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Calculation of Receptance of a Structure Modified by Mass and Grounded Spring

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Abstract— **The dynamic properties of an existing structure are altered, when a mass and/or a spring are added to or removed from the structure. Therefore it is important to know dynamic properties of the structure after modification. One way to determine dynamic properties of the structure is the experimental modal analysis which uses measured Frequency Response Functions (FRFs). In the present study, a general and exact method is presented based on the Sherman-Morrison formula in order to calculate FRF of the structure modified by concentrated mass and grounded spring. The method is very effective and uses only a few FRFs related to modification co-ordinates of the un-modified structure.**

Keywords— **Structural modification, frequency response function, receptance, natural frequency, Sherman-Morrison formula.**

I. Introduction

In the practice, it may be needed to physically modify an existing structure for a design aim. Physical modifications in the form of mass, spring and damping affect the dynamic properties of the structure. In general, the structural modification has been processed in two ways: One is the direct structural modification which deals with how the physical modifications affect the dynamics of structure. Other is the inverse structural modification which deals with the determining necessary modifications to satisfy desired dynamic properties of the existing structure. The developed methods for both direct and inverse structural modifications are based on the use of modal properties which derived from finite elements (FE) solution or experimental modal analysis (EMA) e.g. [1-3] or the use of Frequency Response Functions (FRFs) directly e.g. [4-13]. Some practitioners argue that the direct use of FRFs seems more logical than the indirect use of modal parameters derived from the FRF data because the modal data derived from FE or EMA form an incomplete set of eigenvalues and eigenvectors [3].

An engineer may desire to add a new unit at a specified location on an existing structure due to a design aim. Whenever the unit is added to the structure, due to mass of the unit one of the natural frequencies of the structure may shift to a frequency value such that it may be adjacent to harmonic forcing frequency and consequently the modified structure vibrates at resonance. Therefore it is important to what will be dynamic properties of the structure after modification without modifying it in practice. In this way, for instance it can be estimated whether the resonance phenomenon occurs after modification or not.

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The present study deals with the determination of any FRF of the modified structure using FRFs of the un-modified structure. From this point of view, a method based on the Sherman-Morrison (SM) formula [14] has been presented. First, the modification technique based on SM formula is briefly outlined in the next section. Then, the efficiency of the method is examined by a numerical example.

II. Sherman-Morrison Formula and Structural Modification

The method proposed in this paper is based on the so-called Sherman-Morrison identity which allows a direct inversion of a modified matrix efficiently using the data related to the initial matrix and to the modifications. If $[A]^{-1}$ is the inverse of a non-singular square matrix $[A]$ and the modification is expressed as a product of two vectors such as $\{u\}\{v\}^T$, so that the modified matrix is given by

$$[A^*] = [A] + \{u\}\{v\}^T \ . \tag{1}$$

The inverse of the modified matrix $[A^*]^{-1}$ can be calculated by using the SM formula as:

$$[A^*]^{-1} = [A]^{-1} - \frac{([A]^{-1}\{u\})(\{v\}^T[A]^{-1})}{1+\{v\}^T[A]^{-1}\{u\}} . \tag{2}$$

The SM formula given by (2), has been used in a wide variety of applications in the past, such as, in the fields of statistics, networks, structural analysis, asymptotic analysis, optimization and partial differential equations. A more detailed coverage of this approach and various numerical aspects are discussed by Hager [15] and Akgün et al. [16]. For structural dynamic purposes, the main use of the identity given by (2) is to efficiently analyse the structural modification problems. Level et al. [17] proposed a method, using the receptance strategy in conjunction with the SM formula, to calculate the frequency response of a modified structure. Sanliturk [9] presented a study about using this method for linear and non-linear structural modification purposes. Cakar and Sanliturk [18] adopted the method to remove mass loading effects from the measured FRFs in modal testing. Then, Cakar and Sanliturk [19] applied the method to remove suspension effects. Ozer and Roystone [20] presented a method based on the SM formula for calculation of the optimal absorber parameter values for attachment to a damped multi-degree of freedom system. The objective of the present paper is to develop an exact method for calculation of the receptance of the structure modified by mass(es) and grounded spring(s).

It is known that the dynamic stiffness matrix [Z] of a system is given by

$$[Z]= [K]-\omega^2[M]+j\omega[C] \qquad (3)$$

where [K], [M], [C] represent stiffness, mass and damping matrices, ω is the angular frequency and $j=\sqrt{-1}$. Let [ΔZ] be the modification matrix which includes the mass, stiffness and damping modifications, using the well-known relationship between the receptance and the dynamic stiffness, $[\alpha] = [Z]^{-1}$, and expressing the modified system as $[Z^*] = [Z] + [\Delta Z]$ where $[\Delta Z] = \{u\}\{v\}^T$, the FRFs of the modified structure can be computed using the SM formula in (2) as

$$[\alpha^*]=[Z^*]^{-1}=[\alpha]-\frac{([\alpha]\{u\})(\{v\}^T[\alpha])}{1+\{v\}^T[\alpha]\{u\}}. \qquad (4)$$

The [α^*] matrix above contains the receptances of the modified system. If only one-rank modification is considered at co-ordinate r alone, the all elements of the modification vectors u and v are zero except r^{th} elements. For example, v_r can be expressed for concentrated mass (m), grounded spring (k) and grounded damping (c) modifications respectively as follows while u_r=1:

$$v_r=-\omega^2 m, \quad v_r=k \text{ and } v_r=j\omega c \qquad (5)$$

As it stands, (4) implies that all the elements of the receptance matrix [α] of a test structure are needed. However, it is not desirable in practical applications because it is very consuming time or mostly it cannot possible to measure all of them. Generally, single row or column of the FRF matrix is available in the applications. However, (4) can be written at active co-ordinates only, i.e. excitation, response and modification co-ordinates as shown by Sanliturk [9]. Furthermore, Cakar and Sanliturk [18] expressed an explicit formula for any FRF as follows:

$$\alpha^*_{pq}=\frac{\alpha_{pq}+v_r(\alpha_{rr}\alpha_{pq}-\alpha_{pr}\alpha_{rq})}{1+v_r\alpha_{rr}} \qquad (6)$$

where p, q and r represent response, excitation and modification coordinates, respectively. Equation (6) is a general expression allowing the calculation of the modified receptance α^*_{pq} by using the original receptances and the modification. It should be noted that (6) can be used for more than one modification successively.

Equation (6) has been used for direct structural modification purpose in the early paper of author [18] such that the FRF(s) of the modified system have been calculated by using FRFs of the unmodified structure and modification which is the transducer mass. However, in the present study, (6) is extended to calculate any FRF of the structure after modifying it by masses and springs. The method is outlined hereinafter.

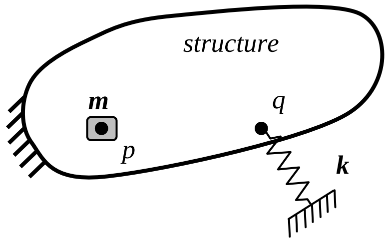

Figure 1. A structure modified by a concentrated mass and a grounded spring.

Let's consider a general structure shown in Fig. 1 to clarify the method. First, consider a consantrated mass m located at the co-ordinate p. The transfer FRF of the modified structure which related to the modification co-ordinates p and q can be calculated via (6) changing r by p and using $v_r=-\omega^2 m$:

$$\alpha^*_{pq}=\frac{\alpha_{pq}}{1-\omega^2 m\alpha_{pp}} \qquad (7)$$

where α_{pq} and α^*_{pq} are the receptances of the original (un-modified) structure and the modified structure by adding mass m at location p, respectively.

Then, consider the grounded spring k located at the co-ordinate q. In this case the modification co-ordinate is r=q and the modification is $v_r=k$. After that, the receptance of the modified structure can be obtained by using (6) successively as:

$$\alpha^{**}_{pq}=\frac{\alpha^*_{pq}}{1+k\alpha^*_{qq}} \qquad (8)$$

where α^{**}_{pq} is the receptance of the modified structure by adding both mass and grounded spring to the co-ordinates p and q, respectively. Where α^*_{pq} can be calculated from (7) directly and α^*_{qq} can be calculated from (6) by writing first p=q, then r=p and $v_r=-\omega^2 m$ as:

$$\alpha^*_{qq}=\frac{\alpha_{qq}-\omega^2 m(\alpha_{pp}\alpha_{qq}-\alpha^2_{pq})}{1-\omega^2 m\alpha_{pp}} \qquad (9)$$

When the mass and the stiffness are added to same location on the structure, say co-ordinate p, the point receptance of the modified structure can easily be calculated as;

$$\alpha^*_{pp}=\frac{\alpha_{pp}}{1-\omega^2 m\alpha_{pp}} \quad \text{and} \quad \alpha^{**}_{pp}=\frac{\alpha^*_{pp}}{1+k\alpha^*_{pp}}. \qquad (10)$$

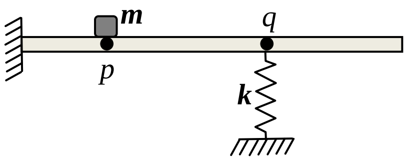

Figure 2. A cantilever beam modified by mass and spring.

III. Numerical Application

A cantilever beam of 80 cm length and 25x1 (cm^2) cross section shown in Fig. 2, is considered to demonstrate the presented method. The material properties; Young's modulus, density and the Poisson ratio are taken as E=207x109 N/m^2, ρ = 7800 kg/m^3 and ν=0.28, respectively. By using finite elements software FINES [21], the un-damped eigen-values and eigen-vectors of the un-modified beam were determined for twelve modes. In the analysis, the beam modelled with a three-node beam element (3BEM03). The beam divided into 24 elements and it has totally 49 nodes. First six natural frequencies of the beam are given in Table 1.

The receptances of the un-modified beam with related to the modification co-ordinates were calculated for frequency band 0-1200 Hz sampling at 0.1 Hz and including six modes using well known mode summation formula as [22]:

$$\alpha_{pq} = \sum_{r=1}^{N} \frac{\phi_{pr}\phi_{qr}}{\omega_r^2 - \omega^2 + i\eta_r\omega_r^2} \tag{13}$$

where p and q are response and excitation co-ordinates, respectively; N is the number of modes; ϕ_{pr} and ϕ_{qr} are eigen-vectors of the response and the excitation co-ordinates for mode r, respectively; ω_r is the natural frequency of the mode r; ω is the frequency of excitation force and η_r is the modal damping of mode r.

First, a point mass of 200 g is added to the node 6. Note that, mass of the beam is 1.56 kg. The FRFs of the structure modified by adding the mass can be calculated by using (7). For example, the calculation of transfer FRF $\alpha^*_{6,42}$ of the modified beam with mass using (7) needs to two FRFs of un-modified beam i.e. point FRF $\alpha_{6,6}$ at the modification co-ordinate and transfer FRF $\alpha_{6,42}$.

TABLE I. FIRST SIX NATURAL FREQUENCIES (HZ) OF THE ORIGINAL AND MODIFIED BEAM.

Modes	Unmodified	Modified by mass m=200g	Modified by mass and spring m=200g, k=10000 N/m
1	13.0	13.0	75.0
2	81.4	81.2	202.0
3	227.7	224.1	228.1
4	445.5	425.1	437.2
5	739.4	676.6	692.2
6	1095	995.6	1010.0

The receptances of the beams un-modified and modified by mass are plotted together in Fig. 3. The modified natural frequencies are also given in Table 1. As expected, the natural frequencies of the modified structure decrease.

Then a spring of 10000 N/m is added to between the node 42 and the ground. The transfer FRF $\alpha^{**}_{6,42}$ of the modified beam is calculated via (8). As can be seen in (8), it needs to transfer FRF $\alpha^*_{6,42}$ of the beam modified by mass as well as the point FRF $\alpha^*_{42,42}$ in order to calculate the transfer FRF of the beam modified by both mass and stiffness. Where $\alpha^*_{6,42}$ already calculated beforehand and $\alpha^*_{42,42}$ can be calculated via (9). However (9) needs to an additional FRF of un-modified beam related to the stiffness modification co-ordinate i.e. $\alpha_{42,42}$.

The calculated transfer receptance $\alpha^{**}_{6,42}$ is plotted with the un-modified receptance $\alpha_{6,42}$ in Fig.(4). As expected, the natural frequencies of the modified structure increase. The natural frequencies of the beam modified by both mass and stiffness are given in third colon of Table 1.

It should be noted that in the each case, some frequencies more affected than others depending on the modification location. A mode is more affected when the movement of modification point is more for that mode.

IV. Conclusion

Structural modification is one of the most important application area of the modal testing. In engineering, when a structure is intended to be modified, it is desired to predict dynamic properties of the modified structure before modifying it. Measured FRFs are effectively used for the determination of dynamic properties of the structures i.e. natural frequencies, mode shapes and structural damping. This study presents a method based on SM formula in order to calculate any receptance of the modified structures. The method is exact and uses only a few receptances of the un-modified structure related to modification co-ordinates and the modification. When more than one modifications are made the number of needed FRFs is increase.

The method can be used when a limited number of receptances are available as in many experimental studies. A simple code can be written to calculate any receptance of the modified structure using present formula without matrix calculations.

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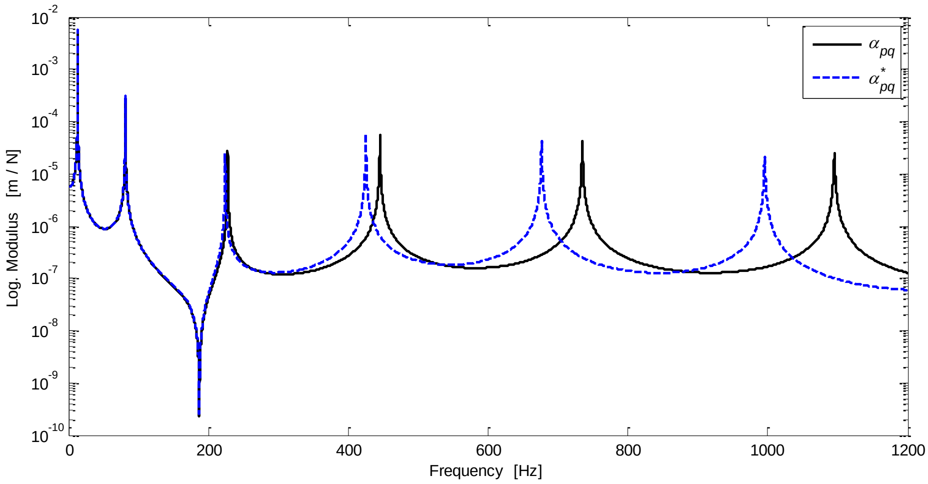

Figure 3. Comparison of modified and unmodified recepances (where *p*=6, *q*=42).

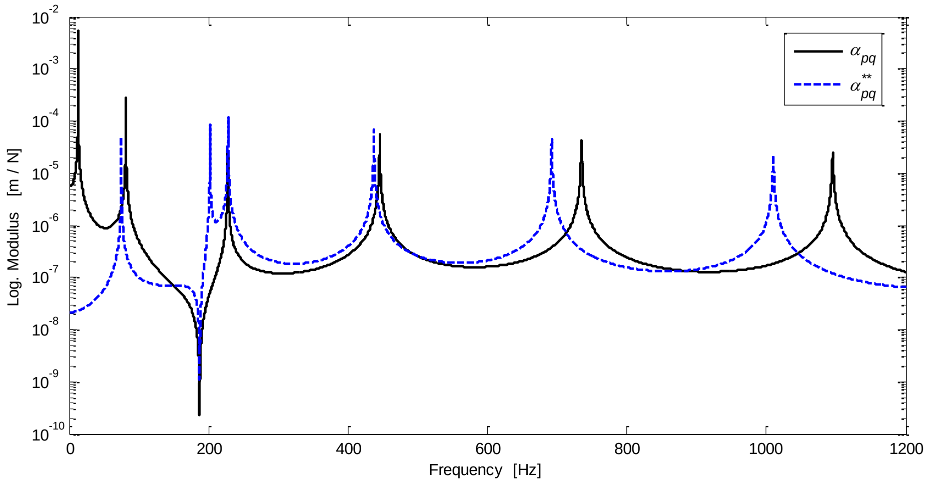

Figure 4. Comparison of modified and unmodified recepances (where *p*=6, *q*=42).

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