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Mathematical Model of Autonomous Underwater Vehicle

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Abstract*—In this paper, we propose an approach of the construction of a mathematical model of AUV, taking into account the hydrodynamic characteristics, which were obtained with CFD software. AUV considered in this article has a torpedo shape with a thrust-vectoring module and two (horizontal and vertical) auxiliary thrusters located in the bow. *(Abstract)

***Keywords*—mathematical model, underwater vehicle, autonomous vehicle**

I. Introduction

Modern underwater vehicles are capable of performing a wide range of tasks, which includes environmental and climate monitoring, oceanographic research, deep-water systems and devices service, underwater fields search, protection of water areas, etc. Autonomous underwater vehicles (AUV) can most effectively address all these tasks. Use of AUVs reduces operating costs and probability of error by decreasing the degree of involvement of the human operator, increasing uptime of mobile systems, reduction in weight and size of underwater vehicles and others. However, to design the control system of such a complex object, it is necessary to construct a mathematical model that would fully take into account all of the parameters unsteadiness, the nonlinearity and multiple-connection of the dynamics of AUV as an object of control, the forces of interaction of the body with a viscous environment.

II. Mathematical Model of AUV

To derive a mathematical model of AUV two Cartesian coordinate systems shown in Figure 1 were used. Mathematical model of AUV can be presented in the following vector-matrix form on the basis of the known equations of a rigid body kinematics and dynamics:

$$\dot{\bar{Y}} = \Sigma(\bar{\theta},\bar{X}) = \Sigma\begin{pmatrix}\Sigma_P(\bar{\theta},\bar{X})\\ \Sigma_\theta(\bar{\theta},\bar{X})\end{pmatrix} \quad (1)$$

$$\tilde{M}\dot{\bar{X}} = \left[\bar{F}_d(\bar{P},\bar{V},\bar{\omega}) + \bar{F}_u(\bar{\delta}) + \bar{F}_v(G,A_\Pi,R_\Gamma)\right] \quad (2)$$

$$T_{uy}\frac{d\bar{\delta}}{dt} + \bar{\delta} = \bar{\Psi}_{uy}(\bar{\delta},\bar{U}) \quad (3)$$

Where T_{uy} is a diagonal matrix of time constants of actuators and Ψ is a vector of nonlinear functions of right sides of the actuators dynamics;

$\bar{\delta}$ - AUV's actuators inputs vector;

$\bar{U}$ - controls vector formed by the control system of AUV.

x - m-vector of internal coordinates (the state variables);

M - ($m \times m$)-matrix of mass and inertial parameters, whose elements are the mass, moments of inertia, added masses of AUV;

F_u - m-vector of control forces and moments, l - vector of parameters of actuators depending on the geometrical arrangement of the actuators;

F_d - m-vector of non-linear elements of the AUV's dynamics;

F_v - m-vector of measured and unmeasured external disturbances;

Y - n-vector of position P and orientation (output coordinates) of body coordinate system relative to the base coordinate system, ;

$\sum(\Theta, X)$ - n-vector of kinematic constraints;

$\sum_P(\Theta, X)$ - vector of linear velocities of the body coordinate system relative to the base coordinate system;

$\sum_\Theta(\Theta, X)$ - the angular velocity of the body frame relative to the base coordinate system.

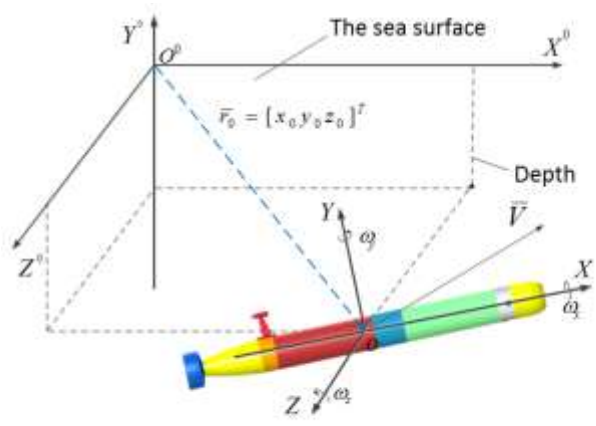

Figure 1. $K^0(OX^0Y^0Z^0)$ and $K(OX\ Y\ Z)$ coordinate frames

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A. Hydrodynamic coefficients and added masses

Expressions for the calculation of hydrodynamic coefficients and added masses are of the form:

$$F_x=k\cdot\left(cx_0+cx_V\cdot V_0+cx_{a_2}\cdot\alpha_0^{\,2}+cx_{b_2}\cdot\beta_0^{\,2}\right),$$

$$F_y = k\cdot\left(V_0^{\,2}\cdot\left(cy_0 + cy_a\cdot\alpha_0 + cy_{a_3}\cdot\alpha_0^{\,3} + cy_{a_2}\cdot\alpha_0\cdot|\alpha_0| + cy_b\cdot|\beta_0| + cy_{b_2}\cdot\beta_0^{\,2}\right) + V_0\cdot 1{,}02\cdot\omega_z\right),$$

$$F_z = k\cdot\left(V_0^{\,2}\cdot\left(cz_b\cdot\beta_0 + cz_{b_2}\cdot\beta_0\cdot|\beta_0| + cz_{b_3}\cdot\beta_0\cdot\beta_0^{\,2} + cz_{ab_2}\cdot\beta_0\cdot|\alpha_0|\right) + V_0\cdot\left(-1{,}19\cdot\omega_y\right)\right),$$

$$M_x = k_m\cdot\left(V_0^{\,2}\cdot\left(mx_b\cdot\beta_0 + mx_{ab}\cdot\beta_0\cdot\alpha_0 + 0\cdot\omega_x + mx_{ab_2}\cdot\alpha_0\cdot\beta_0\cdot|\beta_0|\right) + V_0\cdot(-0{,}090\cdot 1\cdot\omega_x)\right),$$

$$M_y = k_m\cdot\left(V_0^{\,2}\cdot\left(my_b\cdot\beta_0 + my_{b_2}\cdot\beta_0\cdot|\beta_0|\right) + V_0\cdot(-1{,}23\cdot 1\cdot\omega_y)\right),$$

$$M_z = k_m\cdot\left(V_0^{\,2}\cdot\left(mz_0 + mz_a\cdot\alpha_0 + mz_{a_2}\cdot\alpha_0\cdot|\alpha_0| + mz_{ab}\cdot\alpha_0\cdot\beta_0\right) + V_0(-1{,}181\cdot 1\cdot\omega_z)\right),$$

$$\begin{cases}\lambda_{11} = k_{11}\rho U; \quad \lambda_{22} = k_{22}\rho U;\\ \lambda_{33} = k_{33}\rho U; \quad \lambda_{26} = k_{26}\rho UL;\\ \lambda_{35} = k_{35}\rho UL; \lambda_{66} = k_{66}\rho UL^2\\ \lambda_{44} = k_{44}\rho UL^2; \quad \lambda_{55} = k_{55}\rho UL^2;\end{cases} \qquad (4)$$

where $V_0=(V_x^{\,2}+V_y^{\,2}+V_z^{\,2})^{0,5}$, $\alpha_0=arctan(-V_y/V_x)$, $\beta_0=arcsin(V_z/V_0)$, ρ – density of water, U – the volume of AUV ($U=0.2793$), $S=U^{2/3}$, $k=\rho\cdot S\cdot V_0^{\,2/3}$;

$cx_0=0.06805$, $cx_V=-3.57\cdot10^{-3}$, $cx_{a_2}=-0.204$, $cx_{b_2}=-0.069$;

$cy_0=-4{,}011\cdot10^{-4}$, $cy_a=1{,}25$, $cy_{a_2}=0{,}312$, $cy_{a_3}=-0{,}224$, $cy_b=-0{,}00701$, $cy_{b_2}=-0{,}420$;

$cz_b=-1{,}174$, $cz_{b_2}=-0{,}449$, $cz_{b_3}=0{,}34$, $cz_{ab_2}=-0{,}095$;

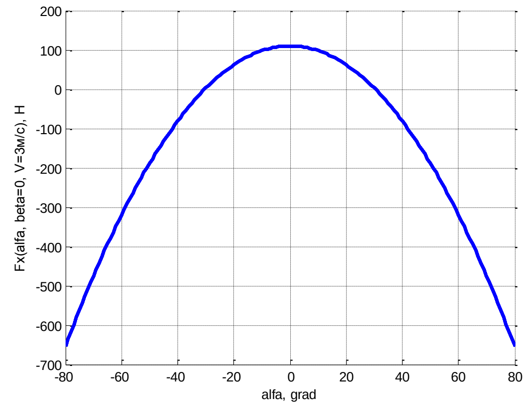
$mx_b=9{,}30\cdot10^{-3}$, $mx_{ab}=0{,}131$, $mx_{ab_2}=-0{,}064$;

$my_b=0{,}0680$, $my_{b_2}=0{,}0389$;

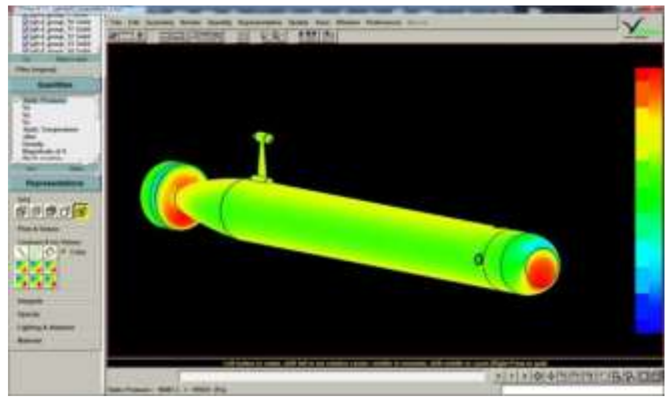
$mz_0=5{,}47\cdot10^{-4}$, $mz_a=0{,}0289$, $mz_{a_2}=0{,}0855$, $mz_{ab}=3{,}02\cdot10^{-5}$;

Pressure distribution on AUV's enclosure with fins (see Fig. 2 (b)) and dependence of the coefficient of drag (see Fig. 2 (a)) on the angle of attack were investigated by means of a numerical simulation and hydrodynamic characteristics study package by NUMECA International.

Figures 3-4 are graphs of characteristic dependences of the projection of the hydrodynamic forces and moments on the angle of attack and sliding at speed V = 3 m/s.



a) $F_x(\alpha, V=3м/c, \omega=0)$



b) The distribution of the hydrodynamic pressure on the surface of the vehicle

Figure 2. Dependence of the hydrodynamic force F_x, on the angle of attack α at a speed $V = 3$ m/s

B. Analysis of controllability

It is necessary to formalize the distribution of control forces and moments on a particular arrangement of the actuators (propulsion screw propeller and the bow thruster) for the analysis of controllability.

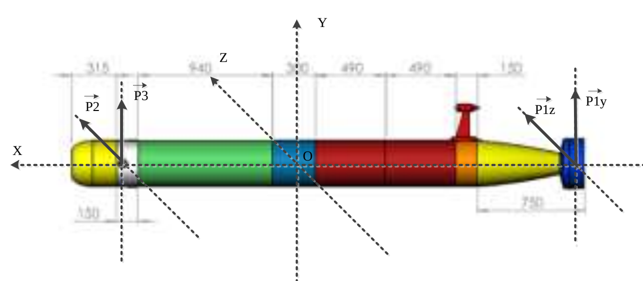


Figure 3. AUV composition

The projections of the thrusts produced by marching screw propeller (thrust-vectoring module): P_{1x}, P_{1y}, P_{1z}.

Thrusts generated by bow thrusters: horizontal (P_2) and vertical (P_3).

Distance along the axis to the point of force application:

axis OX:	axis OY:	axis OZ:
x_{md}=-1.9	y_{md}=0	z_{md}=0
x_{hz}=1.28	y_{hr}=0	z_{hr}=0
x_{vr}=1.18	y_{vr}=0	z_{vr}=0

To create a moment around the axis OY and OZ is necessary that the forces acting on the same axis have opposite signs.

The equations of the forces and moments acting on the AUV:

$$\begin{cases} F_{ux} = P_{1x} \\ F_{uy} = P_{1y} + P_3 \\ F_{uz} = P_{1z} + P_2 \\ N_{ux} = 0 \\ N_{uy} = -P_{1z}x_{md} + P_2 x_{hz} \\ N_{uz} = -P_{1y}x_{md} + P_3 x_{vr} \end{cases} \tag{5}$$

The system (4) is the direct conversion of forces and moments acting on the AUV. Let's write the inverse transformation:

$$P_{1x}=F_{ux}$$

$$P_{1y}=\frac{x_{vr}}{x_{vr}+x_{md}}F_{uy}-\frac{1}{x_{md}+x_{vr}}N_{uz}$$

$$P_{1z}=\frac{x_{hz}}{x_{hz}+x_{md}}F_{uz}-\frac{1}{x_{hz}+x_{md}}N_{uy}$$

$$P_2=\frac{x_{md}}{x_{md}+x_{hz}}F_{uz}+\frac{1}{x_{hz}+x_{md}}N_{uy}$$

$$P_3=\frac{x_{md}}{x_{md}+x_{vz}}F_{uy}+\frac{1}{x_{md}+x_{vz}}N_{uz}$$

To determine the added mass of the AUV body, we use the theoretical nomograms by O.N. Dudchenko. To use them, you need to calculate the width to the height ratio and the length to width ratio of the AUV.

To study the controllability of AUV the following definition was used [2]: the object is called a fully controllable in a certain range, if there is an admissible control on a finite interval of time, taking the object from the starting point into an arbitrarily small neighborhood of the end point for any pair of start and end points of this region.

Let's analyze the controllability of the object. For the object to be controllable according to the Pyatnitskiy theorem [3] it is enough that control thrusts exceeded the required efforts on every coordinate axis. In particular, this condition can be written as follows:

$$P_{1x}^{max} > |F_x|, P_{1y} + P_3 > |F_y|, P_{1z} + P_2 > |F_z|$$

$$-P_{1z}x_{md}+P_2x_{hz}>|M_y|,-P_{1y}x_{md}+P_3x_{vr}>|M_z|$$

where P_{1x}^{max} is a maximal thrust generated by a motor, F_x, F_y, F_z, M_y, M_z - projections of forces and moments Fd+Fv on the axes *OX, OY, OZ.*

For all the control channels either main thrust (thrust-vectoring module) or auxiliary thruster acts, so AUV is controllable in condition of the correct task.

III. Conclusions

Thus, the constructed mathematical model is a system of nonlinear differential equations, the elements of which are determined by the arrangement and parameters of a particular AUV, as well as the structure and nature of the external disturbances. In addition, the distinctive feature of the AUV is non-stationary elements of its dynamic model, depending on the conditions of operation of the AUV and its constructional characteristics. The need to consider the full dynamics of the AUV is determined by the stringent requirements for the quality of the functioning of the AUV.

Specific of the underwater vehicle usage for precise paths following at high speeds requires mandatory accounting and the estimation or measurement of its unsteady parameters (added masses etc.) [4], in addition, it is necessary to develop motion planning system structure and algorithms for the AUV functioning on environmental uncertainty [5].

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