

# Structural Strength Investigation for Concrete Shield Tunnel Linings in Construction

## Material Strength vs. Structural Stability

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**Abstract**—Shield tunnels are widely used in the urban infrastructure facilities, and they are commonly installed through connecting the concrete or steel segments with the longitudinal and circumferential joints. In this paper, the constructing shield tunnel linings mainly subjected to the external pressure are selected to investigate its strength, as well as the effects of concrete material and multi-longitudinal joints. The findings of this study clearly indicates that, the tunnel linings fail in the structure instability rather than the material failure, and thus its structural strength is determined by the buckling strength rather than the material strength. However, the concrete elastic modulus has great impact on the structural strength by changing the structural stability. The joints affect the stability of a tunnel lining significantly; generally, the critical pressure decreases with the decreasing of the flexural rigidity and the increasing of the number of radial joints. Additionally, the arrangement of the joint location can also affect the structural strength.

**Keywords**—Shield tunnel, concrete segment, multi-joints, structural strength, structural stability, material strength, buckling analysis

## I. Introduction

Shield tunnels are widely used in civil engineering as the vital infrastructures such as water and waste pipelines, transportation tunnels, and so on. The applied materials are usually limited in plain concrete, reinforced concrete or ductile steel according to the functional requirement of utilities. As the construction procedure, the shield tunnel linings are assembled with longitudinal (radial) joints and circumferential (ring) joints. Accordingly, the strength evaluation of shield tunnel lining during construction must consider the effects of multi-joints and applied material properties.

Comprehensively, structural strength (load-carrying capacity) depends on both the material property and the geometrics and structural form, and varies with the support

and load conditions. As for a reinforced concrete structure, the material failure is generally considered as the vital criteria governed by material strength. However, for some structures composited with special member configuration and subjected to the sympathetic loads, the elastic instability may govern the critical state. The collapse of the Heathrow Express tunnel (NATM) in October 1994 [1], Gerrards Cross Tunnel (Tesco tunnel, three pin arch) collapse in June 2005 [2], and Kurashiki undersea tunnel (shield tunnel) collapse in February 2012 [3] are some typical failure examples of the concrete tunnel in recent years. Although the failure causes are rather complicated, these tunnel linings lost their stability and collapsed eventually [4, 5, 6]. Particularly, structural joints weakening the stability may explain the failure mechanism of Tesco and Kurashiki tunnels, considering the failure mode at joints. Actually, empirical research on the buckling of cylinders with a longitudinal joint demonstrated that joint could greatly reduce the buckling capacity [7]. The study results also indicated that the transversely jointed cylindrical structure is more vulnerable to lose its stability rather than a common material failure.

In this paper, shield tunnel linings under construction shown in Fig. 1 is targeted, and a numerical study is conducted using the nonlinear finite-element (FE) analysis to account for the effects of concrete material and multi-joints. The shield tunnel linings are modeled as a cylindrical shell with multi-longitudinal joints, and considering the tension-softening and nonlinear mode of concrete. The failure behavior is examined, as well as the impacts of concrete material and joints on its structural strength.

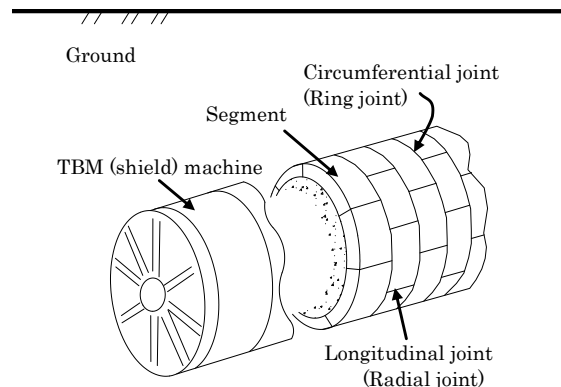


Figure 1. Schematic profile of shielded tunnel under construction

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## II. Concrete Shield tunnel Linings and Related Subjects

### A. Material Failure and Instability

Structural failure varies from both material strength and dimension, and is affected by the support and load conditions. Material strength is usually to decide the load-carrying capacity, but in some cases, the structure with special geometry and structural form may fail in an elastic buckling, e.g. a cylindrical shell subjected to external pressure. Material properties can affect the structure's stability significantly; the higher is the ratio of material strength to elastic modulus, the more vulnerable the instability occurs than material failure. Additionally, the applied loads and supports are usually the key to decide, which will govern the structural strength between material strength and structural instability. Generally, the strength of cylindrical shells is determined by comparison between the material failure and the instability. Many experiments were performed to simulate the tunnel under earth loads, and clarified that structural strength was commonly determined by the material strength. On the other hand, the construction of a deep tunnel or submarine tunnel must consider the dominant groundwater and grouting pressure, and thus the buckling of cylindrical shells subjected to external pressure should be checked to ensure the structural safety.

### B. Buckling of Cylindrical Shells

When a cylindrical shell is subjected to external pressure, the theoretical equation to predict the buckling load can be derived using Timoshenko potential energy method and Ritz-Galerkin method [8, 9]. Based on this method, the equation of buckling critical pressure [10] is obtained for a finite long cylindrical shell with the dimensions of radius  $R$ , thickness  $t$ , and length  $L$  and the material properties of elastic (Young's) modulus  $E$ , Poisson's ratio  $\nu$ ,

$$P_{cr} = \frac{Et}{(n^2 - 1)R} \left[ K(\alpha^2 + n^2 - 1)^2 + \frac{\alpha^4}{(\alpha^2 + n^2)^2} \right] \quad (1)$$

$$\text{where, } \alpha = \frac{\pi R}{L}, K = \frac{t^2}{12(1-\nu^2)R^2}, n = \sqrt[8]{\frac{3\alpha^4}{K}}$$

For an infinite long cylindrical shell,  $\alpha = 0$ , the buckling equation can be simplified by assuming buckling waves  $n = 2$ ,

$$P_{cr} = \frac{Et^3}{4(1-\nu^2)R^3} \quad (2)$$

However, the buckling of cylindrical shells with multi-longitudinal joints is rarely studied in theory, although the model experiments [7] clarified the buckling of a cylindrical

shell with a longitudinal joint is rather different from one without joint. The longitudinal joint reduces the buckling strength, and the cylindrical shells with flexible joint is more vulnerable to lose its stability. The total potential energy method is easily considered to apply by just adding the spring potential energy, but the theoretical equation is difficult to obtain because the buckled deformation expression changes with the number, configuration, and stiffness of joints. Therefore, only the nonlinear FE analysis method is available at present. Additionally, considering the joint modeling is the key to simulate the cylindrical shells with joints, the rotation spring method verified by a simulation of experiment [7] is applied. The flexural stiffness ( $k_\theta$ ) is estimated using Eq. (3), where the effective flexural rigidity factor ( $\eta$ ) commonly used in tunnel engineering is introduced,  $I$  and  $E$  is the area moment of inertia and Young's modulus of segment, respectively.

$$k_\theta = \frac{\log 0.5 EI}{\log \eta R} \quad (3)$$

### C. Concrete Material - nonlinear Properties

Concrete is a mixed material, including cement, aggregate, water, sand and other admixtures, and its properties vary from the proportions of the main ingredients. Generally, concrete has relatively high compressive strength, but a significantly lower tensile strength. Otherwise, concrete expresses a relative constant elastic performance before reaching tensile strength or compressive strength, but the nonlinear performance of tension-softening, work hardening and crushing beyond the strength. In the numerical analysis of concrete structure, the material must be simulated to account for the abovementioned nonlinear behavior. The common stress - strain diagram shown in **Fig. 2** is applied, where the tension-softening modulus  $E_s$  is calculated by the following equation, based on the fracture energy [11].

$$E_s = \frac{h_c E f_t^2}{h_c f_t^2 - 2EG_F} \approx -h_c \frac{f_t^2}{2G_F}$$

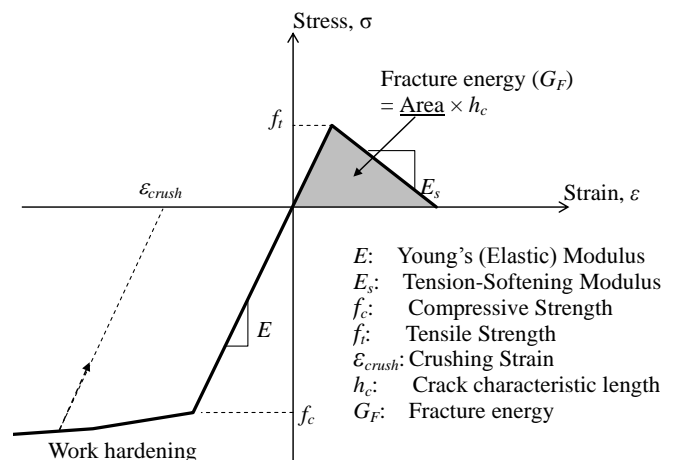


Figure 2. Stress-strain diagram for the concrete material

### III. Target tunnel and finite element analysis approach

#### A. Target Shield Tunnel

A shield tunnel with the outer diameter (O.D.) of 4.95 m is selected as shown in Fig. 3(a). The tunnel linings is assembled by five pieces of precast reinforced concrete segments with a K type (central angle  $\alpha_K = 27.7^\circ$ ), two B type (central angle  $\alpha_B = 83.1^\circ$ ) and two A type (central angle  $\alpha_A = 83.1^\circ$ ), and the radial joints are used to connect two adjacent segments. The dimensions of segment are of thickness  $t = 160$  mm and width  $B = 1200$  mm. The loading condition of a constructing tunnel is targeted as shown in Fig. 3(b), and only the groundwater pressure or grouting pressure is considered in addition to self-weight loading in transverse direction.

The effects of concrete material properties on the structural strength of tunnel linings are investigated in terms of the

concrete strength and the general concrete material in JSCE [12]; the concrete materials (see Table I) are assumed with the same weight density of  $26.0 \text{ kN/m}^3$  and Poisson's ratio of 0.17. Meanwhile, the radial joints are discussed with variables of flexural rigidity. Otherwise, each joint is investigated individually to clarify its effect on the buckling strength.

#### B. FE Modeling and Analysis Approach

Commercial soft package MSC Marc [13] was employed in this study. Numerical modeling was performed using a four nodal thick-shell element to simulate the segment, and the size of meshes was ensured being sufficiently small in order to achieve better convergence. Radial joint was modeled using rotation springs and nodal ties. The finite-element analysis model and details of the joint modeling are described in Fig. 4. The stiffness ( $k_\theta$ ) of the rotation spring was estimated by Eq. (3) for the effective flexural rigidity factor ( $\eta$ ) within the range from 0 to 1. However, the steel reinforcements were ignored herein, considering the facts that the linings was designed

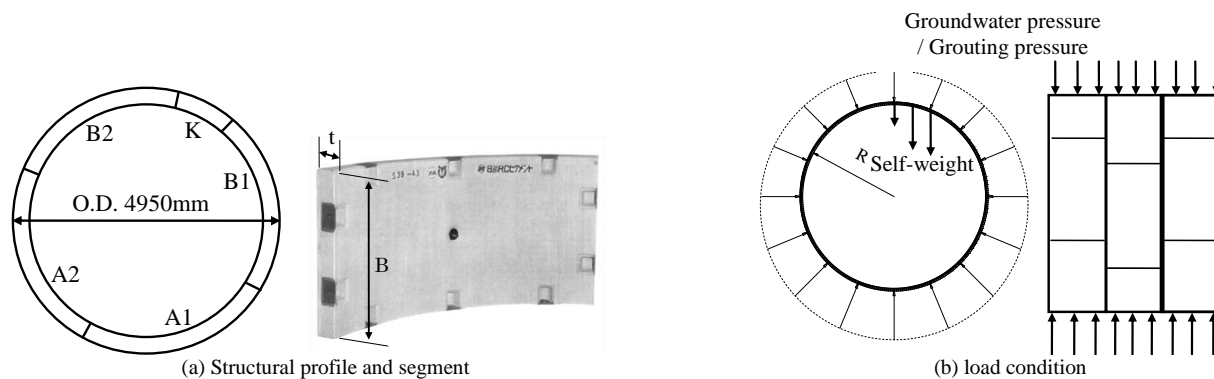


Figure 3. Structural and loading conditions

TABLE I. MATERIAL PROPERTIES OF CONCRETE

Constant \ Material	Elastic	Variable concrete strength			Common concrete material [12]			Others
		※fc42 (E33)	fc51	fc60	E28fc30	E31fc42	E35fc60	
Young's modulus $E_c$ (kN/mm <sup>2</sup> )	33.0	33.0			28.0	31.4	35.0	
Compressive strength $f_c$ (N/mm <sup>2</sup> )	42	42	51	60	30	42	60	
Tensile strength $f_t$ (N/mm <sup>2</sup> )	-	2.78	3.16	3.53	2.22	2.78	3.53	$f_t = 0.23f_{ck}^{2/3}$ [12]
Fracture energy $G_F$ (N/m)	-	94.35	100.66	106.27	84.34	94.35	106.27	$G_F = 10(d_{max} f_{ck})^{1/3}$ [12]
Tension-softening modulus $E_s$ (kN/mm <sup>2</sup> )	-	12.28	14.91	17.54	8.77	12.28	17.54	Eq. (4) $h_c = 300\text{mm}$ ( mesh size)
Crushing strain $\epsilon_{cu}$	0.03							

※is the basic case.

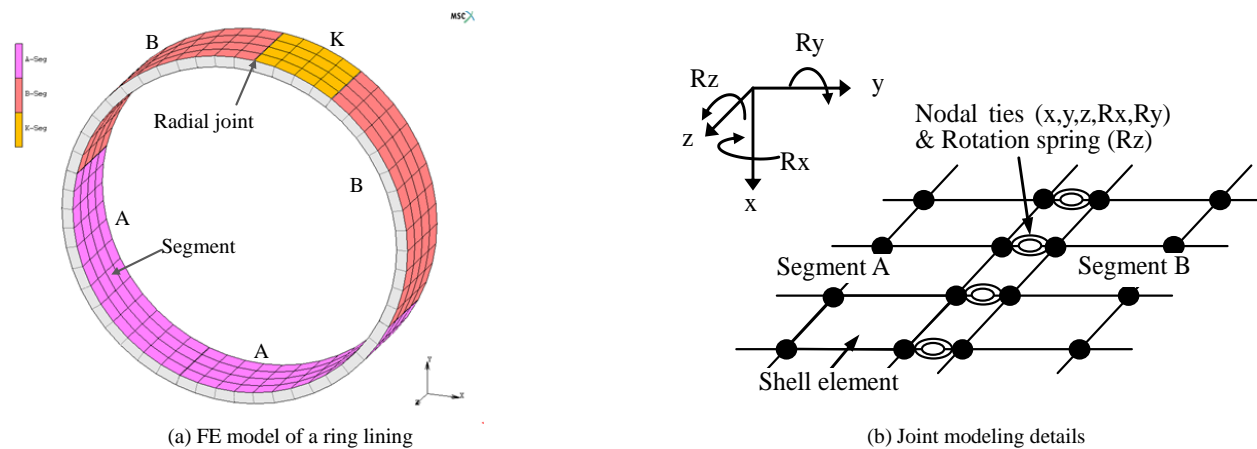


Figure 4. Finite element model and joint modeling details

using the allowable stress method, and the effect of reinforcements on the structural failure is rather small. The concrete material model shown in Fig. 2 was used to simulate the nonlinear behavior of concrete, in which Buyukozturk concrete model [14] is used to predict crack initiation and simulate the tension softening, plastic yielding and crushing. Crack data of critical cracking stress, tension-softening modulus and crushing strain are inputted, as well as the shear retention of 0.2 after cracks developed and fracture energy.

The full Newton-Raphson method was adopted for the solution of the stiffness formulation. An adapted stepping procedure was applied to adjust the step time in the increment automatically, accounting for the both nonlinear material and geometry.

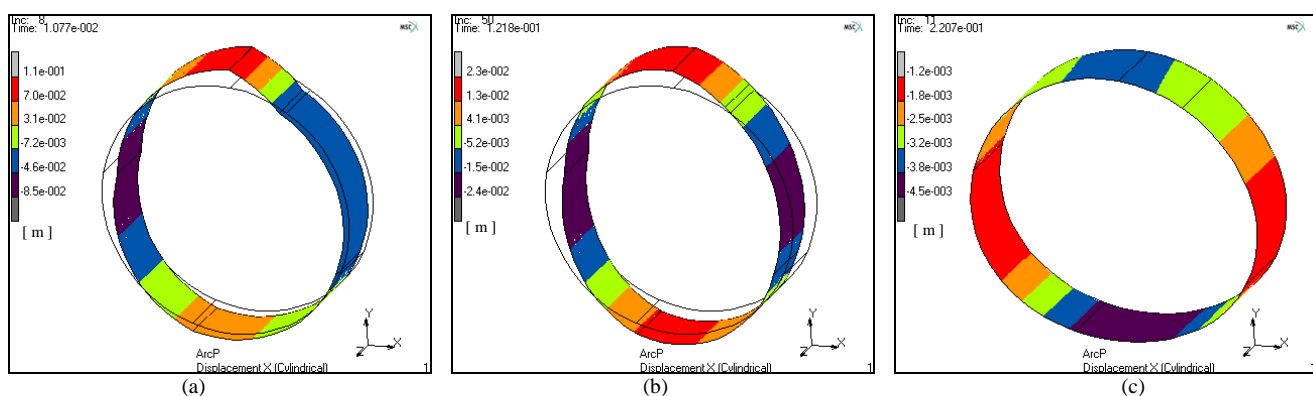
## IV. Results and Discussions

The nonlinear FE analysis was performed, and the critical pressure was obtained using the maximum load. The results of the numerical analyses were discussed with respect to the failure mode, the critical pressure and the cracking and stress condition.

### A. Failure behaviors and Validation

The basic case of the concrete with elastic modulus  $E = 33.0 \text{ kN/mm}^2$  and compressive strength  $f_c = 42 \text{ N/mm}^2$  was investigated firstly, taking into consideration of its nonlinear concrete property. Fig. 5 illustrates the failure mode in the form of radial deformation for three kinds of the effective flexural rigidity factors  $\eta = 0, 0.5$  and  $1.0$ . The critical pressure changing with the flexural rigidity of joint are presented in Fig. 6, also the theoretical critical pressure is plotted to examine the numerical analysis.

Fig. 5 indicates the failure mode changes with the increasing of the flexural rigidity of the joints from a local buckling shape to a two-wave buckling one. Meanwhile, it can be found from Fig. 6, that the critical pressure decreases with the joints becoming flexible. The failure mode presents the reason why the flexible radial joints reduces the structural strength; the local buckling in the vicinity of K segment caused the failure of tunnel lining, and eventually degraded its structural strength. The numerical critical pressure agrees with the theoretical result very well when joints is rigid enough (i.e.  $\eta = 1$ ), as well as failure mode with buckling waves  $n = 2$ . This may validate the present nonlinear FE analysis.



Notice: The negative value means inward deformation.

Figure 5. Failure mode in radial displacement [(a) hinge joint ( $\eta = 0.5$ ); (b) flexible joint ( $\eta = 0.5$ ) and (c) rigid joint ( $\eta = 1.0$ )]

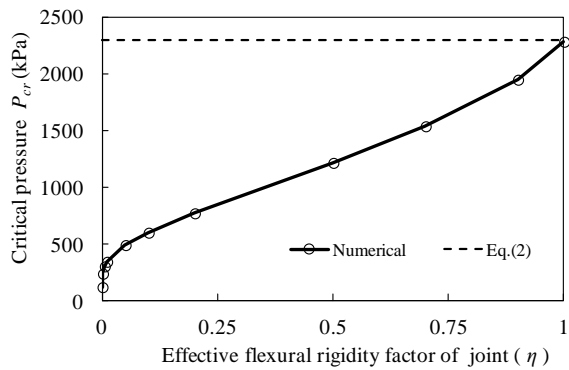


Figure 6. Critical pressure to flexural rigidity of joints  
(Basic case:  $f_c = 42 \text{ N/mm}^2$ ,  $E = 33.0 \text{ kN/mm}^2$ )

### B. Effects of Concrete Material

The strength of tunnel linings is examined accounting for the concrete material properties. The typical propagation of the concrete cracks in basic case is shown in **Fig. 7(a-c)**, by displaying the scalar resultants of the equivalent cracking strain, as well as the change of cracking condition with the flexural rigidity of the joints. Also, the circumferential (hoop) stress of the hinge-jointed tunnel linings under the critical pressure are presented in **Fig. 7(d)** as the elastic analysis result. Additionally, **Fig. 8** and **Table II** present the relation between the critical pressure and the flexural rigidity of joints with the variable material strength of concrete. **Fig. 9** shows the structural strength changes with three kinds of the general concrete having the different compressive strength and elastic modulus.

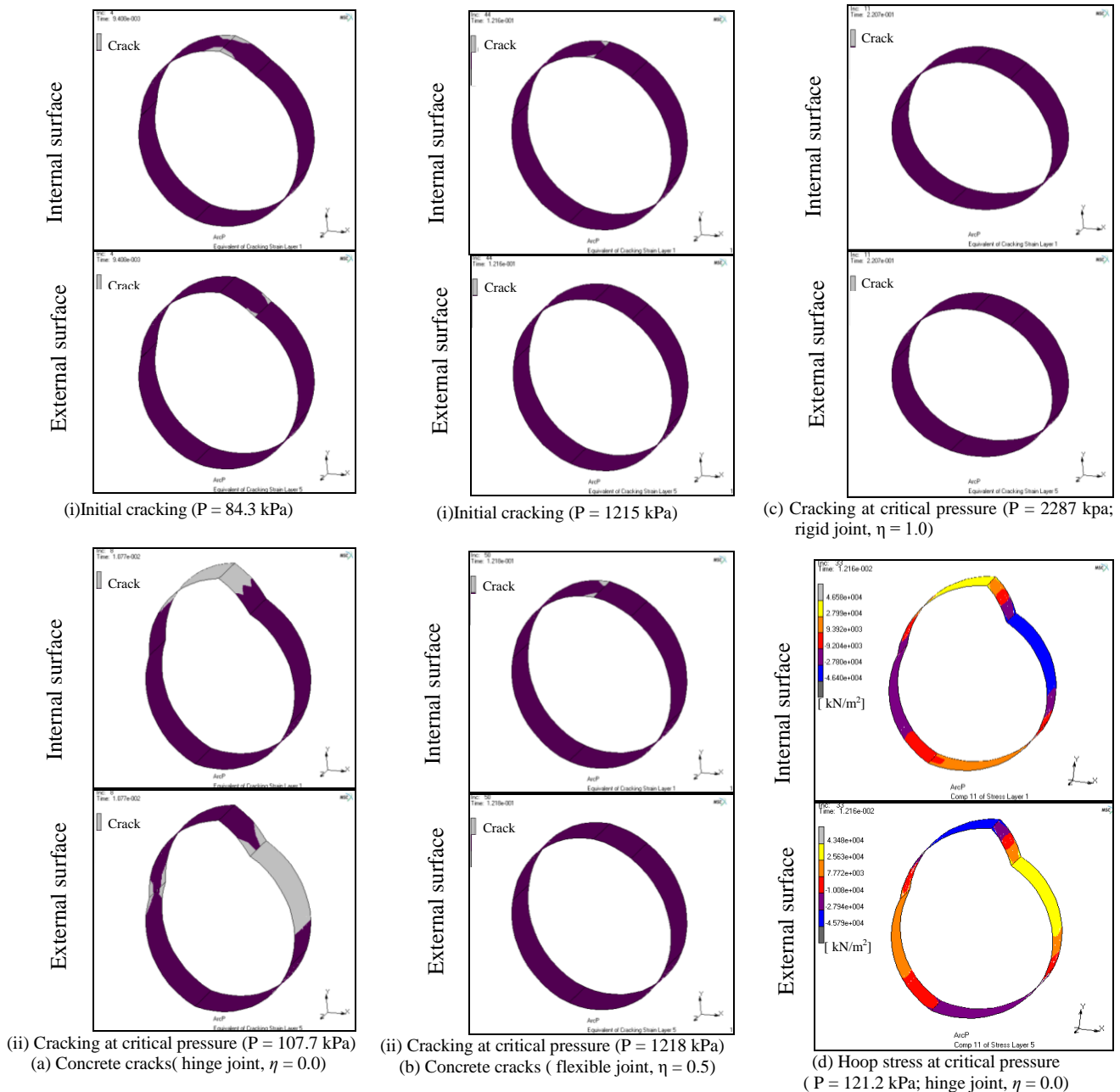


Figure 7. Concrete cracking conditions (basic case) and hoop stress (elastic analysis)

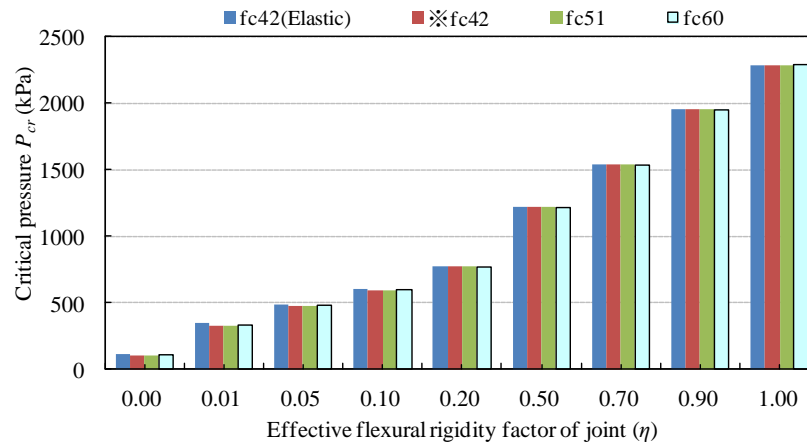
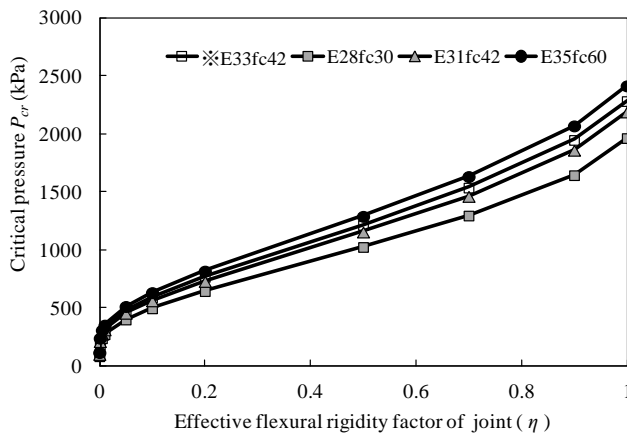


Figure 8. Comparison of critical pressure of different concrete strength with variable flexural rigidity of joints ( Elastic modulus E = 33.0 kN/mm<sup>2</sup>)

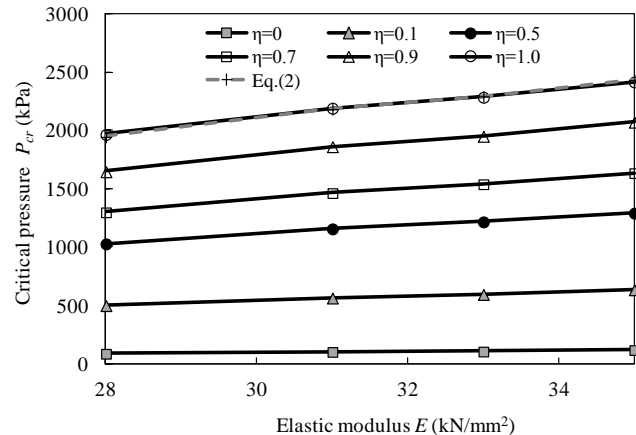
TABLE II. COMPARISON OF CRITICAL PRESSURES

Unit: kPa

η	0.00	0.01	0.05	0.10	0.20	0.50	0.70	0.90	1.00
fc42 (Elastic)	121.2	345.4	492.5	602.0	772.9	1218.0	1541.0	1952.0	2287.0
*fc42	105.7	331.8	482.0	595.5	771.6	1218.0	1541.0	1952.0	2287.0
fc51	108.3	332.6	482.8	596.2	772.0	1218.0	1541.0	1953.0	2287.0
fc60	108.3	332.2	483.5	596.8	772.3	1218.0	1541.0	1953.0	2287.0



(a) Flexural rigidity factor



(b) Elastic modulus

Figure 9. Comparison of the critical pressure of different concrete material with variable flexural rigidity of joints

Fig. 7(a) illustrates that the cracks initially happened at the internal surface in the vicinity of the K segment’s joint near to crown, at the external surface of K segment near to springline, and propagated widely along K segment and the neighboring B segments, for the case of with hinge joints. However, by comparing with Fig. 7(a), (b) and (c), it is easy to find that the scale of cracks decreases with the increasing of joint’s flexural rigidity; and even no cracking happened when the effective flexural rigidity factor  $\eta = 1.0$ . The shape of failure mode (Fig. 5) may give a good interpretation, e.g. the overall tunnel lining

with rigid joints only deformed inward. Generally, the larger is the bending deflection, the greater the scale of cracks occurs. On the other hand, it is known from Fig.7(d) that the elastic analysis results of the compressive and tensile hoop stresses at the critical pressure exceeded the compressive strength of concrete ( $f_c = 42 \text{ N/mm}^2$ ).

Fig. 8 and Table II illustrate that a little degradation of structural strength due to the concrete nonlinear properties can be identified by comparing the elastic critical pressure with the nonlinear results, in the case of with the flexible radial joints.

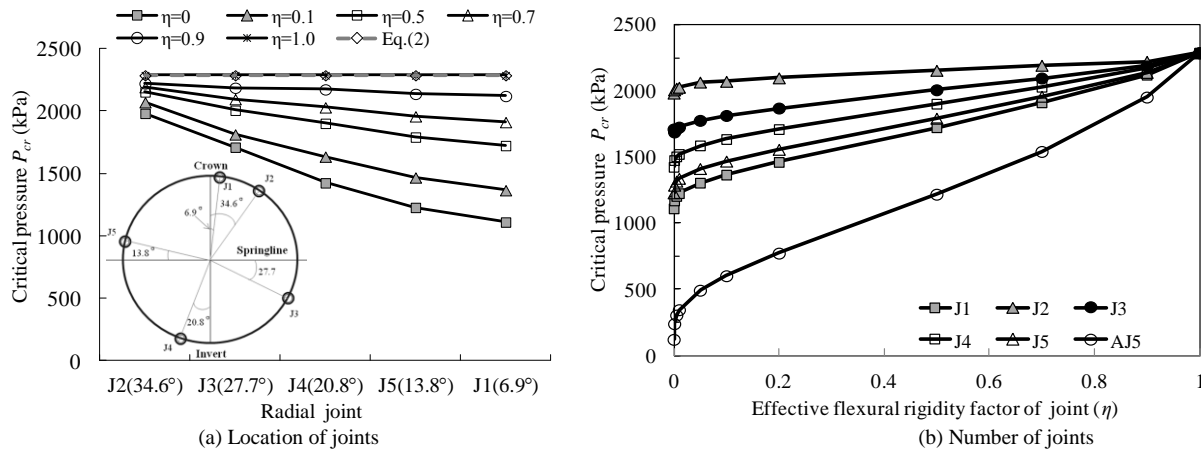


Figure 10. Effects of individual joint on critical pressure with variable flexural rigidity (Basic case: elastic modulus  $E = 33.0 \text{ kN/mm}^2$ ,  $f_c = 42 \text{ N/mm}^2$ )

However, the difference becomes small with the increasing of the flexural rigidity of joints, and the same critical pressures was obtained when the effective flexural rigidity factor  $\eta$  is larger than 0.5. It may imply that the material properties of concrete such as the tension-softening and the compressive strength have no impacts on the critical pressure for rigid jointed tunnel linings under the external pressure. The reason can be considered that the cracking does not happen in the relative rigid jointed cylindrical shells. In addition, it is easy to know that the material strength almost has no influence on the structural strength by comparing the critical pressures of material  $f_{c42}$ ,  $f_{c51}$  and  $f_{c60}$ . Based on this finding, that the critical pressure increases linearly as the elastic modulus became large can be known, by observing Fig. 9. Also, the elastic modulus of concrete affects the critical pressure more greatly as the joints become rigid.

Since the failure mode and the critical pressure of the elastic analysis are almost in agreement with those of the basic case, the material failure has little influence on the strength of overall lining. The elastic modulus of concrete affects the structural strength. This is also in coincidence of the buckling of cylindrical shells under external pressure. Therefore, it can be concluded that the failure of the tunnel linings under external pressure is due to the structural instability rather than the material failure. And the structural strength is determined by the buckling strength rather than the material strength, for a tunnel lining with longitudinal joints under external pressure.

### C. Effects of Joints

Fig. 10 shows the effects of joints on the structural strength of tunnel lining with respect to the flexural rigidity, the location and the number of joints. Fig. 10(a) illustrates that the location of joint has great influence on the critical pressure; the critical pressure is reduced the largest by the joint of J1 closest to the crown, while the smallest by joint J2. The almost linear relation of the location and the critical pressure can be found easily, with respect to the central angle to the nearest crown, inverts and springline. However, the largest reduction of the critical pressure occurs when all joints are assembled, as the illustration in Fig. 10(b).

Generally, the effects of the joint on the critical pressure changes with the variable of the flexural rigidity and the number of joints around the ring; the stiffer and the fewer are the joints, the higher the structural strength is for a shield tunnel lining. Otherwise, the arrangement of assembling joint far from the invert, crown and the springline can increase the strength of lining. All the findings may present an engineering significance to prevent the jointed cylindrical shells subjected to the external pressure from buckling.

## V. Summaries and Conclusions

The strength of the shield tunnel linings subjected to the external pressure was investigated as the cylindrical shells with multiple longitudinal (radial) joints. Structural feature of jointed cylindrical shells and concrete material properties had been taken into account, and their effects on structural strength were examined. Good agreement between results of FE analysis and buckling theory was identified for the cylindrical shells with the joints having the effective flexural rigidity factor  $\eta = 1.0$ . From the discussion above, the conclusion can be drawn as follows:

- The shield tunnel linings subjected to the external pressure failed in the structure instability rather than the material failure. The failure mode changes from locally deformed shape to overall deformed one as the joints become rigid, as well as the critical pressure increases.
- The concrete strength has little influence on the structural strength of the shield tunnel lining, even can be ignored completely for one with completely rigid joints. However, the elastic modulus can linearly raise the structural strength.
- The structural characters of having joints have great influence on the shield tunnel lining subjected to the external pressure. The more flexible in the flexural rigidity, the nearer to invert, crown and springline, and the more in the number are the joints, the more vulnerable to suffer the failure of structural instability.

Rational arrangement of joint's location and reducing the number can improve structural stability, ultimately increase the strength of tunnel lining.

However, we must realize that the effects of the structural feature and material properties on structural strength are principally controlled by target loads. For a shield tunnel mainly subjected to the earth load, the material strength is possible to determine the structural strength.

## **References**

- [1] HSE, "Rep. the collapse of NATM tunnel at Heathrow Airport," Health and Safety Executive, London, 2000.
- [2] Wikipedia: Gerrards Cross tunnel collapse, [http://en.wikipedia.org/wiki/Gerrards\\_Cross\\_Tunnel](http://en.wikipedia.org/wiki/Gerrards_Cross_Tunnel).
- [3] NBP, "Shield machine submerged due to segmental linings collapse," Nikkei Business Publications, Japan, 2012.
- [4] J. G. A. Croll, "Buckling of Cylindrical Tunnel Liners," J. Eng. Mech., 127, No.4, 2001, pp.333-341.
- [5] Railwaysachive.co.UK, [http://www.railwaysarchive.co.uk/documents/NCE\\_GerrardsCross2005.pdf](http://www.railwaysarchive.co.uk/documents/NCE_GerrardsCross2005.pdf).
- [6] J. Wang, W. Zhang and elt, "Mechanism for Buckling of Shield Tunnel Linings under Hydrostatic Pressure," Tunnelling and Underground Space Technology, 2014 (In prep.).
- [7] J. Wang and A. Koizumi, "Buckling of cylindrical shells with longitudinal joints under external pressure. Thin-Walled Structures," 48, No.12, 2010, pp.897-904.
- [8] S. Timoshenko and J. Gere, "Theory of Elastic Stability," 2<sup>nd</sup> ed., McGraw Hill, New York, 1961.
- [9] N. A. Joachim, and A. H. Schatz, "Interior estimates for Ritz-Galerkin methods," Mathematics of Computation, 28, No.128, 1974, pp.937-958.
- [10] J. Wang and A. Koizumi, "Theoretical study on buckling of deep water pipeline under external hydrostatic pressure," JSCE Journals, Division A, Vol.64, No.3, 2008, pp.588-602.
- [11] G. Pijaudier-Cabot, Z. P. Bažant, M. Tabbara, "Comparison of various models for strain-softening," Engineering Computations, Vol. 5, Iss: 2, 1988, pp.141-150.
- [12] JSCE, "Standard specifications for concrete structures-2012, Design, 2012 ed.," Japan society of civil engineers, Tokyo, 2012.
- [13] Marc Analysis Research Corp., "Msc.Marc 2005, Documentation and Manual," Marc Analysis Research Corp., 2005.
- [14] O. Buyukozturk, "Nonlinear analysis of reinforced concrete structures," Computers & Structures 7.1, 1977, pp.149-156.

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