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An Electroelastic Problem of Green Materials Subjected to Surface Friction

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***Abstract*—Aiming at the development of nondestructive evaluation techniques for green materials, we investitgate electroelastic field in a body with D_∞ symmetry subjected to a partially distributed friction across its ∞-fold rotation axis on its surface. By using a potential function method derived by us, the electroelastic field inside the body is formulated. Furthermore, numerical calculation is performed to investigate the field qualitatively and quantitatively.**

***Keywords*—green material, electroelasticity, D_∞ symmetry, theoretical analysis**

I. Introduction

Because of demands to reduce environmental loads, wooden materials and biodegradable polymers have attracted considerable attention as green materials. To ensure their quality, nondestructive evaluation (NDE) techniques need to be developed. Because some of these materials exhibit piezoelectricity [1–3], piezoelectric signals due to mechanical disturbances are one of the candidates for NDE output.

From a mesoscopic viewpoint, wooden materials and poly-L-lactic acid (PLLA), one of the biodegradable polymers, are considered to have the D_∞ symmetry [2, 3], which is characterized by an "∞-fold rotation axis" and a "two-fold rotation axis" perpendicular to it. One of the most striking characteristics of a body with D_∞ symmetry is the coupling between the shear motion in the plane parallel to the ∞-fold rotation axis and the electric poling perpendicular to the plane.

Considering the importance of electroelastic problems in such materials, we developed an analytical technique to obtaining general solutions to electroelastic problems [4], and investigated the response of "elastic quantities", such as stress and strain, resulting from "electric stimulus" by an electric potential on its surface. In order to develop NDE techniques, however, the response of electric quantities due to elastic stimuli need to be elucidated.

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In this paper, therefore, we investigate the electroelastic field in a body with D_∞ symmetry subjected to a partilly distributed friction across its ∞-fold rotation axis on its surface. A semi-infinite body is chosen as an analytical model because it is the simplest shape and enables us to focus attention on the response of electric quantities due to elastic stimuli excluding the effects of other complicated factors. By using the technique mentioned above, we obtain the theoretical solutions of the electroelastic field quantities. Moreover, we perform numerical calculations to investigate the effects of the elastic stimulus modeled by the surface friction on the electric quantities.

II. Theoretical Analysis

A. *Problem*

We consider a semi-infinite piezoelectric body $(x>0)$ with D_∞ symmetry, as shown in Fig. 1, where the z-axis is parallel to the ∞-fold rotation axis of the body. The surface of the body is subjected to surface friction $\tau_0 g(y,z)$ in y-direction, where

$$g(y,z)=\frac{z}{\delta}\exp\left[-\left(\frac{y}{\delta}\right)^2\right]\cdot\exp\left[-\left(\frac{z}{\delta}\right)^2\right]. \qquad (1)$$

The surface $x=0$ is chosen as the reference plane of electric potential. The boundary conditions are then described as

$$x=0:\ \sigma_{xy}=-\tau_0 g(y,z),\ \sigma_{xx}=0,\ \sigma_{zx}=0,\ \Phi=0. \qquad (2)$$

The displacements and electric potential are assumed to vanish at infinity.

B. *Fundamental Equations*

Referring to our previous paper [4], the components of electric displacement and stress are described by elastic displacement potential functions φ_i, piezoelastic displacement potential functions ϑ_i, and electric potential function Φ, respectively, as follows:

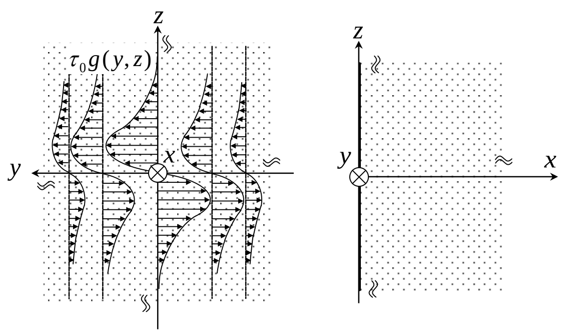

Figure 1. Analytical model

$$\left.\begin{aligned}
\sigma_{xx} &= \sum_{i=1}^{2}\left[\begin{aligned}&\left(c_{11}\frac{\partial^2\varphi_i}{\partial x^2}+c_{12}\frac{\partial^2\varphi_i}{\partial y^2}+k_i c_{13}\frac{\partial^2\varphi_i}{\partial z^2}\right)\\ &+\left(c_{11}-c_{12}\right)\frac{\partial^2\vartheta_i}{\partial x\partial y}\end{aligned}\right],\\
\sigma_{xy} &= \frac{c_{11}-c_{12}}{2}\sum_{i=1}^{2}\left[2\frac{\partial^2\varphi_i}{\partial x\partial y}+\left(-\frac{\partial^2\vartheta_i}{\partial x^2}+\frac{\partial^2\vartheta_i}{\partial y^2}\right)\right],\\
\sigma_{yz} &= c_{44}\sum_{i=1}^{2}\left[\left(1+k_i\right)\frac{\partial^2\varphi_i}{\partial y\partial z}-\frac{\partial^2\vartheta_i}{\partial z\partial x}\right]+e_{14}\frac{\partial\Phi}{\partial x},\\
\sigma_{zx} &= c_{44}\sum_{i=1}^{2}\left[\left(1+k_i\right)\frac{\partial^2\varphi_i}{\partial z\partial x}+\frac{\partial^2\vartheta_i}{\partial y\partial z}\right]-e_{14}\frac{\partial\Phi}{\partial y},\\
D_x &= e_{14}\sum_{i=1}^{2}\left[\left(1+k_i\right)\frac{\partial^2\varphi_i}{\partial y\partial z}-\frac{\partial^2\vartheta_i}{\partial z\partial x}\right]-\eta_{11}\frac{\partial\Phi}{\partial x},\\
D_y &= -e_{14}\sum_{i=1}^{2}\left[\left(1+k_i\right)\frac{\partial^2\varphi_i}{\partial z\partial x}+\frac{\partial^2\vartheta_i}{\partial y\partial z}\right]-\eta_{11}\frac{\partial\Phi}{\partial y}
\end{aligned}\right\}. \quad (3)$$

φ_i, ϑ_i, and Φ are governed by

$$\left.\begin{aligned}
&\left(\Delta_{\mathrm{p}}+\mu_i\frac{\partial^2}{\partial z^2}\right)\varphi_i=0,\ \left(\Delta_{\mathrm{p}}+\upsilon_i\frac{\partial^2}{\partial z^2}\right)\vartheta_i=0,\\
&\frac{e_{14}}{c_{44}}\mu_3\frac{\partial\Phi}{\partial z}=\sum_{i=1}^{2}\left(\Delta_{\mathrm{p}}+\mu_3\frac{\partial^2}{\partial z^2}\right)\vartheta_i
\end{aligned}\right\}, \quad (4)$$

where μ_1, μ_2, υ_1, and υ_2 are the roots of quadratic equations with respect to μ and υ

$$\left.\begin{aligned}
&c_{11}c_{44}\mu^2-\left(c_{11}c_{33}-c_{13}^2-2c_{13}c_{44}\right)\mu+c_{33}c_{44}=0,\\
&\upsilon^2-\left[\mu_3\left(1+k_{\mathrm{couple}}^2\right)+\eta\right]\upsilon+\mu_3\eta=0
\end{aligned}\right\}, \quad (5)$$

$$\left.\begin{aligned}
&\mu_3=\frac{2c_{44}}{c_{11}-c_{12}},\ \eta=\frac{\eta_{33}}{\eta_{11}},\ k_{\mathrm{couple}}^2=\frac{e_{14}^2}{c_{44}\eta_{11}},\\
&k_i=\frac{c_{11}\mu_i-c_{44}}{c_{13}+c_{44}}=\frac{\left(c_{13}+c_{44}\right)\mu_i}{c_{33}-c_{44}\mu_i},\ \Delta_{\mathrm{p}}=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}
\end{aligned}\right\}, \quad (6)$$

and c_{ij}, η_{kl}, and e_{kj} denote the elastic stiffness constant, dielectric constant, and piezoelectric constant, respectively.

C. *Formulation of Electroelastic Field Quantities*

By considering the symmetric and anti-symmetric properties, as found in (1) and Fig. 1, and the finiteness of the electroelastic field and applying the Fourier transform technique [5], the solutions for φ_i and ϑ_i are obtained as

$$\left.\begin{aligned}
&\varphi_i=\int_0^\infty\int_0^\infty\left[A_i(\alpha,\beta)\exp\left(-\gamma_{\mu i}x\right)\sin(\alpha y)\sin(\beta z)\right]\mathrm{d}\alpha\mathrm{d}\beta,\\
&\vartheta_i=\int_0^\infty\int_0^\infty\left[C_i(\alpha,\beta)\exp\left(-\gamma_{\upsilon i}x\right)\cos(\alpha y)\sin(\beta z)\right]\mathrm{d}\alpha\mathrm{d}\beta\\
&\left(\gamma_{\mu i}=\left(\alpha^2+\mu_i\beta^2\right)^{1/2},\ \gamma_{\upsilon i}=\left(\alpha^2+\upsilon_i\beta^2\right)^{1/2}\right)
\end{aligned}\right\}, \quad (7)$$

which in turn give

$$\Phi=\frac{c_{44}}{e_{14}}\frac{1}{\mu_3}\sum_{i=1}^{2}\left\{\int_0^\infty\int_0^\infty\left[\begin{aligned}&\left(\mu_3-\upsilon_i\right)\beta C_i(\alpha,\beta)\\&\cdot\exp\left(-\gamma_{\upsilon i}x\right)\\&\cdot\cos(\alpha y)\cos(\beta z)\end{aligned}\right]\mathrm{d}\alpha\mathrm{d}\beta\right\} \quad (8)$$

from the third equation in (4).

By substituting (7) and (8) into (3), the electroelastic field quantities are obtained as

$$\left.\begin{aligned}
\sigma_{xx}&=\frac{c_{44}}{\mu_3}\sum_{i=1}^{2}\int_0^\infty\int_0^\infty\left\{\begin{aligned}&\left[\begin{aligned}&\left(2\alpha^2+\left(1+k_i\right)\mu_3\beta^2\right)\\&\cdot A_i(\alpha,\beta)\exp\left(-\gamma_{\mu i}x\right)\\&+2\gamma_{\upsilon i}\alpha C_i(\alpha,\beta)\exp\left(-\gamma_{\upsilon i}x\right)\end{aligned}\right]\\&\cdot\sin(\alpha y)\sin(\beta z)\mathrm{d}\alpha\mathrm{d}\beta\end{aligned}\right\},\\
\sigma_{xy}&=\frac{c_{44}}{\mu_3}\sum_{i=1}^{2}\int_0^\infty\int_0^\infty\left\{\begin{aligned}&\left[\begin{aligned}&-2\gamma_{\mu i}\alpha A_i(\alpha,\beta)\exp\left(-\gamma_{\mu i}x\right)\\&-\left(2\alpha^2+\upsilon_i\beta^2\right)\\&\cdot C_i(\alpha,\beta)\exp\left(-\gamma_{\upsilon i}x\right)\end{aligned}\right]\\&\cdot\cos(\alpha y)\sin(\beta z)\mathrm{d}\alpha\mathrm{d}\beta\end{aligned}\right\},\\
\sigma_{yz}&=\frac{c_{44}}{\mu_3}\sum_{i=1}^{2}\int_0^\infty\int_0^\infty\left\{\begin{aligned}&\left[\begin{aligned}&\mu_3\left(1+k_i\right)\alpha\beta\\&\cdot A_i(\alpha,\beta)\exp\left(-\gamma_{\mu i}x\right)\\&+\upsilon_i\gamma_{\upsilon i}\beta C_i(\alpha,\beta)\exp\left(-\gamma_{\upsilon i}x\right)\end{aligned}\right]\\&\cdot\cos(\alpha y)\cos(\beta z)\mathrm{d}\alpha\mathrm{d}\beta\end{aligned}\right\},\\
\sigma_{zx}&=\frac{c_{44}}{\mu_3}\sum_{i=1}^{2}\int_0^\infty\int_0^\infty\left\{\begin{aligned}&\left[\begin{aligned}&-\mu_3\left(1+k_i\right)\gamma_{\mu i}\beta\\&\cdot A_i(\alpha,\beta)\exp\left(-\gamma_{\mu i}x\right)\\&-\upsilon_i\alpha\beta C_i(\alpha,\beta)\exp\left(-\gamma_{\upsilon i}x\right)\end{aligned}\right]\\&\cdot\sin(\alpha y)\cos(\beta z)\mathrm{d}\alpha\mathrm{d}\beta\end{aligned}\right\},\\
D_x&=\frac{c_{44}\eta_{11}}{e_{14}\mu_3}\sum_{i=1}^{2}\int_0^\infty\int_0^\infty\left\{\begin{aligned}&\left[\begin{aligned}&\mu_3k_{\mathrm{couple}}^2\left(1+k_i\right)\alpha\beta\\&\cdot A_i(\alpha,\beta)\exp\left(-\gamma_{\mu i}x\right)\\&+\left(\mu_3\left(1+k_{\mathrm{couple}}^2\right)-\upsilon_i\right)\gamma_{\upsilon i}\beta\\&\cdot C_i(\alpha,\beta)\exp\left(-\gamma_{\upsilon i}x\right)\end{aligned}\right]\\&\cdot\cos(\alpha y)\cos(\beta z)\mathrm{d}\alpha\mathrm{d}\beta\end{aligned}\right\},\\
D_y&=\frac{c_{44}\eta_{11}}{e_{14}\mu_3}\sum_{i=1}^{2}\int_0^\infty\int_0^\infty\left\{\begin{aligned}&\left[\begin{aligned}&\mu_3k_{\mathrm{couple}}^2\left(1+k_i\right)\gamma_{\mu i}\beta\\&\cdot A_i(\alpha,\beta)\exp\left(-\gamma_{\mu i}x\right)\\&+\left(\mu_3\left(1+k_{\mathrm{couple}}^2\right)-\upsilon_i\right)\alpha\beta\\&\cdot C_i(\alpha,\beta)\exp\left(-\gamma_{\upsilon i}x\right)\end{aligned}\right]\\&\cdot\sin(\alpha y)\cos(\beta z)\mathrm{d}\alpha\mathrm{d}\beta\end{aligned}\right\}
\end{aligned}\right\}. \quad (9)$$

By substituting (1) and (9) into (2), a set of simultaneous equations for $A_i(\alpha,\beta)$ and $C_i(\alpha,\beta)$ is obtained as

$$\left.\begin{aligned}
&\sum_{i=1}^{2}\left[\left(2\alpha^2+\left(1+k_i\right)\mu_3\beta^2\right)A_i(\alpha,\beta)+2\gamma_{\upsilon i}\alpha C_i(\alpha,\beta)\right]=0,\\
&\sum_{i=1}^{2}\left[\mu_3\left(1+k_i\right)\gamma_{\mu i}A_i(\alpha,\beta)+\upsilon_i\alpha C_i(\alpha,\beta)\right]=0,\\
&\sum_{i=1}^{2}\left[2\gamma_{\mu i}\alpha A_i(\alpha,\beta)+\left(2\alpha^2+\upsilon_i\beta^2\right)C_i(\alpha,\beta)\right]=\frac{\mu_3\tau_0}{c_{44}}g^*(\alpha,\beta),\\
&\sum_{i=1}^{2}\left(\mu_3-\upsilon_i\right)C_i(\alpha,\beta)=0
\end{aligned}\right\}. \quad \text{... (10)}$$

The solution to (10) is obtained as

$$\begin{Bmatrix} A_1(\alpha,\beta) \\ A_2(\alpha,\beta) \\ C_1(\alpha,\beta) \\ C_2(\alpha,\beta) \end{Bmatrix} = \frac{\tau_0}{c_{44}} g^*(\alpha,\beta) \frac{1}{\Delta(\alpha,\beta)} \begin{Bmatrix} A_1^*(\alpha,\beta) \\ A_2^*(\alpha,\beta) \\ C_1^*(\alpha,\beta) \\ C_2^*(\alpha,\beta) \end{Bmatrix}, \quad (11)$$

where

$$g^*(\alpha,\beta) = \frac{\beta\delta^3}{2\pi} \exp\left[-\frac{(\alpha\delta)^2 + (\beta\delta)^2}{4} \right], \quad (12)$$

$$\left.\begin{array}{l} \Delta(\alpha,\beta) \\ = 2[2\mu_3 - (\upsilon_1 + \upsilon_2)](k_1 - k_2)\alpha^2 \gamma_{\mu 1}\gamma_{\mu 2}(\gamma_{\upsilon 1} - \gamma_{\upsilon 2}) \\ + (\upsilon_1 - \upsilon_2) \left\{ \begin{array}{l} 2(k_1 - k_2)\alpha^2 \\ \cdot [\gamma_{\mu 1}\gamma_{\mu 2}(\gamma_{\upsilon 1} + \gamma_{\upsilon 2}) - \mu_3\beta^2(\gamma_{\mu 1} + \gamma_{\mu 2})] \\ - \mu_3(1 + k_1)(1 + k_2) \\ \cdot (2\alpha^2 + \mu_3\beta^2)\beta^2(\gamma_{\mu 1} - \gamma_{\mu 2}) \\ - 4\alpha^4(k_1\gamma_{\mu 1} - k_2\gamma_{\mu 2}) \end{array} \right\}, \\ A_1^*(\alpha,\beta) \\ = \mu_3\alpha \left\{ \begin{array}{l} (\upsilon_1 - \upsilon_2)[2\alpha^2 + (1 + k_2)\mu_3\beta^2] \\ - 2[\mu_3(\gamma_{\upsilon 1} - \gamma_{\upsilon 2}) + \upsilon_1\gamma_{\upsilon 2} - \upsilon_2\gamma_{\upsilon 1}](1 + k_2)\gamma_{\mu 2} \end{array} \right\}, \\ C_1^*(\alpha,\beta) \\ = -(\mu_3 - \upsilon_2)\mu_3 \left\{ \begin{array}{l} 2\alpha^2[(\gamma_{\mu 1} - \gamma_{\mu 2}) + (k_1\gamma_{\mu 1} - k_2\gamma_{\mu 2})] \\ + \mu_3\beta^2(\gamma_{\mu 1} - \gamma_{\mu 2})(1 + k_1)(1 + k_2) \end{array} \right\} \end{array}\right\}, \quad (13)$$

$A_2(\alpha,\beta)$ and $C_2(\alpha,\beta)$ are obtained by exchanging the subscripts 1 and 2 in $A_1(\alpha,\beta)$ and $C_1(\alpha,\beta)$, respectively. By substituting (11)-(13) into (9), the electroelastic field quantities are formulated.

III. Numerical Calculation

A. *Numerical Specifications*

As a piezoelectric body, we chose Sitka spruce (*Picea sitchensis*), whose material constants were built in our previous paper [4] as

$$\left.\begin{array}{l} c_{11} = 830.84[\text{MPa}], \\ c_{33} = 12.276[\text{GPa}], \\ c_{12} = 294.47[\text{MPa}], \\ c_{13} = 472.07[\text{MPa}], \\ c_{44} = 742.50[\text{MPa}], \\ \eta_{11} = 16.823\times10^{-12}[\text{C}^2\text{N}^{-1}\text{m}^{-2}], \\ \eta_{33} = 22.490\times10^{-12}[\text{C}^2\text{N}^{-1}\text{m}^{-2}], \\ e_{14} = -0.14850\times10^{-3}[\text{Cm}^{-2}] \end{array}\right\}. \quad (14)$$

To show the numerical results, we introduced the following nondimensional quantities:

$$\left.\begin{array}{l} (\hat{x},\hat{y},\hat{z}) = \dfrac{(x,y,z)}{\delta}, (\hat{\sigma}_{yz},\hat{\sigma}_{zx}) = \dfrac{(\sigma_{yz},\sigma_{zx})}{\tau_0}, \\ (\hat{D}_x,\hat{D}_y) = \dfrac{(D_x,D_y)}{\left(\dfrac{|e_{14}|\tau_0}{c_{44}}\right)} \end{array}\right\}. \quad (15)$$

For brevity, we hereafter omit the signs for nondimensional quantities, $\frown$

B. *Distributions of Field Quantities*

Fig. 2 and Fig. 3 show the distributions of stress σ_{yz} and σ_{zx}, respectively, inside the body. From Fig. 2, we see that stress σ_{yz} is maximum at the surface $x=0$ and decreases monotonically toward zero with x to satisfy the finiteness of the field stated in *II. A.* From Fig. 3, we see that stress σ_{zx}, for respective values of y, is zero at the surface $x=0$ to satisfy the boundary condition described by (2), attains the maximum value at a certain x, and decreases toward zero with x. The most important aspect that Fig. 2 and Fig. 3 show is that electroelastic field quantities *inside* the body, which are nearly impossible to obtain experimentally, are obtained by our method.

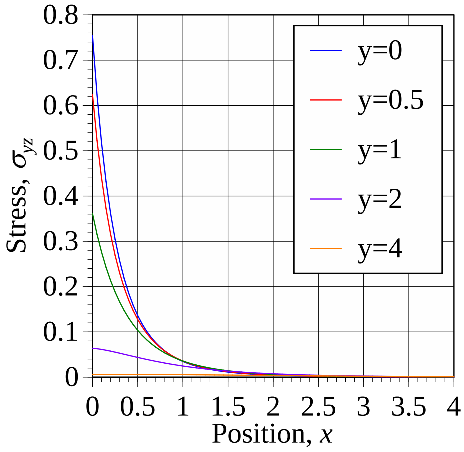

Figure 2. Distribution of stress σ_{yz} inside the body $(z=0)$

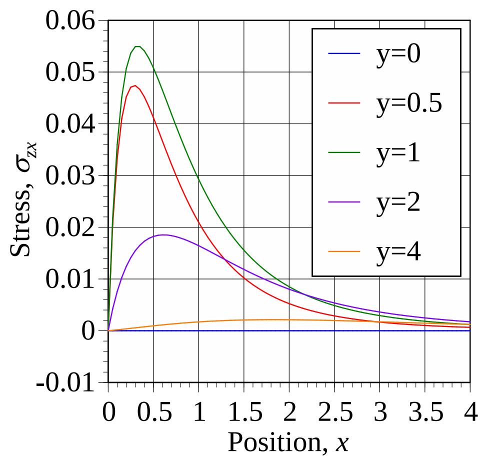

Figure 3. Distribution of stress σ_{zx} inside the body $(z=0)$

Fig. 4 and Fig. 5 show the distributions of electric displacements D_x and D_y, respectively, inside the body. By referring to Fig. 2 and Fig. 3, it is found that the distributions of D_x and D_y are similar to those of σ_{yz} and σ_{zx}, respectively. These similarities reflect the coupling behavior of a body with D_∞ symmetry: the body exhibits the coupling between the shear motion in the plane parallel to the ∞-fold rotation axis and the electric poling perpendicular to the plane. Fig. 4 and Fig. 5 also show the important aspect mentioned with regard to Fig. 2 and Fig. 3 because both figures elucidate experimentally-unmeasurable quantities.

At the same time, experimentally-measurable responses to disturbances need to be investigated from the viewpoint of NDE techniques. Fig. 6 shows the distribution of stress σ_{yz} on the surface $x = 0$, which develops due to the application of the surface friction as such a disturbance. Then, the electric displacement D_x on the surface $x = 0$, as an experimentally-measurable response, develops as shown in Fig. 7. From Fig. 7, it is found that the distribution is roughly proportional to that of stress σ_{yz} shown in Fig. 6, which suggestes the possibility of an NDE technique using piezoelectric signals.

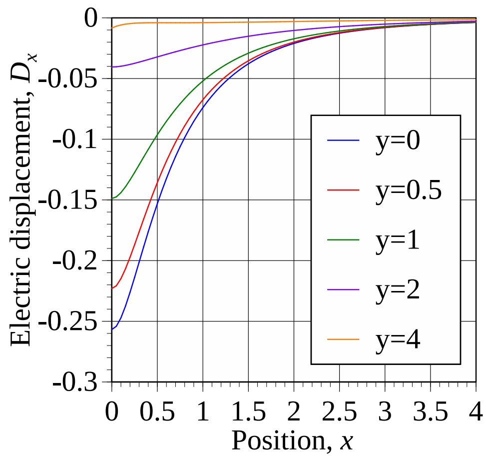

Figure 4. Distribution of electric displacement D_x inside the body $(z = 0)$

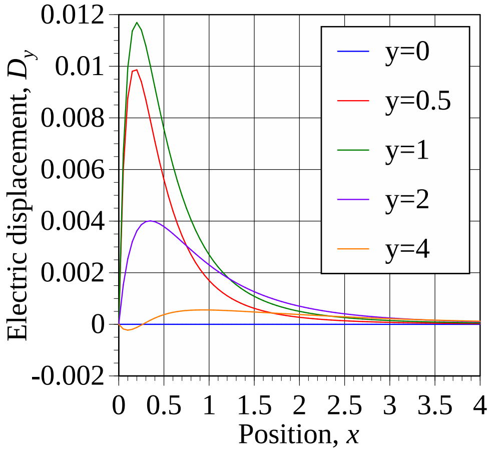

Figure 5. Distribution of electric displacement D_y inside the body $(z = 0)$

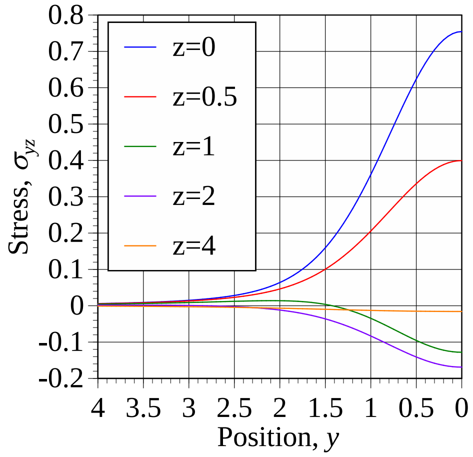

Figure 6. Distribution of stress σ_{yz} on the surface $x = 0$

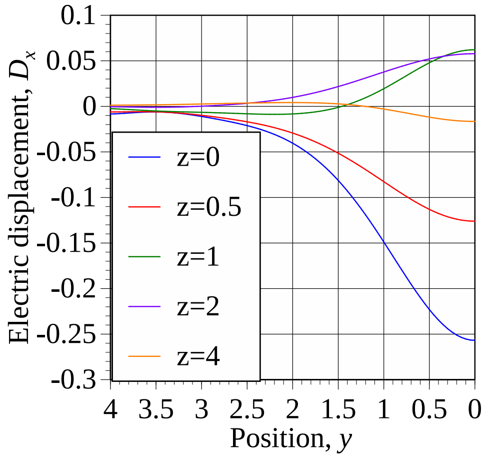

Figure 7. Distribution of electric displacement D_x on the surface $x = 0$

IV. Concluding Remarks

We studied the electroelastic field in a semi-infinite body with D_∞ symmetry subjected to surface friction across its ∞-fold rotation axis on the surface. We applied the potential function method proposed by us to the body and theoretically formulated electroelastic field quantities. Moreover, by numerical calculations, we elucidated the distributions of electroelastic field quantities and suggested the possibility of an NDE technique using piezoelectric signals.

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