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A Modified Adaptive Variable Step Size NLMS Adaptive Filtering Algorithm for Rayleigh Fading Environments

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***Abstract*— This paper presents an adaptive filtering solution for multipath Rayleigh fading channel which is based upon adaptive variable step size parameter. The algorithm is derived from normalized least mean square (NLMS) algorithm to make the step size parameter adaptive of NLMS filter which provides good tracking performance and fast convergence with respect to time varying channel. The simulation results show that the fast convergence as well as low probability of error leads to better signal to noise (SNR) ratio when it is compared to conventional NLMS and Bi-LMS algorithms.**

Keywords: Adaptive filter, NLMS algorithm, fast convergence rate.

I. Introduction

Line of sight signal is necessary for the sake of good radio communication systems. However, in bad urban environments where the line of sight signal is not available due to high rise buildings and the signal receives at receiver through multipath, Which occurs due to the scattering and reflecting phenomenon of the environmental effect between the transmitter and receiver and this can be best modeled by Rayleigh Distribution.

In multipath propagation environment, due to high data rate communications the multiple copies of the same transmitting signal induces delay and interfere with the next coming symbols which causes inter-symbol interference (ISI) which decreases the efficiency of the communication system. Furthermore, band limited channel is another factor that make the cause of ISI.

In order to overcome the ISI from the received signal, various techniques have been proposed in the literature like pulse shaping filters, raised cosine and equalizers [1], [2], [3].

Among these techniques, Finite Impulse Response (FIR) based adaptive equalizer is the better approach according to time varying channel conditions and being a hot research topic due to its simplicity. The performance of adaptive filters is highly dependent on the convergence rate. Thus, this performance parameter has to be considered during the formulation of adaptive filter. Moreover, this may be considered as a tradeoff between the efficiency requirement and complexity, depending upon the needs of the application requirement. In the literature, the gradient based adaptive filters [4], [5], [6] provide the low complexity but the convergence is strictly dependent on the selection of the step size parameter. Moreover, in leaky Least Mean Square (LMS) adaptive filter [7], by using the leak variable to speed up the performance of the filter but still the convergence of the filter is dependent on the step size parameter with the knowledge of output noise variance. Likewise, in bi scale LMS [8] adaptive filter, the step size is dependent on the information of non adaptive noise, signal and interference power which is the time overwhelming process where the channel impulse response changes abruptly due to non stationary behavior of the channel.

In this paper, a modified adaptive variable step size normalize least mean square (AVSS-NLMS) adaptive filtering algorithm introduced which utilizes fast convergence rate with respect to time invariant channel conditions. Furthermore, the convergence performance of modified algorithm is dependent on the adaptive selection of step size parameter. The simulation results show that the proposed algorithm provides fast convergence rate and utilizing low training sequences than the conventional algorithms; named NLMS and Bi-LMS.

The rest of the paper is organized in the following manner.

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In Section II, the modified algorithm is described. In Section III, simulation results are presented and Section IV draws the conclusions.

II. Modified Adaptive Variable Step Size NLMS (VSS-NLMS) For Channel Equalizer

The least mean square error optimization problem at iteration n is

$$\underset{min}{\arg}\ \boldsymbol{e} = \left[d(n) - \boldsymbol{w}^T(n)\boldsymbol{x}(n) \right]^2 \tag{1}$$

Where $d(n)$ is the desired signal and $x(n)$ is the equalizer input signal vector and $w(n)$ is optimal coefficients of filter. The proposed adaptive variable step size normalize least mean square (AVSS-NLMS) algorithm is derived on the bases of the update filter weight equation of fixed step size normalized least mean square (FSS-NLMS) algorithm [9]. The following relations belong to FSS-NLMS algorithm is

$$e(n) = d(n) - \boldsymbol{x}^T(n)\boldsymbol{w}(n-1) \tag{2}$$

$$\boldsymbol{w}(n) = \boldsymbol{w}(n-1) + \frac{\mu\, \boldsymbol{x}(n)e(n)}{\boldsymbol{x}^T(n)\boldsymbol{x}(n)} \tag{3}$$

The constant μ is used to denote the parameter step size of the algorithm, therefore, the value of μ [9] must be between $0 < \mu < 2$. The selection of μ in NLMS is still the promising task like FSS-LMS and other fixed point adaptive filters in non-stationary environments where the channel response changes abruptly. For adaptive step size selection of $\mu(n)$, rearrange the Eq. (3) in different form becomes

$$\boldsymbol{w}(n) = \boldsymbol{w}(n-1) + \frac{\boldsymbol{x}(n)e(n)\mu(\mathrm{n})}{\boldsymbol{x}^T(n)\boldsymbol{x}(n)} \tag{4}$$

The posterior error can be defined as

$$\varepsilon(n) = d(n) - x^T(n)w(n) \tag{5}$$

Substitute eq. (4) into eq. (5) in such a way that the posteriori error $\varepsilon(n)$ that is the combined effect of Inter-symbol Interference (ISI) and AWGN noise could be noticeable

$$\varepsilon(n) = d(\mathrm{n}) - \boldsymbol{x}^T(n)\left[\boldsymbol{w}(n-1) + \frac{\boldsymbol{x}(n)e(n)\mu(\mathrm{n})}{\boldsymbol{x}^T(n)\boldsymbol{x}(n)} \right] \tag{6}$$

$$\varepsilon(n) = \left(1 - \mu(n)\right)e(n) \tag{7}$$

To find the adaptive time varying step size parameter $\mu(n)$, take the expectation and squaring on both sides of Eq. (6)

$$[1-\mu(n)]^2\, \mathrm{E}\left[e^2(n)\right] = \mathrm{E}\left[\varepsilon^2(n)\right] \tag{8}$$

$$\mu(n) = \left[\left(1 - \sqrt{\frac{\mathrm{E}\{\varepsilon^2(n)\}}{\mathrm{E}\{e^2(n)\}}} \right) \right] \tag{9}$$

Where, E(.) is the expectation operator. Now, it seems to be evaluated the eq. (9) in terms of power estimate

$$\mu(n) = \left[\left(1 - \sqrt{\frac{\sigma_\varepsilon^2(n)}{\sigma_e^2(n)}} \right) \right] \tag{10}$$

Where, $\sigma_\varepsilon^2(\mathrm{n})$ and $\sigma_e^2(\mathrm{n})$ are the power estimate of noise plus interference and error, respectively. The power estimate of noise plus interference can be finding by taking the difference of desired signal $d(n)$ and received signal $x(n)$.

$$\sigma_\varepsilon^2(n) = \left(d(n) - x(n)\right)^2 \tag{11}$$

However, the variable in the denominator of Eq. (10) would be computed in a recursive manner [10]

$$\sigma_e^2(n+1) = \alpha\sigma_e^2(n) + (1-\alpha)e^2(n) \tag{12}$$

The value of α could be founded again by using NLMS algorithm by utilizing L_1 norm of the received signal vector to make the process adaptive.

$$\left\|\boldsymbol{w}(n) - \boldsymbol{w}(n-1)\right\|_1 = \left\| \frac{\alpha\, \boldsymbol{x}(n)e(n)}{\boldsymbol{x}^T(n)\boldsymbol{x}(n)} \right\|_1 \tag{13}$$

Impose a time varying upper bound δ_n on the error weight vector

$$\left\|w(n) - w(n-1)\right\|_1 \le \delta_n \tag{14}$$

Select the values of δ_n and α in such a way so that the condition would be satisfied

$$\delta_n = \frac{1}{e(n)} \tag{15}$$

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$$\alpha = \left\| \frac{\mathbf{x}^T(n)\mathbf{x}(n)}{\mathbf{x}(n)e^2(n)} \right\|_1 \tag{16}$$

The selection of α is must be the minimum of $\min\left(\left\| \frac{\mathbf{x}^T(n)\mathbf{x}(n)}{\mathbf{x}(n)e^2(n)} \right\|_1, 1\right)$.

Whereas, in the minimum selection criterion, α is dependent on the square of the error, as the error increases α becomes one and the recursive update in Eq. (12) is dependent on square of error, furthermore, as the error decreases the recursive update is dependent on the previous value. The AVSS-NLMS algorithm is summarized in Tabel.1.

$Initilize: \mathbf{w}(n-1),\ \sigma_\eta^2(n),\ \mu(n),\ \alpha$
$e(n) = d(n) - \mathbf{x}^T(n)\mathbf{w}(n-1)$ $\mathbf{w}(n) = \mathbf{w}(n-1) + \frac{\mu(n)\mathbf{x}(n)e(n)}{\mathbf{x}^T(n)\mathbf{x}(n)}$ $\sigma_\varepsilon^2(n) = \left(d(n) - x(n)\right)^2$ $\mu(n) = 1 - \sqrt{\frac{\sigma_\varepsilon^2(n)}{\sigma_e^2(n)}}$ $\alpha = \min\left(\left\| \frac{\mathbf{x}^T(n)\mathbf{x}(n)}{\mathbf{x}(n)e^2(n)} \right\|_1, 1\right)$ $\sigma_e^2(n+1) = \alpha\sigma_e^2(n) + (1-\alpha)e^2(n)$

Table 1. Implementation of modified AVSS-NLMS algorithm

III. Simulation Results

The simulation results are performed in this section to analyze the performance of proposed modified adaptive variable step size NLMS algorithm. The major concerns are fast convergence and low probability of error provided by the modified algorithm. In the simulations BPSK signal is used as the input signal by considering 30 dB signal to noise ratio. The channel is modeled by 5th order FIR while the filter is modeled by 6th order. Under these conditions the performance of modified AVSS-NLMS in terms of probability of error is compared with FSS-NLMS and Bi-LMS which is clearly shown in Fig. 1. The probability of error of our proposed modified AVSS-NLMS algorithm provides low probability of error at low SNR values. Furthermore, Fig.2 shows the mean square error (MSE) comparison and it is realized that the proposed modified algorithm provides better MSE than FSS-NLMS and Bi-LMS adaptive filtering algorithms.

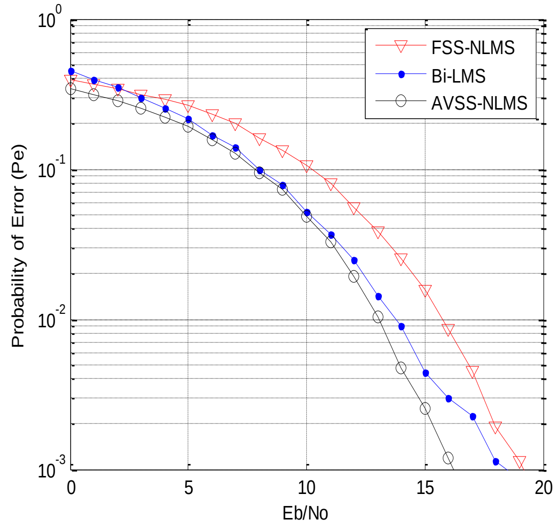

Fig.1 Probability of error vs Eb/No for difference SNR values

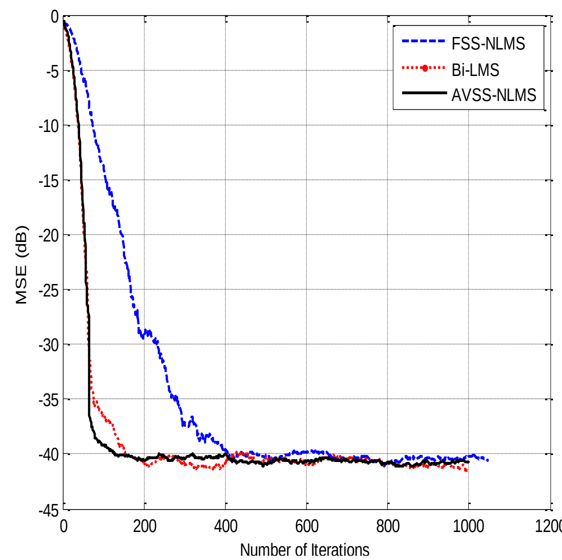

Fig.2 Mean square error vs number of iterations at 30dB SNR

IV. Conclusions

The core objective of this paper is to achieve a adaptive variable step size in FSS-NLMS algorithm which utilizes fast convergence rate in the Rayleigh Fading Environment. In view of simulation results, the modified AVSS-NLMS derived in context of NLMS that uses short sequence to adjust the filter coefficient according to time varying environment. As it shows that the presented adaptive algorithm shows fast tracking capabilities and low probability of error when compared to the existing FSS-NLMS and Bi-LMS adaptive filtering algorithm.

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