 2.376)	1.1747	20
F19	(0.029, 0.11, 0.11, 0.11)	(0.0086, 0.0329, 0.0329, 0.0329)	(2.0536, 7.1306, 7.6059, 60.6773)	23.3020	13
F20	(0.279, 0.36, 0.36, 0.36)	(0.0836, 0.1079, 0.1079, 01079)	(1.7324, 3.7082, 4.0558, 7.6732)	4.4141	16

$$(ORF_ɤ)_i = \frac{\int_0^1 x ORF_i(x) d(x)}{\int_0^1 ORF_i(x) d(x)} \qquad (21)$$

where i is each one of the risks at the bottom level of the hierarchy. *Fig. (2)* shows the risk ranking starting from the most important (F6) to the lowest one (F18).

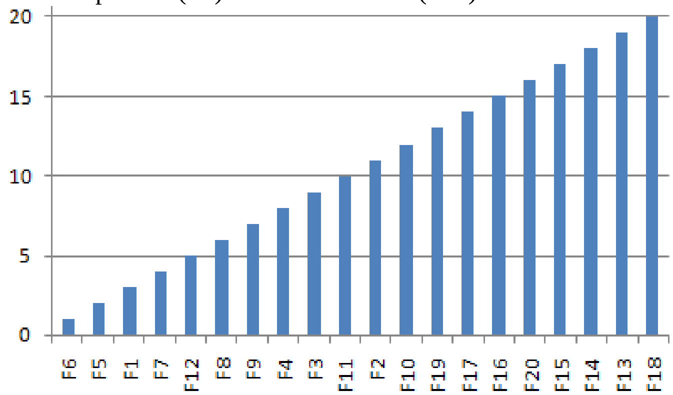

Figure 2. Risk ranking in Nieto-Morote and Ruz-Vila [1] method

C. *Steps applied in Kuo and Lu* [3] *method*

1) Calculate RD value

For the elements in each group and level, the values of RD are obtained by converting RDC values obtained from Eq.(16) which are the aggregated trapezoidal fuzzy numbers for pair wise comparison of risks for diagonal elements only into single numerical values using Equation (21). The relative impacts were then further synthesized using Equation (22). The values of RD for group (D1) are shown in Table V.

$$a_{ij} \cdot a_{ji} = 1 \quad \forall i, j \in \{1, \ldots, n\} \tag{22}$$

2) Consistent fuzzy preference relations

Fuzzy preference relations (P_{ij}) is the same as risk discrimination (RD_{ij}) where, p_{ij} is interpreted as the level of preference for risk r_i over r_j Equation (23) is used to obtain the reciprocal fuzzy preference relation $P=[p_{ij}]$ for $p_{ij} \in [0,1]$ associated with matrix A:

$$P_{ij} = g(a_{ij}) = \frac{1}{2} \cdot (1 + \log_9 a_{ij}) \tag{23}$$

The other preference relation values for the matrix were calculated using Equations (24), (25) and (26). Fuzzy preference relation for group (D1) are shown in Table VI.

TABLE V. MATRIX A VALUES OF RD FOR GROUP (D1)

Risk	F1	F2	F3	F4
F1	1	0.175		
F2	5.7143	1	0.75	
F3		1.3333	1	0.375
F4			2.6667	1

TABLE VI. FUZZY PREFERENCE RELATION P_{IJ} FOR GROUP (D1)

Risk	F1	F2	F3	F4
F1	0.5	0.1034	0.0379	-0.1853
F2	0.8966	0.5	0.4345	0.2113
F3	0.9621	0.5655	0.5	0.2768
F4	1.1853	0.7887	0.7232	0.5

$$p_{ij} + p_{ji} = 1 \quad \forall i, j \in \{1, \ldots, n\} \tag{24}$$

$$p_{ij} + p_{jk} + p_{ki} = \frac{3}{2}, \quad \forall i<j<k \tag{25}$$

$$p_{i(i+1)} + p_{(i+1)(i+2)} + + p_{(j-1)j} + p_{ji} = \frac{j - i + 1}{2}, \quad \forall i<j \tag{26}$$

3) Determining the Priority of Risk Factors:

The matrix was adjusted using Equation (27) when there are preference relations values outside of [0,1] so as to retain reciprocity and additive consistency in the decision matrix.

$$f : [-a, 1 + a] \longrightarrow [0, 1], \; f(x) = (x + a) / (1+2a) \tag{27}$$

This method is utilized to assess the relative impacts on project performance of the risk factors. The obtained assessment decision matrix, $P' = (p'_{ij})$, shows the consistent reciprocal relation. The values of P' matrix for group (D1) are shown in Table VII.

Equations (28) and (29) can now be applied to determine the multiplicative preference relations matrix associated with the relative impacts of risk factors on project performance. The values of matrix A' for group (D1) are shown in Table VIII.

$$a'_{ij} = 9^{(2xp'_{ij} - 1)} \tag{28}$$

$$\boldsymbol{A}' = [a'_{ij}] \tag{29}$$

4) Determining Relative Impact on Project Performance

The last step to explore the relative impact of risk groups and factors on project performance is to investigate the set of eigenvalues λ for pair-wise comparison matrix A′ for weighting solutions. The reflected eigenvector W of the maximum eigenvalue λ max shown in Equation (30) is the set of relative impacts of investigated risk dimensions and factors. The final assessment results can be used to determine the priority of the relative impacts of identified risk dimensions and factors on construction project performance. The values of W are shown in Table IX.

$$\boldsymbol{A}' \times \boldsymbol{W} = \lambda_{max} \times \boldsymbol{W}, \quad \boldsymbol{W} = (w_1, w_2, w_3, \ldots, w_n)^T \tag{30}$$

TABLE VII. P' MATIX FOR GROUP (D1)

Risk	F1	F2	F3	F4
F1	0.5	0.2106	0.1628	0
F2	0.7894	0.5	0.4522	0.2894
F3	0.8372	0.5478	0.5	0.3372
F4	1	0.7106	0.6628	0.5

TABLE VIII. ***A***' MATIX FOR GROUP (D1)

Risk	F1	F2	F3	F4
F1	1	0.2804	0.2773	0.111
F2	3.5669	1	0.8107	0.3963
F3	4.4	1.2336	1	0.4889
F4	9	2.5232	2.0455	1

Risk	W	W(local)	W(overall)	E	R	Rank
D1	0.089	0.048				
F1	0.094	0.056	0.003	0.288	0.0008	20
F2	0.334	0.198	0.009	0.494	0.0047	15
F3	0.412	0.245	0.012	0.394	0.0046	16
F4	0.843	0.501	0.024	0.800	0.0191	9
D2	0.233	0.125				
F5	0.094	0.056	0.007	0.288	0.002	19
F6	0.307	0.184	0.023	0.494	0.0113	13
F7	0.426	0.255	0.032	0.394	0.0125	12
F8	0.846	0.506	0.063	0.800	0.0505	4
D3	0.273	0.146				
F9	0.094	0.056	0.008	0.288	0.0024	18
F10	0.307	0.184	0.027	0.494	0.0133	11
F11	0.426	0.255	0.037	0.394	0.0147	10
F12	0.846	0.506	0.074	0.800	0.0592	3
D4	0.468	0.251				
F13	0.096	0.058	0.015	0.288	0.0042	17
F14	0.303	0.184	0.046	0.494	0.0227	7
F15	0.385	0.233	0.059	0.394	0.023	8
F16	0.866	0.525	0.132	0.800	0.1053	2
D5	0.803	0.43				
F17	0.094	0.056	0.024	0.288	0.0069	14
F18	0.334	0.198	0.085	0.494	0.0422	5
F19	0.412	0.245	0.105	0.394	0.0415	6
F20	0.843	0.501	0.216	0.800	0.1724	1

TABLE IX. VALUES OF W, E,R AND RANK

5) Determining the Probability of Occurrence of Risk Factors

A defuzzification method is used to transform the trapezoidal fuzzy values of risk probability (RP) obtained from Equation (15) into their optimal non fuzzy assessed crisp values, using α-cut approach as in Equation (31). The values of probability (E) are shown in Table IX.

$$LE^{\alpha} = (ME_i - LE_i) \times \alpha + LE_i$$
$$UE^{\alpha}_i = UE_i - (UE_i - ME_i) \times \qquad (31)$$
$$(E^{\alpha}_i)^{\lambda} = \lambda \times LE^{\alpha}_i + (1-\lambda) \times UE^{\alpha}_i, \; 0 \le \lambda \le 1, \; 0 \le \alpha \le 1$$

6) Determine Project Risk

The effects and occurrence probability of risk factors should be integrated to evaluate the overall project risk. This integrated computation is presented as in Equation (32). The values of overall project risk (R) and risk ranking are shown in Table IX.

$$R = E \times W \qquad (32)$$

V. Discussion

The comparison between the two Fuzzy risk assessment methods introduced by Nieto-Morote and Ruz-Vila [1] and Kuo and Lu [2] using the same values for input parameters which are risk probability (RP), risk impact (RI) and risk discrimination (RD) leads to different results. It is observed from *Fig. (4)* that risk ranking in both methods is different. For example, the risk (F6) which has the highest ranking in Nieto-Morote and Ruz-Vila [1] method, ranks thirteen in Kuo and Lu [2] method, and the lowest ranking risk in Nieto-Morote and Ruz-Vila [1] method which is (F18), ranks five in Kuo and Lu [2] method. While the highest ranking risk in the Kuo and Lu [2] method which is (F20), ranks sixteen in the Nieto-Morote and Ruz-Vila [1] method and the lowest ranking risk in the Kuo and Lu [2] method which is (F1), ranks three in the Nieto-Morote and Ruz-Vila [1] method. And this returns to the fact that the Nieto-Morote and Ruz-Vila [1] method depends on minimizing the error; minimization of the difference between the input values and the ideal consistent values in order to reduce the inconsistency of input values and this means that the results obtained are not the optimum results but they are around them because in each trial of minimizing error, different results are obtained. While Kuo and Lu [2] method depends on eliminating the inconsistency in input values actually before going to the next step by transforming the matrices which lie between [-a , 1+a] to be within [0,1]. This transformation process is done to remain the decision matrix with reciprocity and additive consistency because this matrix is a Fuzzy preference relations; a fuzzy set with a membership function lies between [0,1] and any values outside [0,1] are inconsistence. So the results obtained from this method are the optimum results according to this methodology.

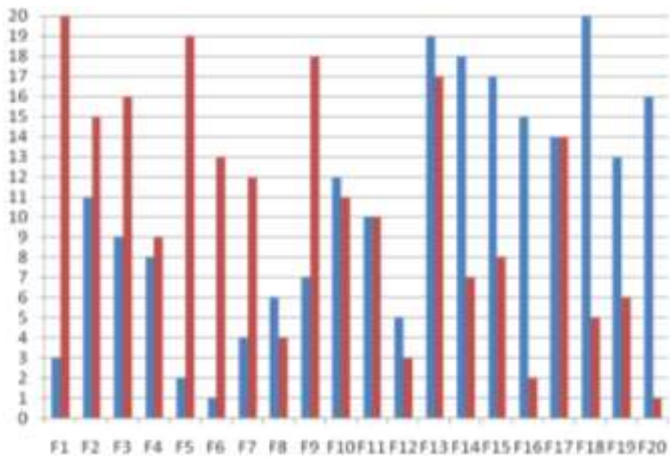

Figure 3. Comparison between risk ranking in both Nieto-Morote and Ruz-Vila method and Kuo and Lu method (dark = Kuo and Lu, light = Nieto-Morote)

It can be also observed that the Nieto-Morote and Ruz-Vila [1] method has the advantages of depending on three parameters which are risk impact (RI), risk probability (RP) and risk discrimination (RD), so the data gathered are more than the Kuo and Lu [2].

VI. Conclusion

Project risk management, has been recognised critical for the construction industry to improve their performance and secure the success of projects. Risk assessment is the most important step in risk management, as if the risk is not assessed, it is difficult to further analyze and respond to this risk. Classical methods for risk assessment are not so accurate and effective. Therefore, new methods such as fuzzy risk assessment methods are need to give more accurate risk ranking. Using fuzzy logic in risk assessment has been introduced in many papers with different methodologies. In this paper, a comparison between two fuzzy risk assessment methods; Nieto-Morote and Ruz-Vila [1] and Kuo and Lu [2] have been done in order to determine if they give the same risk ranking or not, using the same input parameters which are risk impact (RI), risk probability (RP) and risk discrimination (RD). Actually, the comparison results in different risk ranking, because Nieto-Morote and Ruz-Vila [1] method depends on minimization error tool between input values and consistent values, and this tool always doesn't give optimum results, while, we can consider Kuo and Lu [2] method more accurate because it depends on eliminating the inconsistency in results using a transformation process step to remain the decision matrix with reciprocity and additive consistency. However, Nieto-Morote and Ruz-Vila [1] method depends on one more risk parameter than Kuo and Lu [2] method which is risk impact (RI), but Kuo and Lu [2] can be considered more applicable in developing countries.

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