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Reduced Order Modeling of Triple Link Inverted Pendulum Using Particle Swarm Optimization Algorithm

[Mudita Juneja and Sheela Tiwari]

***Abstract*—In recent years, Particle Swarm Optimization has evolved as an effective global optimization algorithm whose dynamics has been inspired from swarming or collaborative behavior of biological populations. In this paper, PSO has been applied to Triple Link Inverted Pendulum model to find its reduced order model by minimization of error between the step responses of higher and reduced order model. Model Order Reduction using PSO algorithm is advantageous due to ease in implementation, higher accuracy and decreased time of computation. The second and third order reduced transfer functions of Triple Link Inverted Pendulum have been computed for comparison.**

***Keywords*—Particle Swarm Optimization, Triple Link Inverted Pendulum, Model Order Reduction, Pole Placement technique.**

I. Introduction

The modeling of complex dynamic systems is one of the most important subjects in engineering and science. A model is often too complicated to be used in real life problems. Simplification of dynamical models that may contain many equations and/or variables is needed in order to perform simulations within an acceptable amount of time and limited storage capacity, but with reliable outcome. Model Order Reduction tries to quickly capture the essential features of a structure. Various methods are available in literature for model order reduction [1-4]. Recently, Particle Swarm Optimization (PSO) technique appeared as a promising algorithm for handling the optimization problems.

Particle swarm optimization (PSO) is a population based stochastic optimization technique developed by Eberhart and Kennedy [5] in 1995, inspired by social behavior of bird flocking or fish schooling. PSO shares many similarities with evolutionary computation techniques such as Genetic Algorithms (GA). PSO is used for Model Order Reduction by initializing the system with a population of random solutions and searching for optima by updating generations to find the optimized value of reduced order model.

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Inverted pendulum system is a highly unstable and non-linear system which is easily available for laboratory usage [6]. It is an ideal model of advanced control theory and typical experiment platform for test control problems.

In this paper, Particle Swarm Optimization is used on Triple Link Inverted Pendulum system to obtain its lower order model which approximates it for step and impulse responses. TLIP model has been reduced to its second and third order approximations which are compared to achieve an efficient reduced order model with minimum error between its step response and the step response of higher order model.

II. Problem Statement

Consider a continuous linear time invariant n^{th} order transfer function model given as;

$$G(s) = \frac{c_m s^m + c_{m-1} s^{m-1} + \cdots + c_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0} \tag{1}$$

where, $n \geq m$.

c_m, c_{m-1},..,c_0 are coefficients of numerator polynomial.

a_n, a_{n-1},..,a_0 are coefficients of denominator polynomial.

The aim is to reduce the above higher order transfer function to its equivalent transfer function of 2^{nd} and 3^{rd} order, $R_2(s)$ and $R_3(s)$ respectively, such that the reduced order model contains the significant characteristics of higher order model with minimized error for impulse and step responses of the system.The reduced order transfer functions are given as;

$$R_2(s) = \frac{B_1 s + B_0}{s^2 + b_1 s + b_0} \tag{2}$$

$$R_3(s) = \frac{B_2 s^2 + B_1 s + B_0}{s^3 + b_2 s^2 + b_1 s + b_0} \tag{3}$$

where, B's are numerator polynomial coefficients, and

b's are denominator polynomial coefficients.

III. Modeling and Stabilization of Triple Link Inverted Pendulum System

A. *Modeling Of TLIP*

The modeling of Triple Link Inverted Pendulum using Lagrange method [7-8] and its schematic diagram is given in fig.1. Triple link inverted pendulum consists of three links: lower, middle and upper pendulums interconnected such that lower link is mounted vertically on a movable cart. It represents a system with single input, u (external force) and four outputs, x, θ_1, θ_2, θ_3 (displacement of cart, angles of lower, middle and upper arm with vertical) hence is a SIMO system. The generalized Euler-Lagrange equation is given as;

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)-\left(\frac{\partial L}{\partial q_i}\right)=-\left(\frac{\partial D}{\partial \dot{q}_i}\right) \tag{4}$$

where, Lagrangian , L = T – V

T = Total Kinetic energy of the system

V =Total Potential energy of the system

D = Work done against friction (Dissipative forces)

$q_1, q_2, \ldots\ldots, q_s$ are the generalized coordinates of the system.

The linear model of the triple link inverted pendulum is represented in state-space form as follows:

$$\dot{X}=\begin{bmatrix}0 & I_{4\times4}\\ E^{-1}H & E^{-1}G\end{bmatrix}X+\begin{bmatrix}0\\ E^{-1}h_0\end{bmatrix}U$$

$$Y=C\,X \tag{5}$$

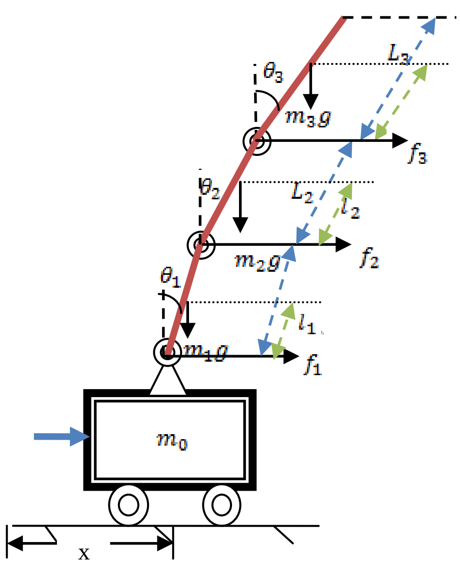

FIGURE 1. Triple Link Inverted Pendulum System Model

where,

$$E=\begin{bmatrix}a_0 & a_1 & a_2 & m_3l_3\\ a_1 & b_1 & a_2L_1 & m_3L_1l_3\\ a_2 & a_2L_1 & b_2 & m_3L_2l_3\\ m_3l_3 & m_3L_1l_3 & m_3L_2l_3 & J_3+m_3l_3^2\end{bmatrix}$$

$$H=\begin{bmatrix}0 & 0 & 0 & 0\\ 0 & a_1g & 0 & 0\\ 0 & 0 & a_2g & 0\\ 0 & 0 & 0 & m_3l_3g\end{bmatrix},$$

$$G=\begin{bmatrix}-f_0 & 0 & 0 & 0\\ 0 & -f_1-f_2 & f_2 & 0\\ 0 & f_2 & -f_2-f_3 & f_3\\ 0 & 0 & f_3 & -f_3\end{bmatrix}$$

The state vector is defined by:

$$X=[x \;\; \theta_1 \;\; \theta_2 \;\; \theta_3 \;\; \dot{x} \;\; \dot{\theta}_1 \;\; \dot{\theta}_2 \;\; \dot{\theta}_3]$$

The coefficient matrices of state equation (5) of cart triple inverted pendulum are evaluated [7] as follows:

A

$$=\begin{bmatrix}0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\ 0 & -12.4298 & -2.0824 & 2.2956 & -5.1127 & 0.0075 & 0.0024 & -0.0053\\ 0 & 67.1071 & 65.2564 & -71.9704 & 14.0176 & 0.0039 & -0.1948 & 0.1659\\ 0 & 144.5482 & -394.2536 & 272.1049 & 5.2021 & -0.4334 & 1.1287 & -0.7492\\ 0 & -300.4564 & 512.8310 & -258.9198 & -10.8077 & 0.6476 & -1.3621 & 0.8260\end{bmatrix}$$

B

$$=[0 \quad 0 \quad 0 \quad 0 \quad 3.6512 \quad -10.0125 \quad -3.7157 \quad 7.7196]^T$$

$$C=\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\end{bmatrix} \tag{6}$$

The roots or eigen values of the unstable TLIP system (6) are given in Table I.

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TABLE I. EIGEN VALUES OF UNSTABLE AND STABLE SYSTEM

S.No.	Eigen Values of unstable system	Eigen Values of stable system
1.	0	-4+5.455i
2.	1.0502 +27.2130i	-4-5.455i
3.	1.0502 -27.2130i	-8+10.91i
4.	11.0345	-8-10.91i
5.	4.2304	-12+16.365i
6.	-1.9248	-12-16.365i
7.	-6.0272	-6.0272
8.	-12.5674	-12.5674

B. *Stabilization Using Pole Placement*

The TLIP model is stabilized using pole placement method. The poles are placed in such a way so that the closed loop system has an overshoot of 10% and settling time of 1 second [9]. So, $\xi = 0.591328$ and w_n=6.7644rad/sec. The dominant poles are at=-4 ± j5.45531, the third and fourth pole are placed 2 and 3 times deeper into the s-plane than the dominant poles. Table I shows the poles of unstable and stable TLIP system. The optimal control problem is solved and the state feedback control parameter *K* is obtained by using place function in MATLAB which places closed loop poles at location *p*;

K= *place*(*A*, *B*, *p*).

The optimal feedback gain matrix is, *K* = [72.4918 6.7674 -38.777 196.9856 39.902 14.7247 20.2290 18.1805].Thus the corresponding closed loop system is given as,

$$\dot{X} = (A - BK)X \tag{7}$$

IV. Particle Swarm Optimization Algorithm

PSO is a member of the wide category of Swarm Intelligence methods [10-11]. In PSO a number of simple entities—the particles—are placed in the search space of some problem or function, and each evaluates the objective function at its current location. Each particle then determines its movement through the search space by combining some aspect of the history of its own current and best (best-fitness) locations with those of one or more members of the swarm, with some random perturbations. The next iteration takes place after all particles have been moved. Eventually the swarm as a whole, like a flock of birds collectively foraging for food, is likely to move close to an optimum of the fitness function [12]. The flowchart of PSO [13] is given in fig. 2.

Most of the population based search approaches are motivated by evolution as seen in nature.PSO, on the contrary, is based on simulation of social behavior. Unlike in genetic algorithms, in PSO selection operation is not performed. The whole population is kept as it is throughout the iterations and only their velocities are updated according to their own personal best and group's or swarm best position attained.PSO is the only evolutionary algorithm that does not implement survival of the fittest.

In the PSO, the speed and position of each particle change according the following equality:

$$v_{id}^{k+1} = wv_{id}^k + c_1 r_1^k\left(pbest_{id}^k - x_{id}^k\right) + c_2 r_2^k\left(gbest_{id}^k - x_{id}^k\right)$$
$$x_{id}^{k+1} = x_{id}^k + v_{id}^{k+1} \tag{8}$$

where,

v_{id}^k represents velocity of ith particle in d-dimension at kth iteration.

x_{id}^k represents position of ith particle in d-dimension at kth iteration.

$pbest_{id}^k$ represents personal best position of ith particle in d-dimension at kth iteration.

$gbest_{id}^k$ represents global best position(swarm best) of ith particle in d-dimension at kth iteration.

w represents inertial weight attached to the particle's previous position.

c_1, c_2 represent acceleration constants.

r_1^k, r_2^k represent random nos. in the range of [0,1] .

The objective or fitness function in PSO algorithm considered for reducing the order of TLIP is given as;

$$error = \sum_{t=0}^{t_\infty} |Y_t - y_t| \tag{9}$$

where,

Y_t and y_t represents step response of higher and reduced order transfer functions, respectively.

t_∞ represents the upper time bound for fitness function.

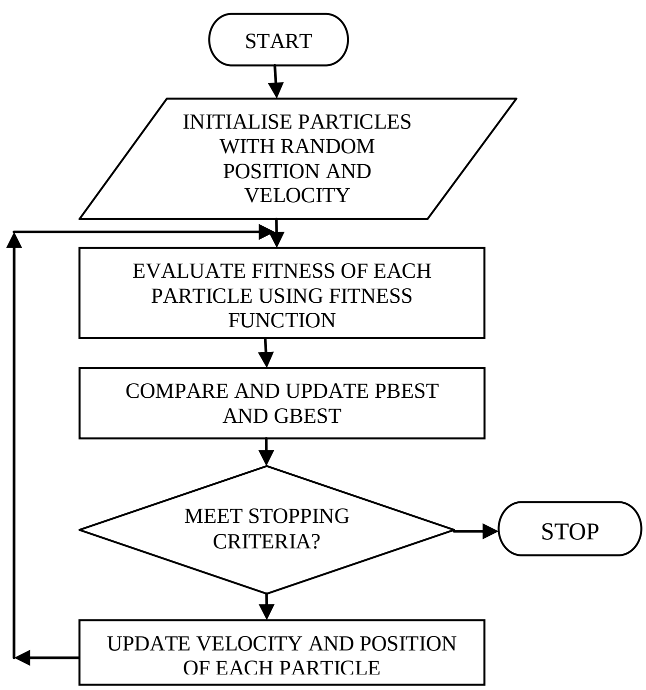

FIGURE 2. Flowchart of PSO

The velocity update rule in PSO consists of three parts:

- Momentum: contribution of previous velocity to current one. It models the tendency of the particle to continue in the same direction as it has been moving.
- Cognitive part: Pull to particle's velocity towards its own personal best (pbest). Referred to as "memory", "self knowledge" or "remembrance".
- Social part: Pull to particle's velocity towards swarm's best (gbest). Referred to as "cooperation", "social knowledge" or "shared information".

V. Simulation Results

The unstable model of TLIP (6) stabilized through pole placement technique by placing poles as given in Table I. The closed loop response obtained (7) is approximated to its second and third order counterparts using PSO algorithm in the MATLAB Environment. The algorithm aims to find B_1, B_0, b_1 and b_0 for second order model and B_2, B_1, B_0, b_2 ,b_1 and b_0 for third order reduced approximation of TLIP system. Table II gives the values of various parameters used in reduction using PSO and its flowchart is given in fig. 2.

The performance measure for analyzing the efficiency of reduced order model is selected to be the summed error between the step responses of higher and lower order transfer functions for some finite time (9). This also forms the fitness or objective function for the various particles of the swarm in PSO algorithm.

TABLE II. PARAMETER VALUES USED BY PSO

Variable name	Value
Swarm size	40
No. of generations	50
c_1,c_2	2
[w_{max},w_{min}]	[0.9,0.4]

Table III and IV shows the results of reduced order modeling of TLIP and the various coefficients of reduced order model have been tabulated. Figures 3, 4, 5, 6 show the step responses of original higher order transfer function with its reduced second and third order transfer function responses. It can be seen that the third order reduced model gives a better approximation to higher order TLIP model than its second order approximation.

VI. Conclusion

The TLIP system is modeled and stabilized by pole placement method. The stable system is then reduced to its second and third order reduced models which are compared. It is observed that, the step responses of third order reduced transfer functions provide a closer approximation to the higher order transfer functions. Moreover, the error reported in the third order reduced transfer functions is also less than the corresponding values for second order transfer functions except for one of the transfer function.

TABLE III. SIMULATION RESULTS FOR SECOND ORDER REDUCED MODEL

TLIP Second Order Reduced Transfer Function	Numerator Polynomial Coefficients		Denominator Polynomial Coefficients		Error
	B_1	B_0	b_1	b_0	
$R_2^1(s)=x/u$	-0.20700	0.48860	6.24100	35.24000	0.79697
$R_2^2(s)=\theta_1/u$	-0.13770	-0.04512	5.94900	107.23350	0.57260
$R_2^3(s)=\theta_2/u$	-0.14070	0.00028	5.54400	87.93000	0.85690
$R_2^4(s)=\theta_3/u$	0.04000	-0.00220	2.57570	24.98470	0.25850

TABLE IV. SIMULATION RESULTS FOR THIRD ORDER REDUCED MODEL

TLIP Third Order Reduced Transfer Function	Numerator Polynomial Coefficients			Denominator Polynomial Coefficients			Error
	B_2	B_1	B_0	b_2	b_1	b_0	
$R_3^1(s)=x/u$	0.00063	0.25630	0.08450	3.53830	18.79850	6.23490	0.89928
$R_3^2(s)=\theta_1/u$	-0.08200	1.27420	0.00940	16.34990	80.85520	486.39250	0.29876
$R_3^3(s)=\theta_2/u$	-0.02510	1.00540	-0.02674	12.76990	72.61900	344.40000	0.36660
$R_3^4(s)=\theta_3/u$	0.07577	0.03240	0.00106	5.90600	31.4400	50.39000	0.20750

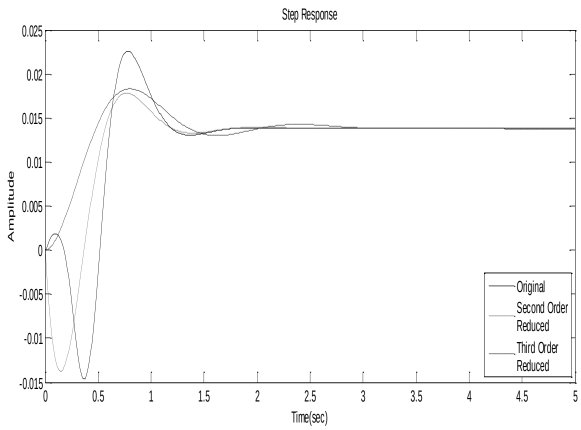

FIGURE 3. Step Responses for x/u

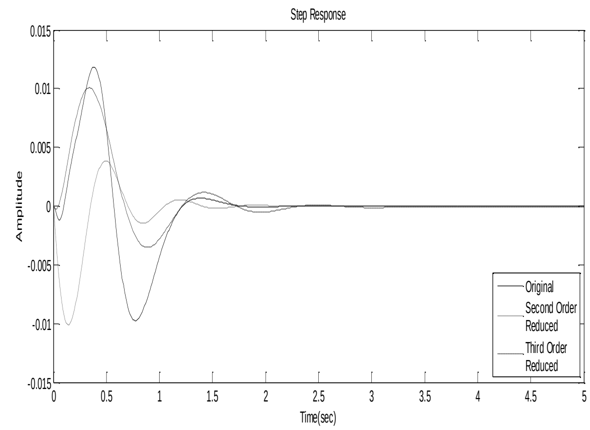

FIGURE 5. Step Responses for θ_2/u

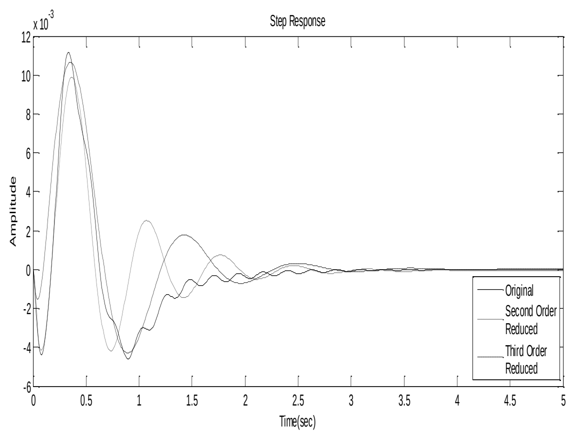

FIGURE 4. Step Responses for θ_1/u

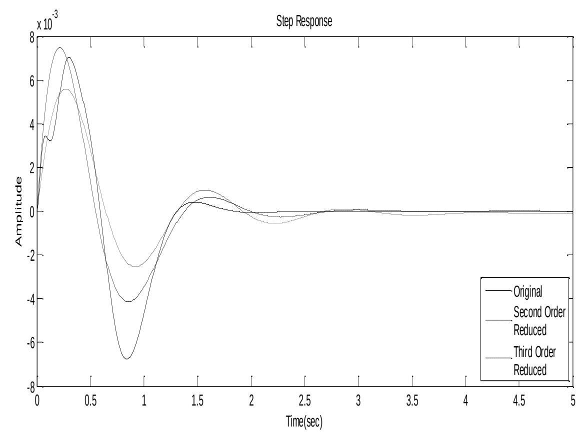

FIGURE 6. Step Responses for θ_3/u

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