International Journal of Civil and Structural Engineering– IJCSE
Volume 2 : Issue 1 [ISSN : 2372-3971]
Publication Date: 30 April, 2015

# The implementation of developed firefly algorithm in multireservoir optimization in continous domain

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***Abstract*—this paper presents the performance of developed firefly algorithm which is formed to be used in the problems where decision variables stem from each other. In this regard, the proposed algorithm so-called "DFAORO" is used to a benchmark multi-reservoir problem in continuous domain. So as to examine the results of DFAORO, linear programing (LP) and standard firefly algorithm (SFA) are applied. Final results based on the value of $R^2$ indicate that DFAORO differs from LP by 0.0015 percent. Whereas SFA is about 37 times worse than DFAORO compared to LP. All in all, it can be asserted that DFAORO has proved more robust and effective.**

***Keywords*—optimization, multireservoirs operation, Firefly algorithm.**

## I. Introduction

Of all the gifts with which God has blessed us, water is the greatest. Therefore, every effort must continue to be made to develop this resource. In this regard, one of the most important steps in developing the usage of water is operation strategies from which obtaining the optimal ones are more popular and controversial.

Given the fact that the issue of optimal reservoir operation is mostly a complex and nonlinear problem, many researchers have tried to use new solving methods called evolutionary or meta-heuristic algorithms (EMA) in the field of reservoir operation optimization recently. Among most known EMA such as pattern search (PS) algorithm, genetic algorithm (GA), simulated annealing (SA) algorithm, tabu search (TS) algorithm, ant colony optimization algorithm (ACO), differential evolution (DE) algorithm, particle swarm optimization (PSO) algorithm, honey-bee mating optimization (HBMO) and so forth, Firefly algorithm (FA) has not been examined in the field of optimal reservoir operation to date. Hence, the four-reservoir benchmark problem is chosen in the present study.

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The roots of the four reservoir benchmark problem can be traced to the work of Chow and Cortes-Rivera (1974) who introduced the problem and solved it by applying LP method. Then, this problem was suited by Murray and Yakowitz (1979) that used differential dynamic programming (DDP). Also, this problem can be found on the work of Wardla and Sharif (1999) who applied GA to solve the problem. Recently, the aforesaid problem has been determined by Bozorg Haddad et al. (2011) who proposed the HBMO algorithm and compared the results of LP, GA and HBMO.

Yang (2007) developed SFA in Cambridge University for the first time. After that, in order to show the capability of SFA, Yang applied his proposed algorithm in various optimization test problems some of which can be found in what follows. Yang (2009) applied GA, PSO and SAF to solve 10 multi optimization test problems. He stated that the success rate of SFA in obtaining the global optimum solution was more that GA and PSO. Moreover, Yang (2010a) tried to improve the performance of SFA by entering the searching approach of levy flight in order to solve multi optimization test problems. Although Yang reported the great performance of SFA in his studied, Yan et al. (2012) developed adaptive FA (AFA) for the problems which are of many decision variables. They stated that considering the amount of attractiveness parameter as a constant value is not proper and showed the better performance of AFA toward SFA. Also, many studies have been conducted to improve the searching accuracy of SFA. All in all, the advantages of SFA and other improved forms of SFA in terms of convergence speed caused it to be used in complex and nonlinear problems in many different science fields.

Since SFA has not been used as a new technique for multi-reservoir optimization as of yet, the potential of developed SFA in terms of its formulations for multi-reservoir operation problems is arguably needed to be examined. Thus, the present study was conducted to address the implementation of developed SFA named DFAORO for the aforesaid problems. It is noted that the mentioned problems have special characteristic due to which DFAORO is provided and is basically appropriate for these kind of problems. In addition, so as to examine the performance of DFAORO, SFA and LP methods are used. The comparisons between the results of LP, SFA and DFAORO are shown as convergence curves and time series of releases plots. Moreover, to better understanding the graphical results, three statistical criteria are used according to which the solution reported by these methods can be checked to show the effectiveness and efficiency of DFAORO versus SFA.

# II. Methodology

## A. SFA

SFA is one of the meta-heuristic methods inspired from the natural behavior of fireflies. In this regard, the fireflies with lower light intensity (attractiveness) move toward those with higher light intensity. The details and basic theory of SFA can be found completely in the studies of Yang (2009), Yang (2010a) and Yang (2010b).

In general, SFA process is conducted through some equations as follows: The attractiveness of fireflies $\beta$ is computed by (1), the distance $r$ between two fireflies which are located in positions $x_i$ and $x_j$ is calculated by (2) and the new position of fireflies after moving is defined as (3).

$$\beta = \beta_0 e^{-\gamma r^2}. \tag{1}$$

in which $\beta$ and $\beta_0 =$ firefly's attractiveness and the attractiveness at a distance of $r = 0$, respectively, $\gamma =$ light absorption coefficient and $r$ distance between two fireflies.

$$r_{ij} = \|x_i - x_j\| = \sqrt{\sum_{k=1}^{d}(x_{i,k} - x_{j,k})^2}. \tag{2}$$

in which $r_{ij} =$ Cartesian distance between fireflies $i$ and $j$, $\| \; \| =$ distance vector between fireflies $i$ and $j$, $x_{i,k} =$ and $x_{j,k} =$ $k$ th dimension of the spatial coordinate of the $i$ th firefly's position and $j$ th firefly's position; respectively, and $d =$ number of dimensions (decision variables).

$$x_{i_{new}} = x_i + \beta_0 e^{-\gamma r_{ij}^2}(x_j - x_i) + \alpha(rand - 0.5). \tag{3}$$

in which $x_{i_{new}}$ and $x_i =$ new position of firefly $i$ with the less brightness and current position of firefly $i$ with less brightness, respectively; $x_j =$ position of firefly $j$ with more brightness; $\beta =$ attractiveness of firefly, $\alpha =$ a randomized parameter; and $rand =$ a randomized value in the range [0, 1].

According to the above equations, it is obvious that SFA is of three important parameters whose values can affect strongly on the performance of SFA. Tuning the values of these parameters which cause to obtain the best solution is sometimes a time consuming and onerous process in some problems spatially those of which have lots of decision variables. Moreover, SFA has some limitations which call it into the question whether it can be effectively used in some problems like the mentioned problem of this paper or not.

## B. DFAORO

To overcome some limitations of SFA in solving problems like optimization operation of reservoir systems, DFAORO which takes account of five important remarks is proposed in the present study as follows.

Tip 1. Unlike SFA in which the value of $\beta$ is calculated by (1), it is suggested that the value of $\beta$ should change in the range of [0, 1]. Thus, the terms of (1) needs to be changed in order to provide the value of $\beta$ in the mentioned range.

Tip 2. In SFA, the process of reforming the position of fireflies is done only for those which are lower in terms of light intensity. In other words, if two fireflies are of the same light intensity, no movement is conducted. Whereas in DFAORO it has been considered.

Tip 3. The most important development of SFA is this tip in which the randomization movement defined in (3) is changed. As in optimization operation of reservoir problems the decision variables are dependent to each other, a little change of one decision variable can cause many changes in next variables. Thus, a little change of one variable can cause great change in the value of evaluation function. Therefore it is better to provide a chance for the algorithm to produce different rang of numbers from small to large which will definitely decrease in amounts during the process. In other words, contrary to SFA which the randomization movement is done identically on all the variables, in DFAORO the variables are categorized randomly and then for each category a separate range of numbers is defined. In this respect, there would be a chance of producing both large and small random numbers in each iteration of DFAORO.

Tip 4. Another important tip of developing is related to the mathematical definition of the distance between two fireflies defied in (2) previously. As the amount of decision variables in reservoir operation problem (release) are generally between 0 to a three-digit number, the amount of $r_{ij}$ calculated by (2) would be very large and this will cause the term $e^{-\gamma r_{ij}^2}$ to be approximately equal to 0 regardless of whether the amount of $\gamma$. Thus, according to the type of the four-reservoir operation problem in which the objective function is to maximize the benefits, the definition of $r_{ij}$ in DFAORO, is presented as (4).

$$r_{ij} = \frac{EF_j - EF_i}{EF_j}. \tag{4}$$

in which $EF_j$ and $EF_i$ are the objective function of fireflies $i$ and $j$, respectively. It is noted that firefly $i$ is better than $j$ in terms of the evaluation function value.

Tip 5. The final development is related to the process in which the last sorted solutions of each iteration that will be used in the next iteration are chosen uniquely. In other words, the same and the duplicated solutions are chosen once. This process will provide more different solutions for the nest iterations. Thus the quick attenuation is prevented.

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### C. Statistical Criteria

In order to measure and examine the results of SFA and DFAORO toward LP, three statistical criteria are used in this study as follows.

$$R=\frac{\sum_{t=1}^{T}\left(\left[ReLP_t-\overline{ReLP}\right]\times\left[ReMH_t-\overline{ReMH}\right]\right)}{\sqrt{\sum_{t=1}^{T}(ReLP_t-\overline{ReLP})^2\times\sum_{t=1}^{T}(ReMH_t-\overline{ReMH})^2}}. \quad (5)$$

$$NSE=1-\frac{\sum_{t=1}^{T}(ReLP_t-ReMH_t)^2}{\sum_{t=1}^{T}(ReLP_t-\overline{ReLP})^2}. \quad (6)$$

$$RMSE=\sqrt{\frac{\sum_{t=1}^{T}(ReLP_t-ReMH_t)^2}{T}}. \quad (7)$$

in which $R=$ correlation coefficient, $NSE=$ Nash-Sutcliffe efficiency, $RMSE=$ root mean square error, $ReLP=$ the release of LP, $\overline{ReLP}=$ the average of the releases obtained from LP, $ReMH=$ the release of meta-humoristic methods and $\overline{ReMH}=$ the average of releases obtained from meta-humoristic methods.

## III. Case study

### A. Four-reservoir benchmark problem

So as to determine the performance of DFAORO in the problem of optimal operation of reservoirs systems, a four-reservoir operation problem in continuous domain is applied.

The aim of the aforesaid problem is to maximize the benefit stemmed from the release of the four-reservoir system considering some constrained which are provided with the following equations. The objective function is computed by (8).

$$Maximize\ \ B=\sum_{r=1}^{nRes}\sum_{t=1}^{T}b_r(t)Re_r(t). \quad (8)$$

in which $B=$ the objective function, $r=$ the number of reservoirs, $nRes=$ the total number of reservoirs, $t=$ the number of operational period, $T=$ the total number of periods, $b_t(t)=$ the benefit function of reservoir $r$ in period $t$ and $Re_r(t)=$ the release of reservoir $r$ in period $t$.

The most important constraint of reservoir operation problems is continuity equation which is defined as (9).

$$S_r(t+1)=S_r(t)+Q_r(t)+RCM_{n\times n}Re_r(t). \quad (9)$$

in which $S_r(t+1)=$ reservoir storage volume of reservoir $r$ in period $t+1$, $S_r(t)=$ reservoir storage volume of reservoir $r$ in period $t$, $Q_r(t)=$ monthly inflow volume of reservoir $r$ in period $t$ and $RCM_{n\times n}$ matrix of reservoirs connections indices. Moreover, other constraints are presented in (10) to (13).

$$Re_{\min_r}(t)\le Re_r(t)\le Re_{\max_r}(t). \quad (10)$$

$$S_{\min_r}(t)\le S_r(t)\le S_{\max_r}(t). \quad (11)$$

in which $Re_{\min_r}(t)=$ and $Re_{\max_r}(t)=$ minimum and maximum limits of release of reservoir $r$ in period $t$, respectively, $S_{\min_r}(t)=$ and $S_{\min_r}(t)=$ minimum and maximum limits of storage volumes of reservoir $r$ in period $t$, respectively. Equation (9) denotes the carryover condition which is used in short-term operation.

$$S_r(1)=S_{initial_r}. \quad (12)$$

$$S_r(T+1)>S_{t\arg et_r}. \quad (13)$$

in which $S_{initial_r}=$ the initial storage of reservoir $r$ in the first period of operation and $S_{t\arg etl_r}=$ the target storage f reservoir $r$ in the last period of operation.

Fig. 1 illustrates the schematic of the four-reservoir problem in which the connection of reservoirs are shown. Furthermore, it is evident from Fig. 1 that reservoirs 3 and 4 are not of monthly inflow. Moreover, releases from reservoirs are used for two purposes (1) producing hydropower energy and (2) supplying irrigation demands.

It is noted that the operation period for the aforesaid problem is 12 month.

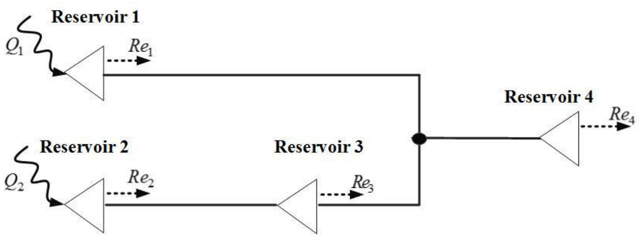


Figure 1. Schematic of the four-reservoir problem

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### B. *Penalty Function*

To consider the constraints of the problem in evolutionary or meta-hubristic methods the strategy of penalty functions should be used.

The combination of penalty functions and objective function is named function evaluation which is described as (14) in the mentioned benchmark problem. It is obvious that the method of applying penalty functions to the objective function is considered additive in this study.

$$EF = B + 40 \times \sum_{p=1}^{P}\sum_{t=1}^{T} Penalty_{p,t} \cdot \quad (14)$$

in which $EF =$ evaluation function, $p =$ the index of penalty, $P =$ the total number of penalties and $Penalty_{p,t} =$ the penalty $p$ in period $t$.

It is noted that penalties for the problem of four-reservoir operation are as follows.

$$Penalty_{1,t} = (S_{\min_r} - S_r(t))^2 \quad (15)$$

$$Penalty_{2,t} = (S_r(t) - S_{\max_r})^2 \quad (16)$$

$$Penalty_{3,t} = (S_{target_r} - S_r(T+1))^2 \quad (17)$$

## IV. Results

In this study, the value of the objective function resulted from LP solver of LINGO 14 is 308.2915.

So as to solve the mentioned problem with SFA and DFAORO, a preliminary sensitivity analysis is conducted for the parameters' values and are reported in Table 1.

According to the results of Table 1 it can be said that the number of function evaluation is 500,050.

TABLE I. RESULTS OF SENSIVITY ANALYSIS OF SFA AND DFAORO PARAMETER

| **Method** | **SFA** | | | | | | |
|---|---|---|---|---|---|---|---|
| **Feature** | ***Number of Decision Variables*** | ***Iteration*** | ***Random Function*** | $\beta_0$ | $\gamma$ | $\alpha$ | $\theta$ |
| **Value** | 50 | 10,000 | Uniform | 2 | 0.1 | 1 | 0.99 |
| **Method** | **DFAORO** | | | | | | |
| **Feature** | ***Number of Decision Variables*** | ***Iteration*** | ***Random Function*** | ***Type of Function*** | ***Number of Class*** | $\beta_0$ | $\gamma$ |
| **Value** | 50 | 10,000 | Proposed | Uniform | 10 | 1 | 5 |
| **Feature** | ***First Values of*** $\alpha_1,\dots,\alpha_{10}$ | $\theta_1$ | $\theta_2,\theta_3$ | $\theta_4,\theta_5$ | $\theta_6,\theta_7$ | $\theta_8,\theta_9$ | $\theta_{10}$ |
| **Value** | 1 | 1 | 0.99 | 0.98 | 0.97 | 0.96 | 0.95 |

The results of evaluation function for five independent runs of SFA and DFAORO are reported in Table 2.

TABLE II. RESULTS OF THE FUNCTION EVALUATION OBTAINED FROM 5 INDEPENDENT RUNS OF SFA AND DFAORO

| **Method** | **Maximum** | **Average** | **Minimum** | **Standard deviation** | **Difference percentage from LP** |
|---|---|---|---|---|---|
| SFA | 306.35 | 305.51 | 304.72 | 0.66 | 0.63 |
| DFAORO | 308.25 | 308.21 | 308.13 | 0.05 | 0.01 |

According to Table 2, it is obvious that the maximum result of DFAORO only defers 0.01 percent from LP while the maximum result of SFA toward LP is 63 times worse than the results of DFAORO toward LP.

Fig. 2 and 3 illustrate the convergence of SFA and DFAORO, respectively. Each figures enjoys three curves which show the maximum, the average and the minimum amount of function evaluation during the iterations. As it can be seen in Fig. 3 and 4, DFAORO has better performance compared to SFA in terms of producing solutions with high variance. Also, according to the results of Table 2 and Fig. 4, it can be stated that DFAORO was more effective in terms of obtaining the better function evaluation amount. Because the curves of Fig. 4 has got closer to the absolute optimal solution (308.2915) compared to the curves of Fig. 3. In addition, the curves of Fig. 4 enjoy more convexity than those of Fig. 3. Finally, it can be said that the curves in Fig. 4 are converging faster than SFA. It is worth mentioning that SFA has been converged approximately at the number of function evaluation 10,000. Whereas DFAORO has been converged at 1,000.

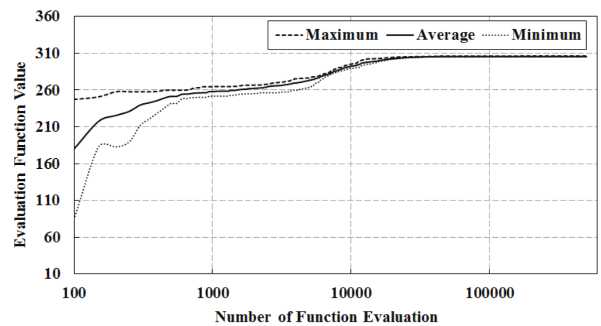


Figure 2. The convergence curve of SFA

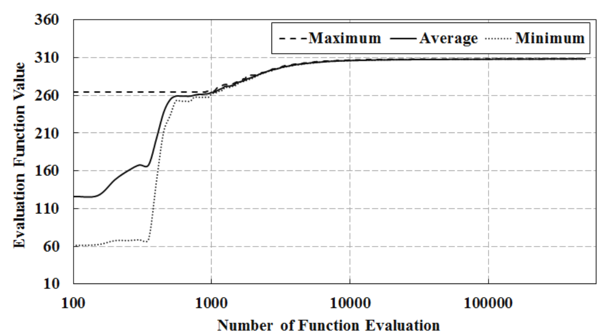


Figure 3. The convergence curve of DFAORO

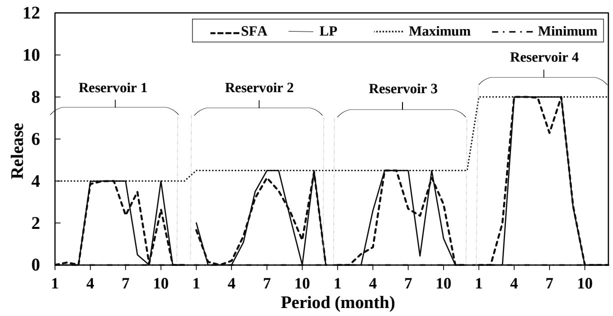


Figure 4. Example of a figure caption. *(figure caption)*

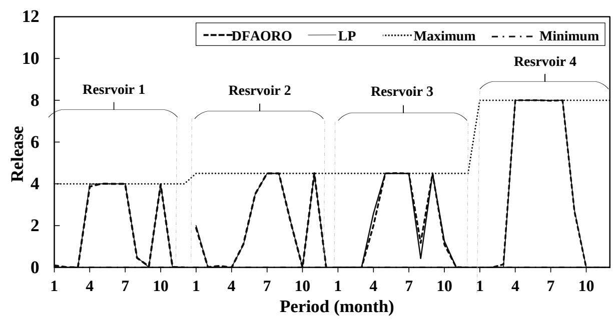


Figure 5. Example of a figure caption. *(figure caption)*

Figures 4 and 5 show the result of release of the four-reservoir system obtained from SFA and DFAORO, respectively, along with the results of release obtained from LP. It is evident that the releases of SFA and DFAORO are in the range of the minimum and maximum limitations. However, the releases of DFAORO shown in Fig. 5 are more similar to the release reported from LP compared to the releases of SFA in Fig. 4. In other words, concerning Fig.4 it can be mentioned that the results of SFA for reservoir 1, 2 and 3 differs remarkably from the results of LP.

So as to better understanding of the results, Table 3 is presented which provides the amounts of the statistical criteria mentioned in this paper.

TABLE III. RESULTS OF SENSIVITY ANALYSIS OF SFA AND DFAORO PARAMETER

| **Method** | ***Statistical Criteria*** | ***Number of Reservoir*** | | | | ***Whole Four-reservoir System*** |
|---|---|---|---|---|---|---|
| | | ***1*** | ***2*** | ***3*** | ***4*** | |
| **SFA** | $R^2$ | 0.7038 | 0.9438 | 0.7289 | 0.9631 | 0.9450 |
| | *NSE* | 0.7005 | 0.9291 | 0.7276 | 0.9608 | 0.8906 |
| | *RMSE* | 0.0202 | 0.0278 | 0.0470 | 0.0806 | 0.8707 |
| **DFAORO** | $R^2$ | 0.9996 | 0.9996 | 0.9809 | 0.9998 | 0.9985 |
| | *NSE* | 0.9994 | 0.9996 | 0.9808 | 0.9998 | 0.9970 |
| | *RMSE* | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.1431 |

As in Table 3, the statistical criteria according to which DFAORO is asserted to be more efficient than SFA are calculated not only for each reservoir separately but also for the whole four-reservoir system.

# V. Conclusions

This paper proposed a developed version of SFA, named DFAORO, which is adjusted for the optimization operation of reservoir problems. Concerning the special characteristics of reservoir operation problems the most important of which is the relation between decision variables, the authors attempted to show the disability of SFA in solving such problems efficiently. The results shows that DFAORO has proved effective in the problems of optimal operation of reservoirs.

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