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Heterogeneous Population, Economic Growth and Income Distribution

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Abstract - We present an over lapping generation model of an economy that consists of three classes of workers. Agents might invest in education or capital. As people are more educated, their population growth rate is lower. The poorest class cannot borrow in order to invest in education or capital and its population is growing faster. As capital grows, educated agents can be mobilized into a state of firm owners. We show that under such conditions, while the wage per worker of the poorest class can increase, income distribution worsens as the economy grows.

Key Words: **Heterogeneous Population, Economic Growth, Income Distribution**

1. Introduction

Many countries around the world have a heterogeneous population consisting of several ethnic and cultural groups. Variety of population is usually accompanied by very large differences in population growth rates for different classes, leading to drastic changes in population and income distribution. The relationship between population and economic growth is complex and the historical evidence is ambiguous, particularly concerning causes and impacts (Thirlwall 1994, p. 143). Becker, Glaeser, and Murphy (1999, p. 149) demonstrated in a theoretical model that large population growth could have both negative and positive impacts on productivity. A large population may reduce productivity because of diminishing returns from more intensive use of land and other natural resources. Conversely, a large population could encourage greater specialization, and enlarged markets would increase returns to human capital and knowledge. Thus, the net relationship between greater population and economic growth depends on whether the inducements to human capital and knowledge expansion are stronger than the diminishing returns on natural resources. Therefore, it is important to examine the population and economic growth nexus. Rapid population growth tends to depress per capita savings and retard the growth of physical capital per worker. The need for social infrastructure is also broadened and public expenditures must be absorbed in providing for the needs of a larger population rather than in providing productive assets directly (Meier 1995, pp. 276-77). The economic literature discusses various determinants of economic growth. Many researchers focus on human capital accumulation and technology diffusion (see, for example, Creedy and Gemmell (2005), Aiginger (2005) and Zagler (2005)).

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Empirical evidence indicates that, except for very poor countries or households, increases in per capita income tend to reduce fertility. These studies have typically found important linkage between economic variables - such as per capita income, wage rates, level of female and male education, and urbanization - to fertility and mortality (See Wahl (1985), Schultz (1989) and Barro and Lee (1994). Due to these empirical findings, income inequality has been connected to population growth rates. Xavier and Martini (2006) found a spectacular reduction in worldwide poverty. Behind the reduction in poverty indexes, hides the uneven performance of various regions in the world. East and south Asia account for a large fraction of the success. Africa, on the other hand, seems to have moved in the opposite direction. The dismal growth performance of the African continent has meant that poverty rates and head counts increased substantially over the last three decades. The implication is that where poverty was mostly an Asian phenomenon 30 years ago (80 percent of the world's poor lived in east and south Asia), poverty is, today, an essentially African problem (75 percent of the poor live in Africa today, whereas only 19 percent live in Asia). Population changes will most probably have a real impact on economic growth, and therefore, the development of sound models will be increasingly relevant. Several researchers have introduced country-specific population growth rates. For example, Sayan (2005) investigated the implications of the addition of differential population dynamics in two regions assumed to be identical in every respect except for the way their populations evolved over time in a context of a model of international trade within an Overlapping Generations framework. Naito and Zhao (2008) formulated a two-country, two-good, two-factor, two-period-lived overlapping generations model to examine how population aging determines the pattern of gains from trade. Yakita (2012) examined the effects of different aging speeds on international trade patterns in an open Overlapping Generations model.

However, we couldn't find papers allowing for different growth rates for different sections of society supplying different factors of production.

2. Model Structure

All individuals live during two periods of time. An individual works during the first period and retires at the beginning of the second. Each individual saves part of his/her first period income so that the savings, including the return, finance his/her second period consumption. During each period, both young and old people are alive.

2.1 Supply and demand for capital

In each generation, young entrepreneurs are without means and must borrow in order to finance the capital that is needed by their new firms. The younger generation borrows from the older generation an amount that equals the value of the depreciated capital of the last generation plus a share λ_t of the savings of the last generation. Each generation would invest in human capital a share $(1-\lambda_t)$ of the savings. The capital stock at time t-1 is K_{t-1}, the total savings of the last generation is S_{t-1}, depreciation rate is d and I_t is the net investment at period t. In generation t, the young would borrow from the old an amount of $(1-d)K_{t-1}+\lambda_t S_{t-1}$. We get that the capital stock at time t would be:

$$(1) \qquad K_t = (1-d)K_{t-1} + I_t = (1-d)K_{t-1} + \lambda_t S_{t-1}$$

(Notice that $\lambda S_{t-1} = I_t$, where λ_t is the share of savings invested in capital).

2.2. The sources of financing the capital

At the beginning of period t, younger generation entrepreneurs would borrow from the older generation an amount equal to K_t. We assume that the old generation is paid zero real interest rate for the loan. The current younger generation would repay the loan at the end of period t. Assuming that income of the young generation is Y_t and that their savings are S_t we get that consumption of the young generation in period t, $C^1{}_t$, is:

$$(2) \qquad C^1{}_t = Y_t - \underbrace{[(1-d)K_{t-1} + \lambda_t S_{t-1}]}_{repaid\ \ loan} - S_t$$

While consumption of the old generation is:

$$(2a) \qquad C^0{}_t = [(1-d)K_{t-1} + \lambda_t S_{t-1}]$$

Assuming that investment in real capital is I_t while investment in human capital is E_t

We get that aggregate demand in period t is:

$$(2b) \qquad C^1{}_t + C^0{}_t + I_t + E_t = \{Y_t - [(1-d)K_{t-1} + \lambda_t S_{t-1}] - S_t\} + \{[(1-d)K_{t-1} + \lambda_t S_{t-1}]\} + I_t + E_t = Y_t$$

$$\longrightarrow I_{t+1} + E_{t+1} = S_t = \lambda_t S_t + (1-\lambda_t)S_t$$

Where $I_t = \lambda_t S_t$. We should notice that current young generation repaid the loan to the old generation and at the end of period t they will lend to the new young generation born at t+1 an amount of $(1-d)K_t + \lambda_{t+1}S_t$, which is the amount of capital in period t+1.

2.3 Production

L_{it} *for* $i = 0,1,2$ - represents the number of agents in class i.

$L_{0,t}$ is the number of entrepreneurs and each firm has one firm owner, such that $L_{0,t}$ is also the number of identical active firms at the market in period t.

$L_{1,t}$ is the number of workers in class 1.

$L_{2,t}$ is the number of workers in class 2.

E_{it} - is total years of education of workers belonging to class i.

We assume that each firm produces according to the Cobb–Douglas constant return to scale function.

The output produced by one firm at time t is:

$$(3) \qquad Y^j{}_t = k_t{}^{\alpha} L_{1t}{}^{\beta_1} L_{2t}{}^{\beta_2} E_{0t}{}^{\gamma_0} E_{1t}{}^{\gamma_1} E_{2t}{}^{\gamma_2}$$

Where: $\beta_1 < \beta_2$

and $\sum_{i=1}^{2}\beta_i + \sum_{i=0}^{2}\gamma_i + \alpha = 1$.

k_t - is the amount of capital in firm j and each firm employs the same optimal amount of capital. The amount of capital of each firm is

$$(4) \qquad k_t = \frac{K_t}{L_{0,t}}$$

where total amount of capital in the economy is K_t.

2.4 Income distribution

The product, $Y^j{}_t$, in each firm j is divided between the entrepreneurs and the workers. If labor is perfectly mobile, then workers in each class will get their marginal product. Workers' marginal product is equal to additional production due to an additional unit of labor plus marginal product due to an additional amount of education used by the firm when it employs an additional worker, so that:

$$(5a) \qquad w_{1t} = \beta_1 k_t{}^{\alpha} L_{1t}{}^{\beta_1 - 1} L_{2t}{}^{\beta_2} E_{0t}{}^{\gamma_0} E_{1t}{}^{\gamma_1} E_{2t}{}^{\gamma_2} + \gamma_1 K_t{}^{\alpha} L_{1t}{}^{\beta_1} L_{2t}{}^{\beta_2} E_{0t}{}^{\gamma_0} E_{1t}{}^{\gamma_1 - 1} E_{2t}{}^{\gamma_2}$$

$$w_{2t} = \beta_2 k_t{}^{\alpha} L_{1t}{}^{\beta_1} L_{2t}{}^{\beta_2 - 1} E_{0t}{}^{\gamma_0} E_{1t}{}^{\gamma_1} E_{2t}{}^{\gamma_2} + \gamma_2 K_t{}^{\alpha} L_{1t}{}^{\beta_1} L_{2t}{}^{\beta_2} E_{0t}{}^{\gamma_0} E_{1t}{}^{\gamma_1} E_{2t}{}^{\gamma_2 - 1}$$

Where w_{it} is the wage of a worker in class i (each worker's wage consists of marginal product of labor plus marginal product of education). For each firm, we get that total income of the workers in class 1 is:

$$(5b) \qquad L_{1t}MPL_{1t} + E_{1t} * MPE_{1t} = \beta_1 k_t{}^{\alpha} L_{1t}{}^{\beta_1 - 1} L_{2t}{}^{\beta_2} E_{0t}{}^{\gamma_0} E_{1t}{}^{\gamma_1} E_{2t}{}^{\gamma_2} * L_{1t} + \gamma_1 k_t{}^{\alpha} L_{1t}{}^{\beta_1} L_{2t}{}^{\beta_2} E_{0t}{}^{\gamma_0} E_{1t}{}^{\gamma_1 - 1} E_{2t}{}^{\gamma_2} * E_{1t} = (\beta_1 + \gamma_1) Y^j{}_t$$

Where MPE_{1t} is the marginal product of human capital of a worker belonging to class 1, while the total income of the workers in class 2 is:

$$(5c) \qquad L_{2t}MPL_{2t} + E_{2t} * MPE_{2t} = \beta_2 k_t{}^{\alpha} L_{1t}{}^{\beta_1} L_{2t}{}^{\beta_2 - 1} E_{0t}{}^{\gamma_0} E_{1t}{}^{\gamma_1} E_{2t}{}^{\gamma_2} * L_{2t} + \gamma_2 k_t{}^{\alpha} L_{1t}{}^{\beta_1} L_{2t}{}^{\beta_2} E_{0t}{}^{\gamma_0} E_{1t}{}^{\gamma_1} E_{2t}{}^{\gamma_2 - 1} * E_{2t} = (\beta_2 + \gamma_2) Y^j{}_t$$

The income of the entrepreneur is:

$$(6) \qquad W_{0,t} = k_t{}^{\alpha} L_{1t}{}^{\beta_1} L_{2t}{}^{\beta_2} E_{0t}{}^{\gamma_0} E_{1t}{}^{\gamma_1} E_{2t}{}^{\gamma_2} - (\beta_1 + \gamma_1 + \beta_2 + \gamma_2) Y_t = (1 - \beta_1 - \gamma_1 - \beta_2 - \gamma_2) Y^j{}_t$$

2.5 Distribution of Total production

The total production of the economy $Y_t = L_{0,t} Y^j{}_t$ is divided between the workers and the entrepreneurs. The lowest income class of workers would get a total income of $L_{0t}(\beta_2+\gamma_2)Y^j{}_t$, the second class would get $L_{0t}(\beta_1+\gamma_1)Y^j{}_t$, while the total income of the entrepreneurs would be $L_{0t}(1-\beta_1-\gamma_1-\beta_2-\gamma_2)Y^j{}_t$. As we can see, given the level of capital, the production technology and the number of workers in each class, the total production in the economy is divided between the workers, the entrepreneur and the old generation and the Gini index can be calculated.

2.6 Utility

Defining $C_{0t}{}^i$ - as consumption of agent i when s/he is young while $C_{1t}{}^i$ is consumption of agent i when s/he is old, let us assume that the utility function of each agent in the economy, $U=\ln(C_{0t})+\gamma\ln(C_{1t})$, is similar to agents belonging to all categories.Each agent tries to maximize utility, subject to his/her budget constraints:

$$(7)\qquad Max \qquad \ln(C_{0t}{}^i)+\gamma\ln(C_{1t}{}^i)$$

$$s.t \qquad C_{ot}{}^i+\frac{C_{1t}{}^i}{1+r}=W_{it}$$

Solving the problem, we get:

$$(8)\qquad C_{ot}{}^i=\frac{1}{1+\gamma}W_{it}$$

$$(9)\qquad S_t^i=\frac{\gamma}{1+\gamma}W_{it}$$

Notice that agents who acquired education use part of their savings to finance the acquired education.

The aggregate savings in the economy would be:

$$(10)\qquad S_t=L_{0,t}\sum_{i=0}^{2}S_t^iL_{it}=L_{0,t}\sum_{i=0}^{2}\frac{\gamma}{1+\gamma}W_{it}L_{it}$$

Substituting $Y^j{}_t=\sum_{i=0}^{2}W_{it}L_{it}$ into (10) we get:

$$(11)\qquad S_t=L_{0,t}\sum_{i=0}^{2}S_t^iL_{it}=\frac{\gamma}{1+\gamma}L_{0,t}Y^j{}_t=\frac{\gamma}{1+\gamma}Y\ {}_t$$

2.7 Education

The education of an agent belonging to classes 0 and 1 is determined by the rules

$(12)\quad MP_{E0}=MP_k$ and $(12a)\quad MP_{E1}=MP_k$.

From (12) we get:

$$(13)\qquad \gamma_0k_t{}^{\alpha}L_{1t}{}^{\beta_1}L_{2t}{}^{\beta_2}E_{0t}{}^{\gamma_0-1}E_{1t}{}^{\gamma_1}E_{2t}{}^{\gamma_2}=\alpha k_t{}^{\alpha-1}L_{1t}{}^{\beta_1}L_{2t}{}^{\beta_2}E_{0t}{}^{\gamma_0-1}E_{1t}{}^{\gamma_1}E_{2t}{}^{\gamma_2}$$

$$\frac{\gamma_0Y^j{}_t}{E_{0t}}=\frac{\alpha Y^j{}_t}{k_t}\rightarrow E_{0t}=\frac{\gamma_0k_t}{\alpha}$$

and from (12a) we get:

$$(13a)\qquad \gamma_1k_t{}^{\alpha}L_{1t}{}^{\beta_1}L_{2t}{}^{\beta_2}E_{0t}{}^{\gamma_0}E_{1t}{}^{\gamma_1-1}E_{2t}{}^{\gamma_2}=\alpha k_t{}^{\alpha-1}L_{1t}{}^{\beta_1}L_{2t}{}^{\beta_2}E_{0t}{}^{\gamma_0-1}E_{1t}{}^{\gamma_1}E_{2t}{}^{\gamma_2}$$

$$\frac{\gamma_1Y^j{}_t}{E_{1t}}=\frac{\alpha Y^j{}_t}{k_t}\rightarrow E_{1t}=\frac{\gamma_1k_t}{\alpha}$$

We assume an initial fixed education level for class 2. Due to their low income, agents of class 2 do not invest in education ($\gamma_2=0$) or capital due to inability to borrow. Let assume that the cost of 1 year of education is fixed and equals to $P_E=1$, then the total amount invested in education is:

$$(14)\qquad (1-\lambda_t)S_t=L_{0t}[E_{0t}*L_{0t}+E_{1t}*L_{1t}]=L_{0t}*\frac{(\gamma_0+\gamma_1)*(L_{0t}+L_{1t})k_t}{\alpha}=\frac{(\gamma_0+\gamma_1)*(L_{0t}+L_{1t})K_t}{\alpha}$$

2.8 Growth Path

According to equation 1, the amount of capital in period t+1 is:

$$(1a)\qquad K_{t+1}=(1-d)K_t+I_t$$

The net investment is financed by the savings of the current generation, so that $\lambda S_t=I_t$.

We can present equation $(1a)$ as:

$$(1a')\qquad K_{t+1}=K_t(1-d)+\lambda_tS_t=K_t(1-d)-(1-\lambda_t)S_t+S_t$$

Substituting (11) for St in (1a') and substituting (14) for $(1-\lambda_t)S_t$, we get:

$$(15)\qquad K_{t+1}=(1-d)K_t-\frac{(\gamma_0+\gamma_1)*(L_{0t}+L_{1t})K_t}{\alpha}+\frac{\gamma}{1+\gamma}L_{0,t}Y_t$$

Substituting the production function (3) into (15) we get:

$$(15a)\qquad K_{t+1}=(1-d)K_t-\frac{(\gamma_0+\gamma_1)*(L_{0t}+L_{1t})K_t}{\alpha}+\frac{\gamma}{1+\gamma}L_{0,t}K_t{}^{\alpha}L_{1t}{}^{\beta_1}L_{2t}{}^{\beta_2}E_{0t}{}^{\gamma_0}E_{1t}{}^{\gamma_1}E_{2t}{}^{\gamma_2}$$

2.9 Economic growth

Production is determined by education level, population size and capital amount.

Whenever the stock of capital grows, new firms will be created. This is due to the assumption that the optimal size of each firm will remain constant in all generations and no technological changes occur (long run equilibrium in production, which determines an optimal size of a firm in each generation is also assumed).

The firms optimal amount of capital in all generations would be k_t for $k_t=\frac{K_t}{L_{0,t}}=\frac{K_{t+1}}{L_{0,t+1}}=......=\frac{K_{t+k}}{L_{0,t+k}}$, where $L_{0,t+j}$ is the number of firm owners in period $t+j$.

In each generation, when the cumulative amount of capital grows, the number of firms grows. Given that each individual can own only one firm, we would get that the number of firm owners in each generation $t+j$ would be

$$L_{0,t+j}=\frac{K_{t+j}}{k_t}\qquad for \qquad j=0,1,2,3,......$$

The assumption that one individual cannot be an owner of more than one firm lies in the idea that risk management of savings would encourage the older generation to spread the loans (their savings) among the new entrepreneurs. Given that the characteristics needed for being a good entrepreneur are distributed among the population, we get that in each generation there would be mobility of individuals from workers belonging to class 1 into the firm owners' class. In each generation, we get that the number of firms and the number of firm owners is determined by the level of capital. If capital level grows, then the number of firms and the number of firm owners grow. Since population grows according to equation (16), we will get a lower population growth rate for classes 0 and 1 and a higher population growth rate for class 2.

In each time period, whenever capital grows, we get a change in income distribution. Production growth, population growth and income distribution are determined simultaneously.

5. Summary

The main aim of this paper is to suggest a theoretical framework for analyzing the existence of different population growth rates, for different income classes, on population and income distribution along the growth path of the economy. Using the framework of an overlapping generation model, savings finance investment in capital and education. The poorest class, class 2, is not able to invest in capital or education due to credit constraints. The other two classes, i.e. class zero (firm owners) and class one (higher class workers) invest in capital and in education. The population growth rate is affected by education and other factors which are not specified here. Given a higher education for the richer classes, their population growth rate is lower. As the economy evolves, the relative size of the poorest population increases and its wage per worker decreases due to a reduction in its marginal product. Assuming that the optimal size of a firm is given (according to risk considerations of the lenders), as the cumulative amount of capital increases, the number of firms and the number of firm owners increase, leading to the mobility of some of the highest income classes' workers into the firm owner class. A fast increase in the number of firms reduces the number of workers per firm, leading to a change in the wage per worker of the two working classes and a change in the firm owners' income. We show an example with a specific production function for a firm that uses labor capital and education. Given the equalization of the marginal product of capital and education, the level of education for the two highest income classes is determined. Education is financed by borrowing from the previous generation. The rest of the previous generation's savings finance new acquired capital. For each generation, we can calculate production, capital, savings, education, wages per worker of each class, mobility of workers from a higher income class into the firm owners' class and the income distribution.

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