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Yield Line Analysis of Rectangular Slabs by Finite Element Method

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Abstract— **This paper presents the Finite Element Method (FEM) for analyzing the failure pattern of rectangular slab with various edge conditions. The convergence of maximum central deflection in square and rectangular plates with the improvement in fineness of mesh has been carried out. Comparison has been made between FEM (ANSYS software) results and yield line theory Classical results.**

Keywords— Rectangular Slab, ANSYS, Classical Approach1.

I Introduction

Yield line is defined as a line in the plane of the slab across which reinforcing bars have yielded and about which excessive deformation (plastic rotations) under constant ultimate moment, continues to occur leading to failure. Yield lines are the narrow zones of localized yielding that occur as a result of this plastic behavior. The yield-line method may be used to predict the collapse load of slabs and plates. It is applicable to steel plates and under-reinforced concrete with uniform distribution of reinforcement. The yield line method is an upper bound approach to limit analysis of reinforced concrete slab systems. The failure mode is arrived at on the basis of certain assumptions such as failure is due to complete yielding of reinforcing steel along the yield lines and the yield line occurs on the tension face, slab deforms**.** The two primary methods employed to solve for the unknown dimensions defining the actual yield pattern are analysis by the principle of virtual work or analysis using the equations of equilibrium. Displacements will generally be the plastically but the individual segments behave elastically, plastic deformations are much greater than the elastic deformations which can be considered as negligible***Dr*** most ***1.*** accurate response quantity computed and will converge faster than stresses, with the exception of some elements derived with hybrid stress formulation of the plate. It is worth noting that the largest error in the entire process is often in the boundary conditions. Objective of this paper is providing an alternative technique for finding failure pattern by FEM (ANSYS software). Some of the work already commenced on yield line theory are by Famiyesin et al. (2001), Kumar and Prakash (2001) on ultimate load of two way rectangular Reinforced concrete (RC) slabs. Phuvoravan and Sotelino (2005) have proposed a new finite element for the non linear analysis of reinforced concrete RC slabs, Zhang and Zhu (2010) developed a simple shear-flexible rectangular layered Fibre reinforced polymer-reinforced concrete slab element, Ibrahim et al. (2011) performed numerical simulations using ANSYS to study the response of waffle slabs with and without openings.

II Methodology

Usual FEM equilibrium equation has been used in ANSYS software for finding the failure pattern and analysis of the plate problems.

A Description of elements used in ANSYS software

SHELL 63 and SOLID 65 elements have been used in analysis of the plates. Description of the elements is given below

SHELL 63: SHELL63 has both bending and membrane capabilities. Both in-plane and normal loads are permitted. The element has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z-axes. Stress stiffening and large deflection capabilities are included.

SOLID65 (3-D Reinforced Concrete Solid): Element Description
SOLID65 is used for the 3-D modeling of solids with or without reinforcing bars (rebar). The solid is capable of cracking in tension and crushing in compression. In concrete applications, for example, the solid capability of the element may be used to model the concrete while the

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rebar capability is available for modeling reinforcement behavior. The element is defined by eight nodes having three degrees of freedom at each node: translations in the nodal x, y, and z direction.

III Numerical Examples

A Steel plate

FEM (ANSYS software) has been used in simply supported steel plate with following dimensional parameters.

Side of the square steel plate; a = 3m, thickness of the steel plate; h = 10mm, Poisson's ratio; $\nu = 0.3$, uniformly distributed load; $q = 500\ N/m^2$. The maximum out plane deflection by classical theory in above mentioned plate is 8.977 mm. Validation check for selecting the proper element has been performed and SHELL 63 has been selected on the basis of best performance. The fig 1 shows the convergence of results of maximum central deflection with fineness of mesh.

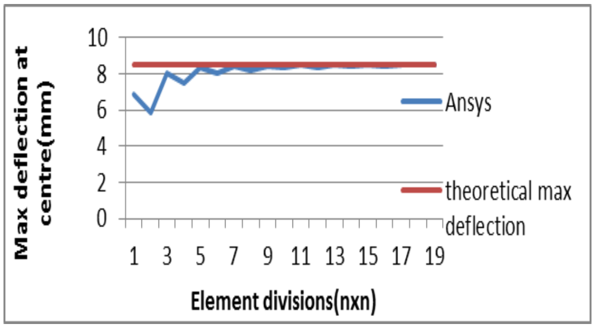

Figure 1. Graph showing the converging value of maximum central deflection with the increase in the fineness of mesh

B. Concrete Slab

FEM (ANSYS software) has been used in simply supported concrete slab with following dimensional parameters. Side of the square steel plate; a = 3m, thickness of the steel plate; h = 154mm, Poisson's ratio; $\nu = 0.17$, grade of concrete, f_{ck} = M25, uniformly distributed load; $q = 5000\ N/m^2$. The maximum out plane deflection by classical theory in above mentioned plate is 0.21 mm. Validation check for selecting the proper element has been performed and SOLID65 has been selected on the basis of best performance. The fig 2 shows the convergence of results of maximum central deflection with fineness of mesh

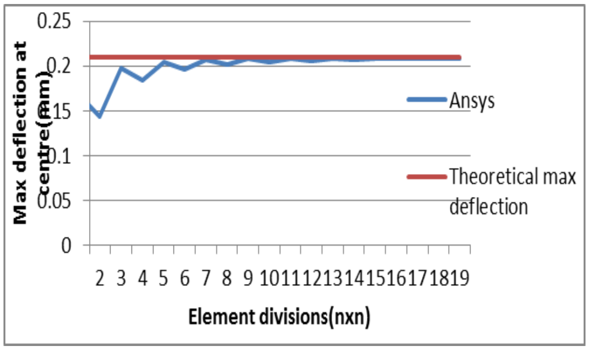

Figure 2 Graph showing the converging value of maximum central deflection with the increase in the fineness of mesh

FEM (ANSYS software) has been used in concrete slab with different edge conditions and following dimensional parameters. Side of the square steel plate; a = 3m, thickness of the steel plate; h = 154mm, Poisson's ratio; $\nu = 0.17$, grade of concrete, f_{ck} = M25, uniformly distributed load; $q = 25000\ N/m^2$. The maximum out plane deflection by classical theory for different aspect ratio are tabulated below in table 1 and table 2. Validation check for selecting the proper element has been performed and SOLID65 and SHELL63 has been selected on the basis of best performance.

TABLE 1 Maximum deflection at centre with all sides simply supported

ASPECT RATIO	α	δ_{max}(mm)
1.5	0.00772	2.159
2	0.01013	2.833

TABLE 2 Maximum deflection at centre with all sides fixed/clamped

ASPECT RATIO	α	δ_{max}(mm)
1.5	0.00220	0.615
2	0.00254	0.710

IV Results and Discussions

Prediction of ultimate collapse load through the study of Load v/s Deflection graph and cracking pattern observed at sequential loading step.

Case 1: All sides simply supported (Aspect Ratio: 2)

A: Load Step 12

B: Load Step 13

A. Cracking pattern observed at Load Step 12

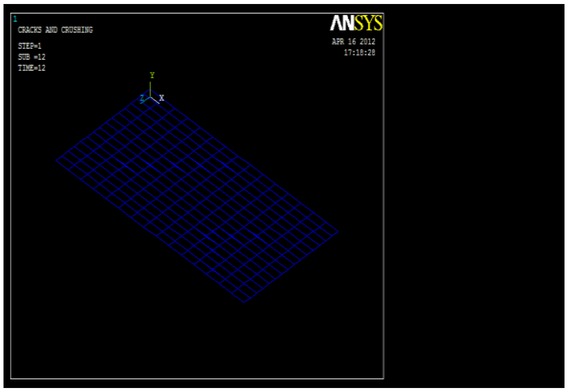

Figure 3. Point A: Cracking pattern @Load Step 12 (Top Surface)

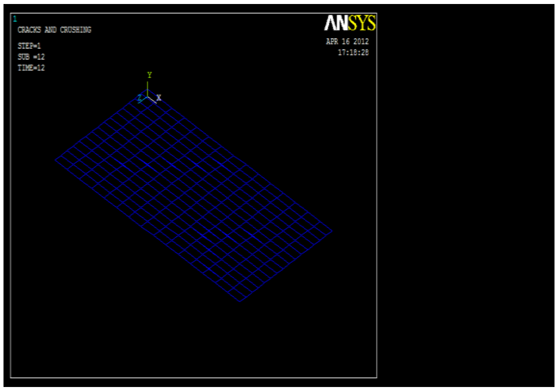

Figure.4 Point A: Cracking pattern @Load Step 12 (Bottom Surface)

B. Cracking pattern observed at Load Step 13

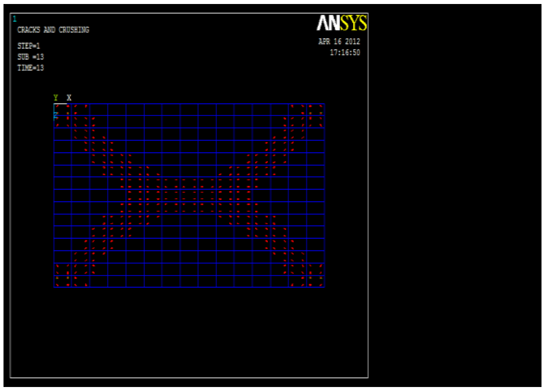

Figure.5. Point B: Cracking pattern @Load Step 13 (Top Surface)

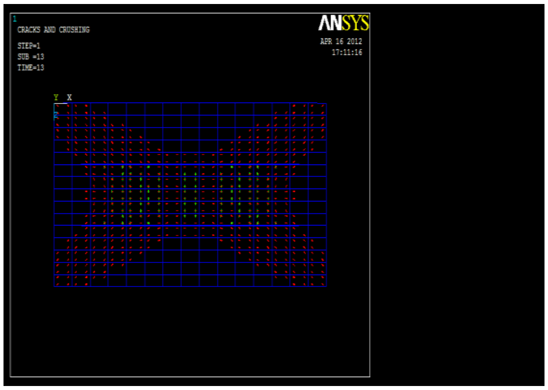

Figure.6. Point B: Cracking pattern @Load Step 13 (Bottom Surface)

At time step 12 the total uniformly distributed load acting on the slab is 12 kN/m^2. Until this loading, slab behaves elastically. The deformation is small and upto this point "A" the Hooke's law is valid.
The slab reaches its ultimate collapse load in between 12-13 kN/m^2 and the transverse deflection suddenly increases 3-4 times as the load step increases from 12 to 13. The collapse of slab can also be confirmed by the study of cracking pattern which has been generated in the highly stressed elements just as the ultimate load has been reached.
The figures (3,4,5,6) are enlisted above showing the difference in the top and bottom surface at load step 12 and load step 13. It is conspicuous in the figure that a complete fracture has occurred at load step 13 as the cracks (explicitly representing the yield lines) have reached to the boundaries of the slab.

Case 2: All sides Fixed (Aspect Ratio: 2)

A. Load Step 15
B: Load Step 16
C: Load Step 26
D: Load Step 27

A. Cracking pattern observed at Load Step 15

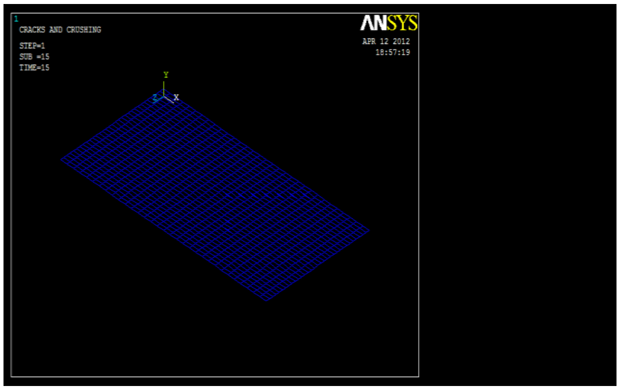

Figure 7. Point A: Cracking pattern @Load Step 15 (Bottom Surface)
Cracking pattern observed at Load Step 16

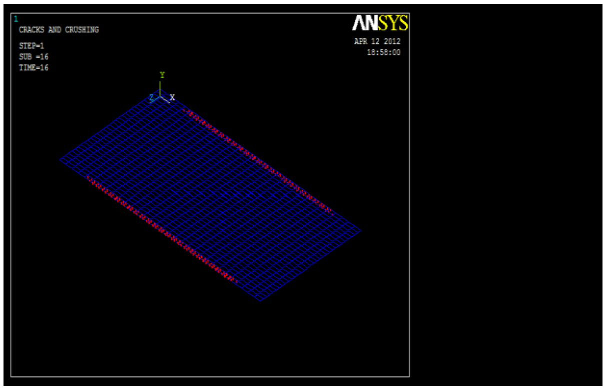

Figure.8. Point B: Cracking pattern @Load Step 16 (Top Surface)

B. Cracking pattern observed at Load Step 16

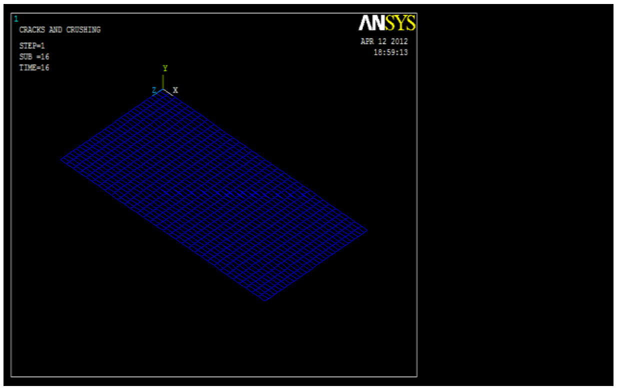

Figure 9. Point B: Cracking pattern @Load Step 16 (Bottom Surface)

C. Cracking pattern observed at Load Step 26

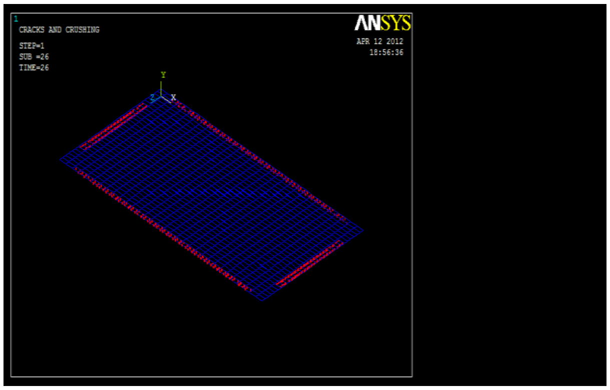

Figure10.Point C: Cracking pattern @Load Step 26 (Top Surface)

D.Cracking pattern observed at Load Step 27

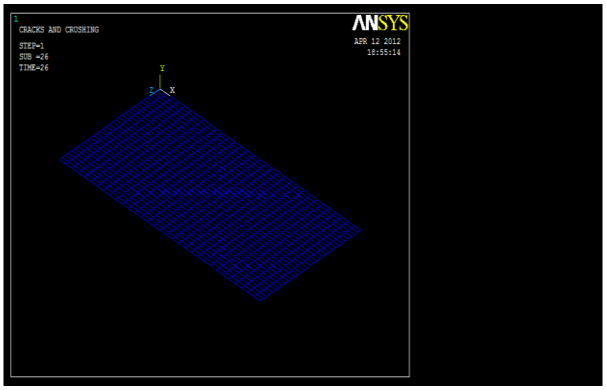

Figure 11. Point C: Cracking pattern @Load Step 26 (Bottom Surface)

Cracking pattern observed at Load Step 27

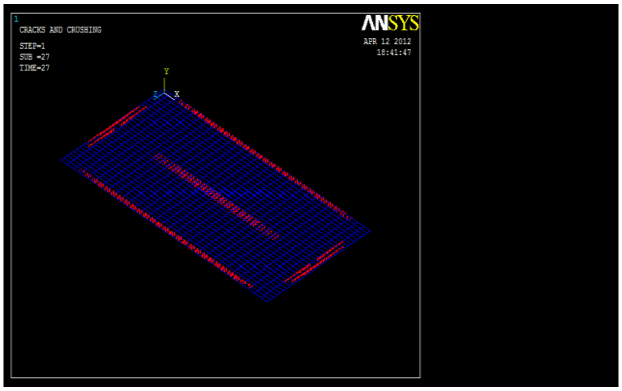

Figure 12. Point D: Cracking pattern @Load Step 27 (Top Surface)

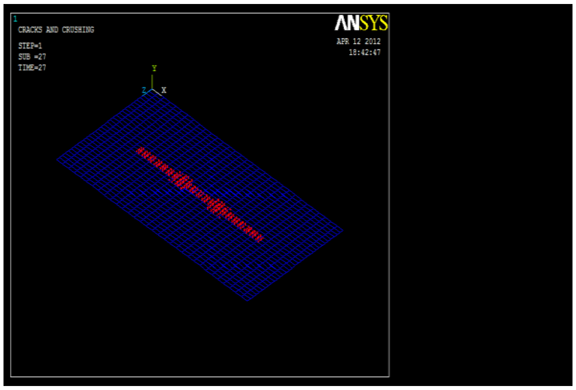

Figure 13.Point D: Cracking pattern @Load Step 27 (Bottom Surface)

The sequential failure patterns in the case of all sides fixed slab are shown in the figures (7, 8, 9, 10, 11, 12 and 13). The description of failure patterns are same as simply supported slab except failure patterns occur at edges as well as at centre at larger load values.

V Conclusions

1. The equations derived by Timoshenko are valid only till the elastic limit is reached. Hence, for large deflection problems where the structure behaves inelastically the conventional equations cannot be used accurately. This limits the use of conventional equations in the yield line theory where large deformation is involved. Therefore, the comparison done between the maximum deflections obtained through ANSYS and the conventional approach is restricted to the loading conditions within which the slab behaves elastically.
2. The results obtained for yield line pattern are in accordance with the past prediction done by the classical approach. The propagation of cracks in the slab confirmed the sequence of yield line formation as the load was increased towards the ultimate collapse load.
3. Due to limitation in the element characteristics, SHELL-63 is restricted to the calculation of bending moments only and SOLID-65 element is used to confirm the cracking pattern for different slabs.
4. With the refinement of mesh improved results were observed but displacements were generally the most accurate response quantity computed and converged faster than stresses.
5. The limited computer memory RAM obstructs the refinement of mesh size.
6. Towards the boundary edges the displacement derivative functions like stresses and moments did not converge with the same rate as at the centre attributing to the steep stress gradient towards the edge region.

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This paper presents application of FEM in finding the failure pattern of steel plate and concrete slabs under the sequential increasing loading. This study is having practical application in knowing the behavior of steel plate and concrete slabs under the static loading which leads to design aid.