

# Analysys of the Ramp Profile Functions of Buried Asymmetric Dielectric Target for Non-Axial or Oblique Incidence of the EM Wave.

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**Abstract**— The object of this study is to analyze the ramp profile functions of buried asymmetric dielectric targets for non-axial or oblique incidence of the electromagnetic wave.

This approach can be used, with due limitations, in satellite SAR radar to scan large areas of land on which it intends to identify buried metallic or dielectric objects (such as landmines, etc..).

The results contained in this dissertation are based on (a scattering computation using a Method of Moments (MoM) solutions with SCILAB simulation tool. The targets used in the simulation include metallic and dielectric spheres and rectangular metallic and dielectric blocks.

**Keywords**—ramp function, buried object, oblique incidence, asymmetric target.

## I. Introduction

He in [1] proposed a theoretically exact approach to creating a two-dimensional image of a buried dielectric scatterer by reconstructing the induced surface currents on the scatterer surface. He, using multiple plane wave illuminations, reconstructed distinctive images of a buried dielectric cylinder and a buried rectangular air cavity from theoretically simulated data. Application of this imaging technique to discriminate buried targets from experimentally measured radar data could be informative. However, this technique of image reconstruction could be time consuming if one considers that ultimately large areas of land need to be cleared, and some estimates in the literature give false alarm rate of the order of 100:1. Also, the incidence angle of the backscattered fields (traveling from the ground into air) on the ground-air interface at the critical angle will restrict the possible target look angles. Chen [2] has demonstrated that as the incidence angle is changed from normal incidence to the air-ground interface, the time duration of the clutter from even mild rough surfaces is increased so that the buried target scattering is masked by the surface scatter. These difficulties would pose limitations on the applications of the technique in [1].

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Starting from a first order Kirchoff's approximation or Physical Optics approximation, Lewis [3] proposed an approach based on spatial transform to generate the size and shape of the target, the backscattered fields need to be measured at all frequencies and all incident angles, which is not easily feasible.

## II. Ramp Response

### A. Size and shape of the target

The ramp response signature overcomes these limitations. A ramp signature obtained from a single backscattered frequency response of a target already provides some information about the size and shape of the target, and so extensive measurement can be avoided. This means that a large frequency band is not required to generate a valid ramp response signature of a buried target. Essentially, the ramp response of a target is a signal processing technique. This technique can be applied to the identification of an unknown body in a given medium. The ramp response of a radar target is defined as the far-zone backscattered time domain waveform resulting from illumination by a plane electromagnetic wave with a time domain ramp wave shape. The ramp response gives information about the target size, geometry and orientation.

### B. Kennagh and Moffatt idea

Kennagh and Moffatt [4] showed that the scattered field due to a ramp driving function traces out the cross sectional area profile of a metallic targets as a function of time along the line of sight. Using this concept, Young [5], [6] compared the measured ramp response signature for different metallic targets of simple shapes with the corresponding profile functions. In addition, Young generated low frequency images of conducting targets of simple shapes in free space using the ramp response for only three orthogonal observation angles. The ramp response signature is defined as the far-zone backscattered time domain waveform of a target illuminated by a traveling planar transverse electromagnetic wave whose time dependence is that of a ramp function.

The maximum amplitude of the ramp response gives the approximate maximum value of the transverse physical cross

sectional area of a scatterer in the direction of propagation of the incident wave.

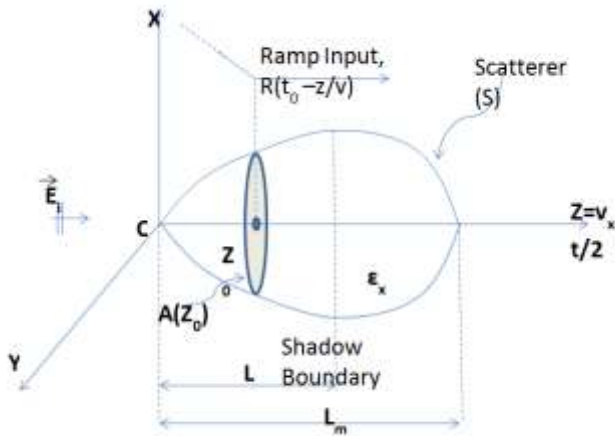


Figure 1 Scattering Geometry of a Target

### III. Analysis of the Ramp Response

The ramp response duration gives an estimate of the scatterer thickness in the direction of EM wave propagation, and the rise time (shape) of the ramp response yields information about the shape of the target in the given direction of incident wave.

If  $f_i(t)$ ,  $f_u(t)$ ,  $f_r(t)$ , are respectively the impulse response, the step response and the ramp response of a system, and  $F_I(s)$ ,  $F_U(s)$ ,  $F_R(s)$  are their corresponding Laplace transforms, then [8]

$$F_R(s) = \frac{1}{s^2} F_I(s) + \frac{1}{s^2} \int_{-\infty}^{0^-} f_i(\tau) d\tau + \frac{1}{s} \int_{-\infty}^{0^-} \left\{ \int_{-\infty}^{\tau} f_i(t') dt' \right\} d\tau \quad (1)$$

Since we know that  $\int_{-\infty}^{0^-} f_i(\tau) d\tau = 0$ ,  $\int_{-\infty}^{0^-} f_u(\tau) d\tau = 0$  and  $f_u(0^-)$ , hence equation (1) reduces to

$$F_R(s) = \frac{1}{s^2} F_I(s) + \frac{1}{s} \int_{-\infty}^{0^-} f_u(\tau) d\tau = \frac{1}{s^2} F_I(s) \quad (2)$$

So the ramp response  $f_r(t)$  is obtained by taking the inverse Fourier transform of a target's backscattered frequency response  $F_I(j\omega)$  weighted by  $\frac{1}{(j\omega)^2}$ .

Young et al [9] observed that since the ramp response of a scatterer has a finite duration, a sequence of ramp excitations in the form of a periodic plane wave is more practical than an isolated ramp function excitation. The periodic ramp excitations should be spaced such that the spacing is larger than the duration of the ramp response waveform.

In Figure 2 is shown a ramp excitation  $R_i(t)$  obtained using a weighted sum of harmonic (c.w.) plane waves. The excitation is obtained with an Inverse Fourier Transform (IFT) of a unit amplitude and zero phase band limited signal weighted by  $1/(j2\pi f_{GHz})^2$ , where  $f_{GHz}$  is the frequency  $f$  expressed in GHz.

$$R_i(t) = \frac{-1}{\pi\omega_0} \sum_{n=1}^{\infty} \frac{1}{n^2} \cos(n\omega_0 t) \quad (3)$$

In order to obtain a valid ramp response of a scatterer, Young observed that the frequency band must include a part of the Rayleigh region (typically  $L_m < \lambda_x/5$ ) (see Figure 1) of the backscattered fields of the scatterer due to a ramp input.

If this is achieved, the scattered fields produced by the ramp excitation is almost independent of frequency as the minimum frequency of the operational frequency band goes to zero. In fact, the backscattered fields could be extended down to minimum frequency  $f_{min} = f_0 = 0$  so that the response would be the one produced by the ramp excitation in Figure 2.

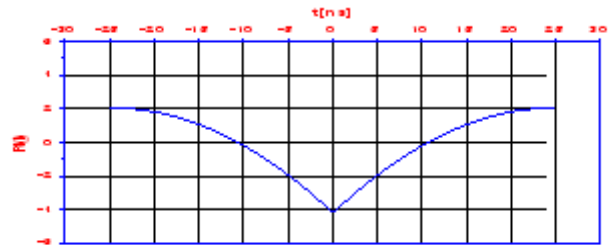


Figure 2 One period of a periodic train of ramp excitation with a fundamental period of T=50ns

#### A. Physical Optics (PO) solution for the ramp response of a target

Kennaugh and Moffatt [4] were the first to discuss the applications of EM field waveforms produced by scattering of transient plane waves from finite objects to obtain target identification. They obtained the PO solutions for the ramp response of metallic targets of simple convex geometry.

The PO approach is valid only for the lit region of the target. Young [5] observed that it is approximately correct for the shadowed region as the ramp discontinuity passes over the target.

Try to consider a plane transverse electromagnetic wave (time harmonic) propagating in  $+\hat{z}$  direction which is incident on a scatterer with the Area of the surface S (Figure 1).

Suppose the incident wave is a ramp waveform both in the domain of time and space, such as:

$$R(z, t) = R\left(t - \frac{z}{v_x}\right) \quad (4)$$

Where  $t$  is time, and  $v_x$  is the velocity of the wave in the ambient lossless medium. In Figure 1 is shown the geometry of the target for a given time instant. the input ramp function is equal to zero for  $z > v_x t$  whereas the input signal is non-zero when  $z < v_x t$ .

The scatterer can be either a perfectly electric conductor or penetrable (dielectric) body with relative permittivity  $\epsilon_{x*}^{rel}$ .

Supposing the scatterer is a PEC (Perfectly Electrical Conducting), using PO, the impulse response approximation for a Metallic target of simple shape is given by [4]:

$$f_i^M(t) = -\frac{1}{\pi v_x^2} \frac{d^2 A\left(\frac{v_x t}{2}\right)}{dt^2} \quad (5)$$

Where  $A(z)$  is the area of the scatterer between the  $xy$  plane and a cutting plane at  $z$ , projected orthogonally on the  $xy$  plane.  $v_x$  is the velocity of propagation in the ambient medium.  $A(z)$  is a monotonic function, having a constant value 0 for  $z \leq 0$  and a limiting value  $A(L)$  for  $z \geq L$ .

We can observe that the impulse response calculated with PO is obtained with a time scale such that the cutting plane which determine  $A(z)$  moves with one-half the velocity of the incident impulsive wave ( $v_x/2$ ), starting at  $t = 0^-$  and reaching the shadow boundary at  $t = 2L/v_x$ . In other words, the equation (5), which defines the ramp response as a function of time for the illuminated portion of a simple shape metallic target, is proportional to the cross sectional area  $A(z)$  along the line-of-sight. This transverse cross sectional area of a target is also known as its profile function.

Using PO, the backscattered impulse response obtained for a dielectric scatterer of simple shape having two finite radii of curvatures, is given by:

$$f_i^D(t) = \frac{1}{\pi v_x^2} \frac{d}{dt} \left[ R_{12,avg} \left( \frac{v_x t}{2} \right) \frac{d}{dt} A \left( \frac{v_x t}{2} \right) \right] \quad (6)$$

Where the polarization of the backscattered field is parallel to that of incident wave.  $R_{12,avg}\left(\frac{v_x t}{2}\right)$  is the average reflection coefficient of the wave incident on the scatterer, and is given by:

$$R_{12,avg}\left(\frac{v_x t}{2}\right) = \frac{1}{2} \left[ R_{12}^\perp \left( \frac{v_x t}{2} \right) + R_{12}^\parallel \left( \frac{v_x t}{2} \right) \right] \quad (7)$$

$R_{12}^\perp\left(\frac{v_x t}{2}\right)$  and  $R_{12}^\parallel\left(\frac{v_x t}{2}\right)$  are Fresnel reflection coefficients corresponding to perpendicular and parallel components of  $\vec{E}_i$  respectively with respect to plane of incidence.

The ramp response is the double integral of the impulse response, that is:

$$f_r^M(t) = \int_{t'=0^-}^t \int_{\tau=0}^{t'} f_i^M(\tau) d\tau dt' \quad (8)$$

Then the ramp response for a metallic target can be obtained from equation (5) as:

$$f_r^M(t) = -\frac{1}{\pi v_x^2} A\left(\frac{v_x t}{2}\right) \quad (9)$$

Whereas, for a dielectric target the ramp response is given by

$$f_r^D(t) = \frac{1}{\pi v_x^2} \int_{0^-}^t R_{12,avg} \left( \frac{v_x t'}{2} \right) \frac{d}{dt'} A \left( \frac{v_x t'}{2} \right) dt' \quad (10)$$

The meaning of the equation (9) is that the backscattered field from a ramp incident wave is proportional to the transverse cross-sectional area corresponding to the time and position on the target such that the argument of the incident ramp function is zero.

For a given time instant  $t_0$  the ramp input at its slope discontinuity point traces out the cross sectional area  $A(z_0) = A(v_x t_0/2)$ . Thus, as the ramp input passes over the scatterer at different time instants we get the transverse cross sectional areas of the body at the corresponding points in the space domain. The equation (6) has been obtained by integrating the PO electric and magnetic surface current distributions  $\vec{J}_i$  and  $\vec{M}_i$  respectively over the scatterer surface S. The ramp response initiates when incident wave illuminates the front interface of the scatterer. Hence these current densities include the reflection coefficients  $R_{12}^{\perp,\parallel}$  of the incident wave hitting the scatterer, and do not include the transmission coefficients and the internal reflection coefficients of the wave inside the penetrable scatterer. The surface current densities used in the PO solution are only approximate and they do not represent the total scattered fields from the dielectric body.

$A\left(\frac{v_x t}{2}\right)$  gets its maximum value at the shadow boundary of the target. Also, from equations (9) and (10) we find that a variation in relative permittivity of external medium ( $\epsilon_x^{rel}$ ) causes a change in the duration of the ramp response because affect the velocity of the EM Wave. But a change in relative permittivity of the target ( $\epsilon_i^{rel}$ ) does not affect the ramp response duration because  $A(z) = A\left(\frac{v_x t}{2}\right)$  is a function of  $v_x$ , which only depends on  $\epsilon_x$ . However the amplitude of the ramp response of a given scatterer depends on the values of ( $\epsilon_x^{rel}$ ) and ( $\epsilon_i^{rel}$ ). In the current analysis, the duration of the ramp response is defined between  $z = 0$  and the first intersection point of the  $A(z)$  curve on the  $z$  axis for  $z > 0$ .

## B. Determination of $A(z)$ of a Scatterer from the backscattered field

For a linear system, the frequency response  $F_I(j\omega)$  and the impulse response  $f_i(t)$  are Fourier transform pairs:

$$f_i(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F_I(j\omega) e^{j\omega t} d\omega \quad (11)$$

$$F_I(j\omega) = \int_0^{\infty} f_i(t) e^{-j\omega t} dt \quad (12)$$

In scattering applications, the frequency response can be defined as the ratio of the scattered field intensity  $E_s(j\omega)$  (Volts/meter) to the incident plane wave field intensity  $E_i(j\omega)$  (volts/meter), multiplied by a range factor, at a far zone observation distance  $r$  from the scatterer:

$$F_I(j\omega) = \frac{1}{v_x} \frac{E_s(j\omega)}{E_i(j\omega)} (e^{j\omega r/v_x}) (2r) \equiv e_s(j\omega) \quad (13)$$

The ramp response  $f_r(t)$  is related to the phasor response  $F_I(j\omega)$  by:

$$f_r(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{F_I(j\omega)}{(j\omega)^2} e^{j\omega t} d\omega \quad (14)$$

The transverse cross-sectional area  $A(z)$  of a PEC scatterer for the given incident fields can be obtained by:

$$A(z) = -\pi v_x^2 f_r^M(t) \quad (15)$$

Now, for a (lossless) dielectric scatterer having a flat interface in the lit region,  $R_{12,avg}(z)$ , given in equation (10) is independent of  $z$  for a given polarization. Then the ramp response, via PO, takes the following simpler form:

$$f_r^D(t) = \frac{1}{\pi v_x^2} R_{12} A \left( \frac{v_x t}{2} \right) \quad (16)$$

$A(z)$ , for a dielectric body having a flat interface in the lit region, can be simply obtained as:

$$A(z) = \pi v_x^2 f_r^D(t) / R_{12} \quad (17)$$

Where  $R_{12}$  is the reflection coefficient from a flat dielectric interface for a given linear polarization of the incident wave. Here, will be presented the ramp response signatures of PEC and lossless dielectric targets obtained from numerically computed and experimentally measured backscattered frequency responses.

## C. Ramp Response of a Pec Sphere

Now we consider the ramp response of a PEC sphere embedded in free space for plane wave incidence. For a sphere of diameter  $d$ ,  $A(z)$  is given by:

$$A(z) = \begin{cases} 0 & \text{for } z < 0 \\ \pi \left[ \left( \frac{d}{2} \right)^2 - \left( \frac{d}{2} - z \right)^2 \right] & \text{for } 0 \leq z \leq d \\ 0 & \text{for } z > d \end{cases} \quad (18)$$

In Using equation (5), (9) and (21), we can easily calculate the impulse response and the ramp response of a PEC sphere given by PO:

$$f_i^M(t) = \frac{1}{2} - \frac{d}{2v_x} \delta(t) \quad (19)$$

for  $0 \leq t \leq 2d/v_x$ .

$$f_r^M(t) = \frac{-4}{v_x^2} \left[ \left( \frac{d}{2} \right)^2 - \left( \frac{d}{2} - \frac{v_x t}{2} \right)^2 \right] \quad (20)$$

for  $0 \leq t \leq 2d/v_x$ .

Figure 4 shows the plot of the transverse cross sectional area  $A(z)$  as a function of  $z$  for the 76.2mm diameter PEC sphere in air, obtained from the exact backscattered field solutions.

From Figure 3 and Figure 4, we may observe that the ramp response due to exact solution is in excellent agreement with the desired waveform obtained from PO through the lit region of the target. Soumya et al. [7] showed that  $A(z)$  of the dielectric sphere obtained from the ramp response is in excellent agreement with that of the PEC sphere and the physical profile function especially up to the shadow boundary. Thus, using  $R_{12}(0)$  instead of  $R_{12}(z)$  in equation (10) enables us to estimate the transverse cross sectional area of a dielectric body, with two finite radii of curvatures, embedded in a less dense dielectric medium.

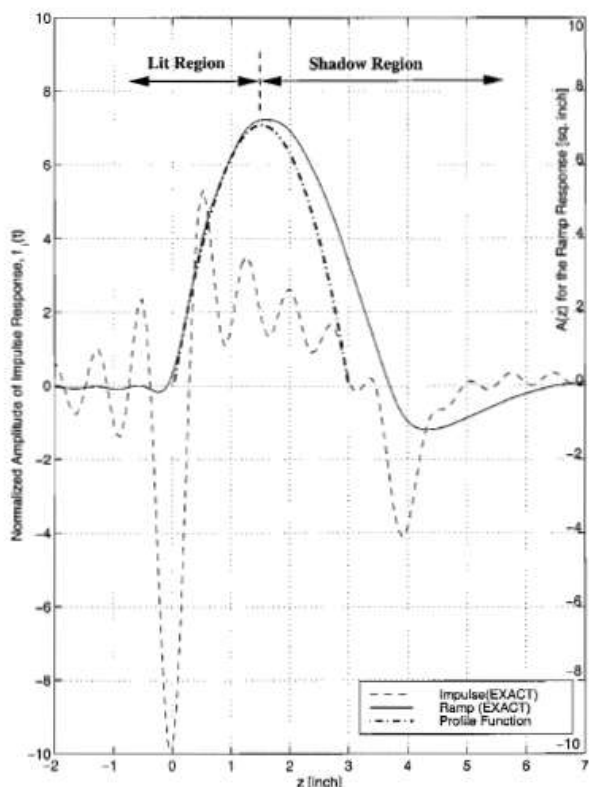


Figure 3 The Plots of  $A(z)$  obtained from the backscattered fields, the profile function, together with the normalized impulse response of a 76.2mm diameter PEC Sphere in free space.

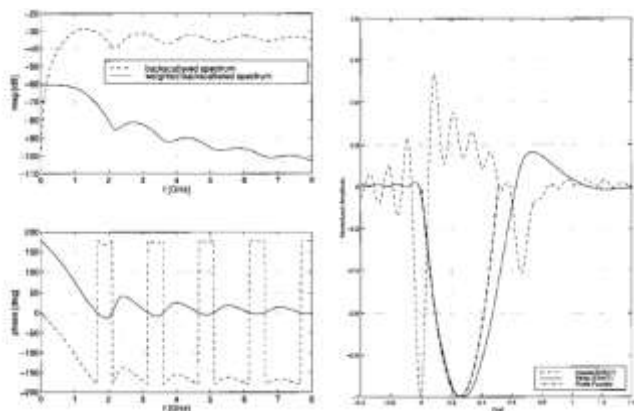


Figure 4 Impulse and Ramp waveforms, together with their corresponding frequencies responses, of a 76.2mm PEC sphere in free space for 0.02-8.02GHz

The ramp profile functions of a conducting and dielectric spheres and a dielectric cube (for normal incidence) immersed in free space have been obtained from the numerically computed backscattered electric fields.

#### IV. Non-Axial or Oblique Incidence of the EM Wave in case of a dielectric Sphere

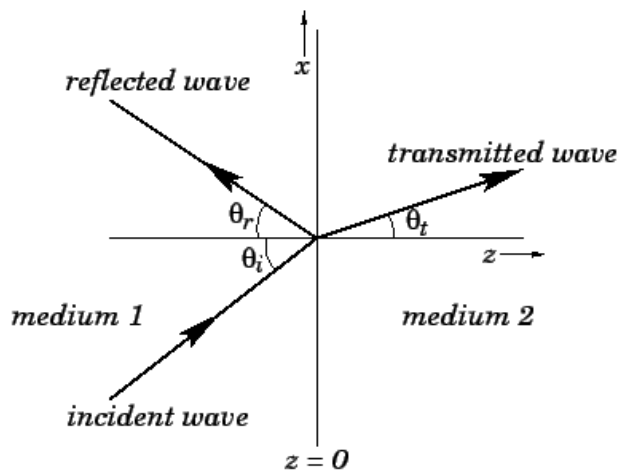


Figure 5 Oblique incidence plane wave

Let us now consider the case of incidence oblique to the boundary (see Figure 1). Suppose that the incident wave subtends an angle  $\theta_i$  with the normal to the boundary, whereas the reflected and transmitted waves subtend angles  $\theta_r$  and  $\theta_t$  respectively. The incident wave can be written:

$$E(r, t) = E_i e^{i(k_i r - \omega t)} \quad (21)$$

$$B(r, t) = B_i e^{i(k_i r - \omega t)} \quad (22)$$

Where  $E_i$  is the incident electric field, and  $E_r$  and  $E_t$  are respectively the reflected and transmitted EF. Now, it is convenient to define the parameters

$$\alpha = \frac{\cos \theta_t}{\cos \theta_i} \quad (23)$$

$$\beta = \frac{v_1}{v_2} = \frac{n_2}{n_1} \quad (24)$$

Where  $v_1 = c/n_1$  is the phase-velocity in medium 1 and  $n_1$  the refractive index. Using PO we can obtain that

$$E_r = \left( \frac{1-\alpha\beta}{1+\alpha\beta} \right) E_i \quad (25)$$

$$E_t = \left( \frac{2}{1+\alpha\beta} \right) E_i \quad (26)$$

the associated coefficients of reflection and transmission take the form:

$$R = \left( \frac{\alpha-\beta}{\alpha+\beta} \right)^2 \quad (27)$$

$$T = \alpha\beta \left( \frac{z}{\alpha+\beta} \right)^2 \quad (28)$$

Note that at oblique incidence the Fresnel equations for the wave polarization in which the electric field is parallel to the boundary are different to the Fresnel equations for the wave polarization in which the magnetic field is parallel to the boundary. This implies that the coefficients of reflection and transmission for these two wave polarizations are, in general, different.

### Conclusions

Now considering that for a dielectric target the ramp response is given by:

$$f_r^D(t) = \frac{1}{\pi v_x^2} \int_{0^-}^t R_{12,avg} \left( \frac{v_x t'}{2} \right) \frac{d}{dt'} A \left( \frac{v_x t'}{2} \right) dt'$$

And from (27) we can put  $R_{12,avg} = R$  which is dependent from  $\alpha$  and  $\beta$  that are non dependent from  $z$ .

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