

Defined Contribution Pension Fund Management In A Jump Diffusion Market

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Abstract

This paper investigates multi-period defined contribution fund management under mean-variance criteria in a jump diffusion market. Besides, we consider both stochastic income and mortality risk in a more general market with multiple assets that can all be risky.

Currently, there are two main types of pension funds: defined benefit (DB) pension fund and defined contribution (DC) pension fund. DB fund is a pension fund where the retirement benefits are calculated by a predetermined formula. Retirement benefits are usually calculated using average salary over the last few years before retire and the number of years one has worked in the company or public service. In a DC plan, both employee and employer do contribution to the pension account, and upon retirement, the member can receive regular income from the pension account or the member can withdraw the lump sum from the pension account. Compared with Defined Benefit plan, Defined Contribution plan is fairer because the more contribution one has made in his or her career, the more retirement income he or she will get after retirement. So in our paper, we concentrate on DC pension fund management problem.

As the contribution is often a fixed percentage of salary, the contribution is also stochastic. Therefore, it is crucial to take the income risk into consideration. A pension fund member may die before his retirement. In such a case, his pension fund has to be terminated due to mortality risks. Furthermore, since the real market is incomplete, the jump diffusion process is introduced to describe the incompleteness with the condition that all the assets can be risky.

Mean-Variance criteria, which was first proposed by Markowitz, is concerned with the allocation of wealth among various securities so as to seek the optimal trade-off between the expected return of the portfolio and its risk over a certain or uncertain planning horizon. It has become the foundation of modern finance theory and inspired hundreds of extensions and applications in literature. By applying the Lagrange method and stochastic dynamic programming techniques, we derive analytical closed-form expressions for optimal investment strategy and the efficient frontier. Then we discuss three special cases. Finally, we numerically examine the general solution and special cases.

Keywords

Defined Contribution Pension; Stochastic Income; Mortality Risk; Jump diffusion