International Journal of Advancements in Mechanical and Aeronautical Engineering– IJAMAE
Volume 2: Issue 1 [ISSN : 2372-4153]

Publication Date : 30 April, 2015

VIBRATION ANALYSIS OF NANOBEAM CARRYING NANOPARTICLE BASED ON NONLOCAL ELASTICITY THEORY

Necla Togun

***Abstract*— In this study, vibration of effect of nanoparticle on an Euler-Bernoulli nano-beam is investigated using nonlocal elasticity beam theory. Equation of motion of nanobeam is obtained using Hamilton's principle. The dimensionless equations are solved for the various boundary conditions. Approximate solutions of the equations are obtained by using the method of multiple scales a perturbation technique. The first terms of the perturbation series lead to the linear problem. Exact solutions are given natural frequencies for the linear problem. The effects of the different nonlocal parameters (γ) and mass ratios (α) as well as effects of different boundary conditions on the vibrations are determined.**

Keywords—vibration, nanobeam, nanoparticle, perturbation method, nonlocal elasticity

1. INTRODUCTION

Carbon nanotubes (CNTs) have received considerable attention since they were first discovered by Iijima [1]. Due to their novel electronic, mechanical and other physical and chemical properties, CNTs have potential applications in atomic-force microscopes, field emitters, nano-actuators, nano-motors, nano-bearings, nanosprings, nano-fillers for composite materials, and nanoscale electronic devices.

CNTs are modeled using the continuum mechanics and molecular dynamic simulations. Molecular dynamic simulations are limited to small-scale systems and to short time intervals, continuum mechanics are generally preferred to model the CNTs. But, classical or continuum elasticity theory can not predict the behavior of the nano-scale structures. For this reason, the nonlocal elasticity theory which was formally initiated by the papers of Eringen [2] on nonlocal elasticity can be used for nanotechnology applications. As it is known, in local continuum theory, the stress depends only on the strain at the same point. However, in the nonlocal elasticity theory, the stress at a point is a function of strains at all points in the continuum. Nonlocal beam theory was used to study bending, buckling and vibration of nanobeams [3,4]. Peddieson et al. [5] can be considered to be a pioneering work which first applied the nonlocal elasticity theory of Eringen [2] to the nanotechnology to obtain the static deformations of nonlocal Euler-Bernoulli beam model.

Necla Togun
Vocational School of Technical Sciences in Gaziantep, University of Gaziantep. 27310, Gaziantep, Turkey.

The column buckling of CNTs modeled with the nonlocal Euler-Bernoulli beam theory was studied in [6]. Nonlocal Timoshenko beam model were applied to the studies of wave properties of single and double walled nanotubes [7]. Nonlocal elasticity theory was used to investigate the small scale effects on static response of micro and nano structures [8] and on axial vibration of nanorods [9].

The frequency shift of carbon nanotube-based mass sensor with an attached mass was studied using the nonlocal elasticity theory [10]. They investigated the effects of nonlocal parameter, attached mass and its location on the frequency shift of a cantilevered single walled carbon nanotube (SWCNT). These result indicated that when the location of the attached mass is closer to the free end, the frequency shift is more significant and that makes the sensor reveal more sensitive. When the attached mass is small, a high sensitivity was obtained [10]. The studies on the dynamic analysis of CNTs attached mass are very limited. For instance, the resonant frequency and mode shapes of a SWCNT based mass sensor was studied analytically and using a beam bending model [11,12]. Axial vibration behavior [13] and dynamic behavior [14] of SWCNT based mass sensor was studied using nonlocal elasticity theory. Vibration of CNT based bio sensor was presented to analyze fundamental frequencies using the nonlocal Timoshenko beam theory [15], nonlocal Euler-Bernoulli beam theory [16] and continuum mechanics [17]. The continuum mechanics method and a bending model were applied to obtain the resonant frequency of the fixed free SWCNT where the mass was rigidly attached to the tip as a nanomechanical sensor [18]. The dynamic analysis of nanotube structures under excitation of a moving nanoparticle was carried out using nonlocal continuum theory [19-23].

In this study, an Euler-Bernoulli nanobeam carrying nanoparticle is considered under different boundary conditions. Exact natural frequencies are calculated for different boundary conditions, different nonlocal parameters and mass ratios. The method of multiple scales, a perturbation technique, is used to solve the equations approximately. The first terms in the expansions lead to the linear problem. The natural frequencies and mode shapes are calculated exactly and tabulated for different conditions.

2. Nonlocal Elasticity Theory

According to the Eringen's nonlocal elasticity theory, the nonlocal stress tensor σ at point x can be written as:

$$\sigma_{ij}\left(x^*\right)=\int_V K\left(\left|x^*-x'^*\right|,\gamma\right)C_{ijkl}\,\varepsilon_{kl}\left(x'^*\right)dV\left(x'^*\right) \qquad (1)$$

where σ_{ij} and ε_{ij} is the stress and strain tensors, respectively; C_{ijkl} is the elastic modulus tensor of classical isotropic elasticity, and $K\left(\left|x^*-x'^*\right|,\gamma\right)$ is the kernel function. $\left|x^*-x'^*\right|$ is the Euclidean distance, and $\gamma = \mathrm{e}_0 a/\mathrm{L}$, where e_0 is a constant appropriate to each material, a is an internal characteristic length (e.g, lattice parameter, granular distance) and L is an external characteristic length (e.g, crack length, wavelength). Generally, a conservative estimate of the nonlocal parameter is $0 \le \mathrm{e}_0 a \le 2$ nm for SWCNTs is proposed by Wang [24]. It is very hard to solve the elasticity problems by using the integral constitutive relation in Eq. (1). Therefore, a simplified constitutive relation in a differential form is given by Eringen [2] as follows;

$$\left(1-(e_0a)^2\nabla^2\right)\sigma = T \qquad (2)$$

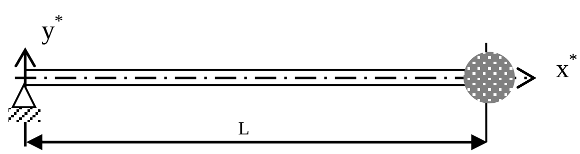
Where ∇^2 is the Laplacian operator. The nonlocal constitutive relation for a homogeneous isotropic Euler-Bernoulli beam takes the following form:

$$\sigma\left(x^*\right)-(e_0a)^2\frac{\partial^2\sigma\left(x^*\right)}{\partial x^{*2}} = E\varepsilon\left(x^*\right) \qquad (3)$$

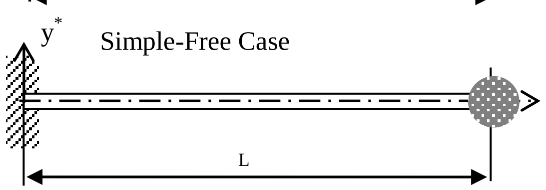
where E is the elasticity modulus.

3. Governing equations for nanobeam carrying a nanoparticle at free end

The present study considers the case of a simple free and fixed free nanobeam with a nano-scale particle attached to its tip, as shown in Fig. 1.



Simple-Free Case



Fixed-Free Case

Fig.1. Boundary conditions for different beam supports

For the system shown in Fig. 1, y^* denotes the transverse displacement of the beam section between supports. L is the length of the beam. t^* is the time. ρA is the mass per unit length, m is the nano-scale particle, *EA* is longitudinal rigidity, *EI* is flexural rigidity and e_0a is the nonlocal parameter of nanobeam, N is the axial force. The Lagrangian can be written as follows

$$\begin{aligned}
\pounds &= \frac{1}{2}\int_0^L \rho A\left(\frac{\partial y^*}{\partial t^*}\right)^2 \mathrm{d}x^* + \frac{1}{2}m\left(\frac{\partial y^*(L)}{\partial t^*}\right)^2 \\
&-\frac{1}{2}\int_0^L\left(EI\frac{\partial^2 y^*}{\partial x^{*2}}+(e_0a)^2N^*\frac{\partial^2 y^*}{\partial x^{*2}}-(e_0a)^2\rho A\frac{\partial^2 y^*}{\partial t^{*2}}\right)\frac{\partial^2 y^*}{\partial x^{*2}}\,dx^* \\
&-\frac{1}{2}\int_0^L N^*\left(\frac{\partial y^*}{\partial x^*}\right)^2\mathrm{d}x^*
\end{aligned} \qquad (4)$$

where dot denotes derivative with respect to t^* and prime denotes derivative with respect to x^*. The first integral and second term are the kinetic energy of the beam section between any successive supports. The second integral is the elastic energy in bending, the third integral is the elastic energy in extension due to stretching of the neutral axis and the last one is the elastic energy due to axial tension. Applying Hamilton's principle and performing the necessary algebra, the equations of motion for the general case for the nanobeam in dimensional form is obtained as follows

$$\begin{aligned}
&EI\frac{\partial^4 y^*}{\partial x^{*4}}+\rho A\frac{\partial^2 y^*}{\partial t^{*2}}\left(1-\frac{\partial^2}{\partial x^{*2}}(e_0a)^2\right) \\
&=\frac{EA}{2L}\left[\int_0^L\left(\frac{\partial y^*}{\partial x^*}\right)^2dx^*\right]\left[\frac{\partial^2 y^*}{\partial x^{*2}}-(e_0a)^2\frac{\partial^4 y^*}{\partial x^{*4}}\right]
\end{aligned} \qquad (5)$$

Dimensionless parameters defined as follow,

$$x=\frac{x^*}{L},\, w=\frac{w^*}{L},\; t=\frac{t^*}{L}\sqrt{\frac{EI}{\rho A}}, \qquad \gamma=\frac{e_0a}{L},\, \alpha=\frac{m}{\rho AL} \qquad (6)$$

The equations and boundary conditions are made dimensionless using the following definitions

$$\frac{\partial^4 y}{\partial x^4}+\frac{\partial^2 y}{\partial t^2}-\gamma^2\frac{\partial^4 y}{\partial t^2\partial x^2}=\frac{1}{2}\left[\int_0^1\left(\frac{\partial y}{\partial x}\right)^2dx\right]\left[\frac{\partial^2 y}{\partial x^2}-\gamma^2\frac{\partial^4 y}{\partial x^4}\right] \qquad (7)$$

Simple-Free Case

$$\begin{aligned}
&y(0)=0, && y'(1)=0 \\
&y''(0)=0, && -\alpha\ddot{y}(1)+\left(1+\gamma^2\right)Ny'''(1)=0
\end{aligned} \qquad (8)$$

Fixed-Free Case

$y(0)=0, \qquad y'(L)=0$

$y'(0)=0, \qquad -\alpha\ddot{y}(L)+\left(1+\gamma^2\right)Ny'''(L)=0$

4. Method of Multiple Scales

The dimensionless form of equation of motion is given before in Eq. 7

$$\frac{\partial^4 y}{\partial x^4}+\frac{\partial^2 y}{\partial t^2}-\gamma^2\frac{\partial^4 y}{\partial t^2\partial x^2}=\frac{1}{2}\left[\int_0^L\left(\frac{\partial y}{\partial x}\right)^2 dx\right]\left[\frac{\partial^2 y}{\partial x^2}-\gamma^2\frac{\partial^4 y}{\partial x^4}\right]$$

The method of multiple scales will be applied to the partial differential equation system and boundary conditions directly [25,26]. There is no quadratic non-linearities, that's why one can write an expansion of the form

$$y(x,t;\varepsilon)=\varepsilon^1 y_1(x,T_0;T_2)+\varepsilon^3 y_3(x,T_0;T_2) \qquad (9)$$

where ε^1 is a small book-keeping parameter representing that the deflections are small. This procedure models a weak non-linear system. $T_0=t$ and $T_2=\varepsilon^2 t$ are the fast and slow time scales. Here only the primary resonance case is considered.

$$\frac{\partial}{\partial t}=D_0+\varepsilon^2 D_2,\ \frac{\partial^2}{\partial t^2}=D_0{}^2+2\varepsilon^2 D_0 D_2 \qquad (10)$$

where $D_n=\partial/\partial T_n$. After expansion, one obtains equations of motion and boundary conditions at order 1 as follows:

Order (ε^1):

$$y_1^{iv}+D_0^2 y_1-\gamma^2 D_0^2 y_1''=0 \qquad (11)$$

Solution of the first order of expansion gives natural frequency values and a solvability condition is obtained from the second order of expansion.

4.1. Exact Solution to the Linear Problem

For Eq. (10) one can assume solutions of the form for any beam segment

$$y_1(x,T_0,T_2)=\left[A(T_2)e^{i\omega T_0}+cc\right]Y(x) \qquad (12)$$

where cc stands for complex conjugate of the preceding terms. Eqs. (11) and (12) give

$$Y^{iv}(x)+\omega^2\gamma^2 Y''(x)-\omega^2 Y(x)=0 \qquad (13)$$

S- F Case:

$$Y(0)=0\,,Y''(0)=0\,,Y(1)=0\,,\alpha\omega^2 Y(1)+\left(1+\gamma^2\right)NY'''(1)=0$$

F-F Case: (14)

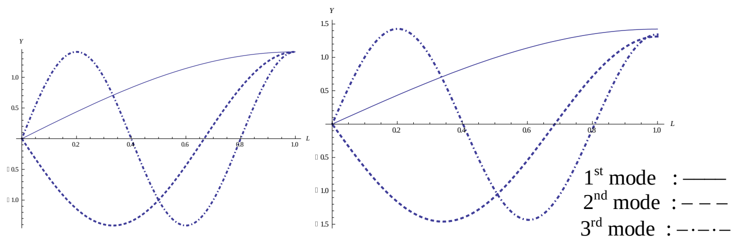
$$Y(0)=0\,,Y'(0)=0\,,Y'(1)=0\,,\alpha\omega^2 Y(1)+\left(1+\gamma^2\right)NY'''(1)=0$$

The solution of the equations can be sought by assuming the following shape function for any beam segment

$$Y(x)=c_1 e^{i\beta_1 x}+c_2 e^{i\beta_2 x}+c_3 e^{i\beta_3 x}+c_4 e^{i\beta_4 x} \qquad (15)$$

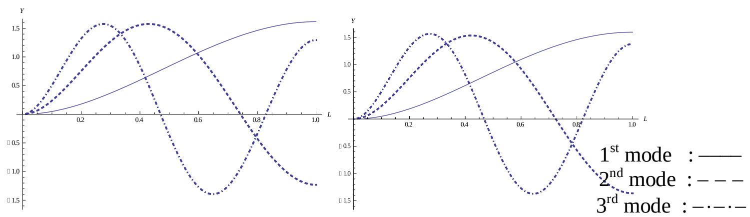
Frequency equations can be obtained when the boundary conditions are applied.

Mode shapes of linear first three frequency are plotted in Figs. 2-3 for different α ve γ values and simple-free and fixed free boundary conditions, respectively.



(a) (b)

Fig.2. First three vibration modes of a simple-free supported nanobeam carrying a nano-scale particle at the free end with a) α=0.1 ve γ=1 b) α=1 ve γ=0.1



(a) (b)

Fig.3. First three vibration modes of a fixed-free supported nanobeam carrying a nano-scale particle at the free end with a) α=0.1 and γ=0.1d) α=1 and γ=0.1

The linear frequencies for the first five frequencies are presented in Table 1 for different case and different γ and α values. The effects of support conditions and material constant (γ) and mass ratio (α) are given. Generally for different support conditions, when the γ values increase, the linear frequencies decrease. When the γ is fixed and α values increase, linear frequencies decrease.

Table 1. The first five frequencies for different γ and α values for different support case

	α	γ	ω_1	ω_2	ω_3	ω_4	ω_5
Simple Free Case	0.1	0.1	2.347	20.016	48.464	81.316	115.397
		0.3	2.167	12.807	24.094	35.073	45.8651
		0.5	1.903	8.6710	15.221	21.636	27.9953
		0.7	1.640	6.4408	11.039	15.577	20.0936
		0.9	1.414	5.0956	8.6405	12.155	15.6596
	0.5	0.1	2.065	19.743	48.278	81.198	115.322
		0.3	1.954	12.739	24.073	35.065	45.8607
		0.5	1.771	8.6525	15.216	21.634	27.9945
		0.7	1.566	6.4344	11.037	15.576	20.0933
		0.9	1.374	5.0929	8.6399	12.151	15.6595
	1	0.1	1.823	19.435	48.059	81.057	115.231
		0.3	1.758	12.660	24.047	35.054	45.8552
		0.5	1.638	8.6303	15.210	21.632	27.9934
		0.7	1.487	6.4266	11.035	15.576	20.0929
		0.9	1.329	5.0897	8.6391	12.155	15.6594
Fixed Free Case	0.1	0.1	5.261	26.954	57.866	92.112	127.017
		0.3	4.777	16.826	28.447	39.605	50.5097
		0.5	4.102	11.356	18.019	24.511	30.9137
		0.7	3.468	8.4597	13.112	17.691	22.2286
		0.9	2.949	6.7151	10.291	17.345	20.8548
	0.5	0.1	4.509	26.562	57.614	91.954	126.913
		0.3	4.229	16.734	28.387	39.579	50.4841
		0.5	3.781	11.333	17.984	24.503	30.8956
		0.7	3.298	8.4542	13.090	17.689	22.2167
		0.9	2.860	6.7137	10.278	17.338	20.8544
	1	0.1	3.899	26.143	57.324	91.767	126.788
		0.3	3.746	16.635	28.319	39.548	50.4543
		0.5	3.464	11.309	17.944	24.494	30.8751
		0.7	3.115	8.4480	13.065	17.687	22.2031
		0.9	2.759	6.7120	10.263	17.329	20.8540

5. Conclusions

The vibrations of an Euler-Bernoulli nanobeam carrying nanoparticle are investigated for different boundary conditions. To obtain approximate solutions, perturbation method is applied to the equations. For the linear problem, exact solutions and numerical values for natural frequencies are obtained. The effects of the γ and α are determined. Generally for different support conditions, when the γ values increase, the linear frequencies decrease. When the γ is fixed and α values increase, linear frequencies decrease.

References

[1] S. Iijima, "Helical microtubles of graphitic carbon," Nature, vol.354, pp.56-58, 1991.

[2] A.C. Eringen, "On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves," J. Appl. Phys., vol.54, pp. 4703-4710, 1983.

[3] J.N. Reddy, "Nonlocal theories for bending, buckling and vibration of beam," Int. J. Eng. Sci., vol.45, pp. 288-307, 2007.

[4] M. Aydogdu, "A general nonlocal beam theory: Its application to nanobeam bending, buckling and vibration," Physica E, vol. 41, pp. 1651-1655, 2009.

[5] J. Peddieson, G.R. Buchanan, R.P. McNitt, "Application of nonlocal continuum models to nanotechnology," Int. J. Eng. Sci., vol. 41, pp. 305-312, 2003.

[6] L.J. Sudak, "Column buckling of multiwalled carbon nanotubes using nonlocal continuum mechanics," J. Appl. Phys., vol. 94, pp.7281-7287, 2003.

[7] P. Lu, H.P. Lee, C. Lu, P.Q. Zhang, "Application of nonlocal beam models for carbon nanotubes," Int. J. Solids Struct., vol. 44, pp. 5289-5300, 2007.

[8] Q. Wang, K.M. Liew, " application of nonlocal continuum mechanics to static analysis of micro and nano structures," Phys. Lett. A, vol. 363, pp. 236-242, 2007.

[9] M. Aydogdu, "Axial vibration of the nanorods with the nonlocal continuum rod model," Physica E, vol. 41, pp. 861-864, 2009.

[10] H.L.Lee, J.C. Hsu, W.J.Chang, "Frequency ahift of carbon nanotube based mass sensor using nonlocal elasticity theory," Nanoscale Res. Lett. Vol. 5, pp.1774-1778, 2010.

[11] D.H.Wu, W.T.Chien, C.S.Chen, H.H.Chen, "Resonant frequency analysis of fixed-free single walled carbon nanotube based mass sensor," Sensor Actuat. A vol. 126, pp. 117-121, 2006.

[12] T.L. Horng, "Analytical solution of vibration analysis on fixed free single valled carbon nanotube based mass sensor", J. Surf. Eng. Mater. Adv. Tech., vol. 2, pp.47-52, 2012.

[13] M. Aydogdu, S.Filiz, "Modeling carbon nanotube based mass sensors using axial vibration and nonlocal elasticity", Physica E, vol. 43, pp. 1229-1234, 2011.

[14] P. Soltani, O. Pashaei, M.M.Taherian, A. Farshidianfar, "Free vibration of a carbon nantube based mass sensor", Adv. Mater. Res., Vol.403, pp.1163-1167, 2012.

[15] Z.B. Shen, L.P. Sheng, X.F.Li, G.J.Tang, "Nonlocal Timoshenko beam theory for vibration of carbon nanotube based biosensor", Physica E, vol. 44, pp.1169-1175, 2012.

[16] I. Mehdipour, A.E.Moghadam, C.Mehdipour, "Application of an electrostatically actuated cantilevered carbon nanotube with an attached mass as a bio-mass sensor", Curr. Appl. Phys. Vol. 13, pp. 1463-1469, 2013.

[17] R. Chowdhury, S. Adhikari, J. Mitchell, "Vibrating carbon nanotube based bio sensors", Physica E, vol. 42, pp. 104-109, 2009.

[18] I. Mehdipour, A.Barari, G. Domairry, "Application of a cantilevered SWCNT with mass at the tip as a nanomechanical sensor", Comput. Mater. Sci. vol. 50, pp.1830-1833, 2011.

[19] K. Kiani, "Application of nonlocal beam models to double walled carbon nanotubes under a moving nanoparticle", Acta Mech., Vol. 216, pp. 165-195, 2011.

[20] K. Kiani, B. Mehri, "Assesment of nanotube structures under a moving nanoparticle using nonlocal beam theories", J. Sound Vib. Vol. 329, pp. 2241-2264, 2010.

[21] K. Kiani, "Longitudinal and transverse vibration of a single walled carbon naotube subjected to a moving nanoparticle accounting for both mechanical and inertial effects", Physica E, vol. 42, pp. 2391-2401, 2010.

[22] M. Şimşek, "Nonlocal effects in the forced vibration of an elastically connected double walled carbon nanotube system under a moving nanoparticle", Comput. Mater. Sci., Vol. 50, pp. 2112-2123, 2011.

[23] A.G. Arani, M.A. Roudbari, S. Amir, "Nonlocal vibration of SWBNNT embedded in bundle of CNTs under a moving nanoparticle", Physica B, vol. 207, pp. 3646-3653, 2012.

[24] Q. Wang, " Wave propagation in carbon nanotubes via nonlocal continuum mechanics", J.Appl. Phys., vol. 98, 2005. 124301.

[25] A.H. Nayfeh, Introduction to Perturbation Techniques, John Wiley, New York, 1981.

[26] A.H. Nayfeh, D.T. Mook, Nonlinear Oscillations, John Wiley, New York, 1979.